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APPENDIX-I

NUMERICAL EXAMPLE ILLUSTRATING CALCULATION OF ULTIMATE STRENGTH OF HYPAR SHELLS

A.1. Data.

Plan area of shell, $2a \times 2a$	$= 20$ m x 20 m
Shell rise, c	$= 2.0$ m
Shell thickness, h	$= 8.0$ cm.
Ridge and edge beams, $b_r \times d$	$= 50$ cm x 50 cm
28 day cube strength of concrete.	$= 250$ kg/cm^2
Yield strength of steel	$= 2600$ kg/cm^2

Other details are given where they are used in the calculations.

A.2. Calculations.

A.2.1. Tie failure (mode 1)

(a) Calculation of ultimate capacity in theabsence of the tie.

Total moment capacity of the shell across one ridge line, assuming the full section to be effective (see Fig.A.1 for details) $= 16.9 \times 10^6$ kg-cm

Using equation 9.4.

$$q_u = \frac{M'_u}{a^3} = \frac{M_u}{a^3} = \frac{16.9 \times 10^6}{(1000)^3} = 0.0169 \text{ kg/cm}^2 = 169 \text{ kg/m}^2$$

(b) Ultimate strength of shell with mild steel tie.

Assume a tie area of 200 cm^2

$$H_u = A.\,\sigma_y = 200 \times 2600 = 5.2 \times 10^5 \text{ kg.}$$

using equation 9.5.

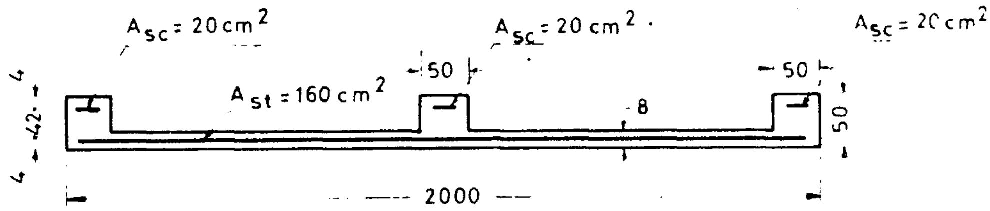


FIG A 1. RIDGE SECTION OF SHELL

(All dimensions in cm)

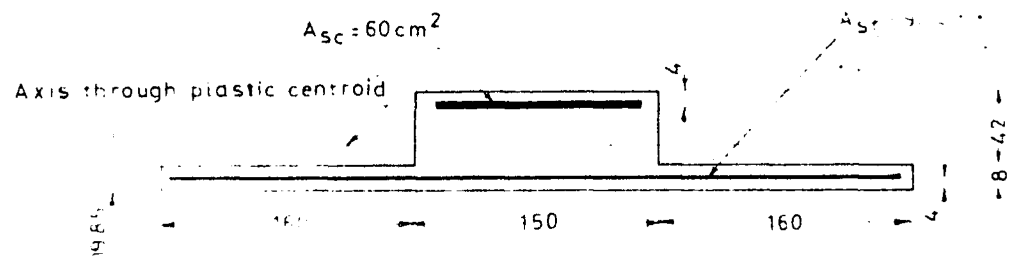


FIG A 2. EQUIVALENT EFFECTIVE SECTION AT RIDGE

(All dimensions in cm)

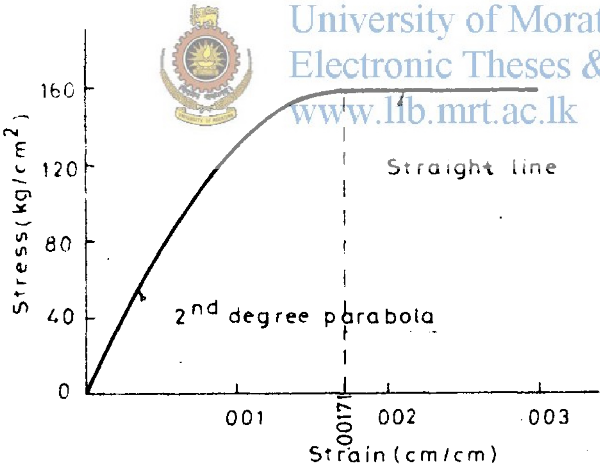


FIG. A.3. STRESS-STRAIN GRAPH FOR CONCRETE

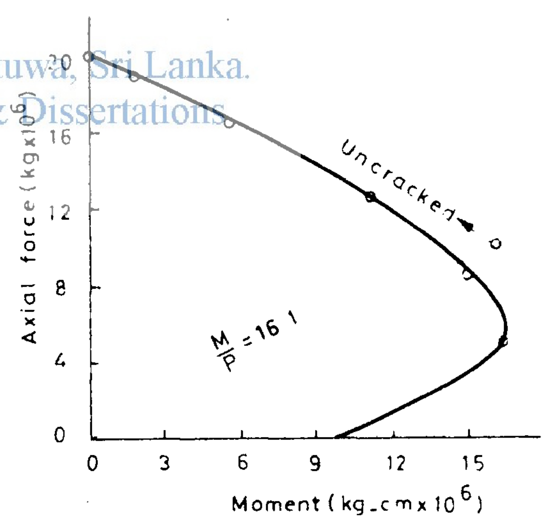
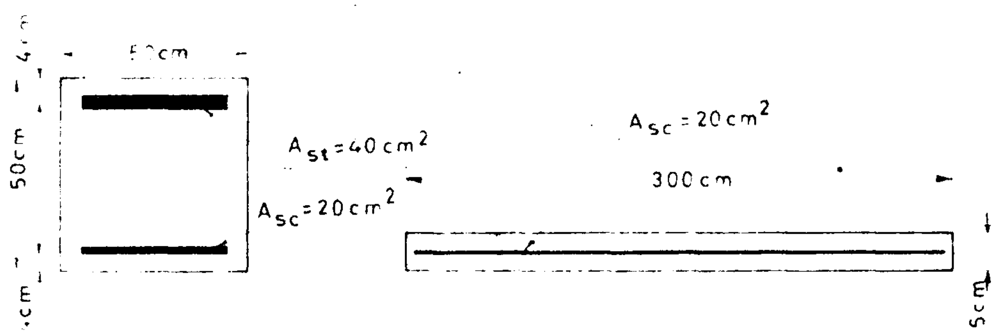


FIG. A.4-M-P INTERACTION DIAGRAM FOR RIDGE SECTION



(a) Beam section

(b) Panel section

FIG A.5. SECTIONS RESISTING MOMENT AT CORNER

$$q_u = \frac{2H_u}{a^2} \left(\frac{c}{a} \right) = \frac{2 \times 5.2 \times 10^5}{(1000)^2} (0.2) = 0.208 \text{ kg/cm}^2 =$$

$$= \underline{2080 \text{ kg/m}^2}.$$

A.2.2. Beam failure (mode 2)

(a) The effective section at ridge and moment-thrust interaction diagram.

The section effective in resitting the axial force and bending moment is shown hatched in Fig.(9.2).For calculation of the moment-thrust interaction diagram for this section, it can be reduced to an equivalent inverted 'T' section as shown in Fig.(A.2). The stress strain graph for concrete is idealised as shown in Fig.(A.3) (C.E.B. recommendation). The moment-thrust interaction diagram at the failure stage for the effective ridge section is shown in Fig.(A.4).

(b) Modulus of elasticity of concrete.

To avoid the trial and error procedure as suggested in section 9.3.2., the following simplifying assumptions, which will give ultimate strengths on the conservative side, are introduced for calculations of axial thrust and bending moment at the ridge section.

(i) Assume the "reduced modulus of elasticity" at the ridge section as the secant modulus corresponding to a strain of 0.0015. Change in "reduced modulus" due to the presence of axial force is neglected. Thus in the present example (see Fig.A.3),

$$E \text{ at the ridge section} = \frac{158}{0.0015}\ 1.05 \times 10^5 \text{ kg/cm}^2$$

(ii) The variation of modulus of elasticity along the length of the effective arch is assumed as linear, with the modulus of elasticity at the supports taken as equal to the initial tangent modulus

Initial tangent modulus from Fig.A.3. $= 2 \times \frac{160}{.00171} = 1.87 \times 10^5$ kg/cm^2

(c) Change in tie force due to known symmetrical horizontal displacement (δ_H).

The procedure of calculation is explained in section 3.3.4. First, the support displacement due to unit load applied at the support of the effective arch is determined. The moment of inertia, bending moment and modulus of elasticity applicable at different sections are tabulated in Table A.1. (see Fig. 3.8 for a typical cross section of the shell).

TABLE A.1. - PROPERTIES OF THE EFFECTIVE ARCH

x (cm)	(I of effective arch) $\times 10^{-4}$ (cm^4)	$E_{reduced}$ $\times 10^{-5}$ (kg/cm^2)	Moment 'm' due to unit horiz. load at support.	EI $\times 10^{-9}$	$\frac{m^2}{EI} \times 10^9$
50	398	1.09	195	434	87.6
150	550	1.17	185	644	53.0
250	980	1.25	175	1225	25.0
350	1550	1.34	165	2080	13.1
450	2380	1.42	155	3380	7.1
550	3420	1.50	145	5130	4.1
650	4700	1.58	135	7420	2.4
750	6200	1.67	125	10700	1.5
850	7950	1.75	115	13900	1.0
950	9900	1.83	105	18100	0.6
			$\sum \frac{m^2}{EI}$		195.4×10^{-9}

$$\left.\begin{array}{l}\text{Horizontal deflect... each support}\\ \text{of the effective ... to unit load}\end{array}\right] = \Sigma \frac{m^2 dx}{EI}$$

$$= dx \Sigma \frac{m^2}{EI} \ldots \times 195.4 \times 10^{-9} = 1.95 \times 10^{-5} \text{ cm.}$$

$$\left.\begin{array}{l}\text{Horizontal reaction ... for}\\ \text{a displacement of } \delta, \text{ ... each}\\ \text{support.}\end{array}\right] = \frac{10^5}{1.95} \times \delta_H = \underline{\underline{51300\,\delta}}$$

The equation for compatibility of deformation between the tie and the shell becomes

$$\frac{2a}{E_t \times 2.A}\left[\frac{qa^3}{c} - 51300\,\delta\right] = 2\,\delta_H$$

For steel tie, taking $E_t = 2.1 \times 10^6$ and solving,

$$\delta_H = \frac{5000q}{(51.3+4.2A)}$$

$$\left.\begin{array}{l}\text{Horizontal reaction } 2H \text{ for}\\ \text{fully rigid ties}\end{array}\right] = \frac{qa^3}{c} = \frac{q(1000)^3}{200} = \underline{5 \times 10^6 \; q.}$$

The axial thrust, bending moment and $\frac{M}{P}$ ratios at the ridge section for various tie areas are tabulated in Table A.2. Having obtained the $\frac{M}{P}$ ratios from Table A.2, the actual values of M_u and P_u for various tie areas can be obtained from the moment-thrust interaction diagram. The ultimate strength corresponding to each tie area can now be calculated as shown in Table A.3. From the details of calculation of moment-thrust interaction diagram, it is seen that for $\frac{M}{P}$ less than 16.1 the ridge section remains uncracked and hence the results from the analysis can be used for tie areas more than 150 cm^2. The last column of Table A.3. brings out the influence of the area of tie on the ultimate stength when the failure is initiated by the crushing of concrete in the beams.

TABLE A.2. - CALCULATION OF $\frac{M}{P}$ RATIOS.

Area of tie (cm^2)	δ_H	Stress in tie	Tie force $(2H) \times 10^{-6}$ (P)	Reduction in tie force $(2H^*) \times 10^{-6}$	$M \times 10^{-6}$	$\frac{M}{P}$
100	10.60q	22200q	4.44q	0.56q	112q	25.20
150	7.33q	15400q	4.62q	0.38q	76q	16.40
200	5.61q	11800q	4.72q	0.28q	56q	11.90
250	4.53q	9530q	4.77q	0.23q	47q	9.85
300	3.81q	8000q	4.80q	0.20q	40q	8.34
400	2.88q	6060q	4.85q	0.15q	30q	6.18
500	2.32q	4880q	4.88q	0.12q	24q	4.91
1000	1.17q	2470q	4.94q	0.06q	12q	2.43

TABLE A.3. - CALCULATION OF ULTIMATE LOAD CAPACITY.

Area of tie (cm^2)	$\frac{M}{P}$	$M_u \times 10^{-6}$ (kg-cm)	$P_u \times 10^{-6}$ (kg)	$M \times 10^{-6}$ (kg-cm)	q_u (kg/cm^2)
150	16.40	14.55	0.888	76q	0.191
200	11.90	12.86	1.079	56q	0.230
250	9.85	11.70	1.190	47q	0.249
300	8.34	10.70	1.275	40q	0.268
400	6.18	8.72	1.418	30q	0.291
500	4.91	7.40	1.510	24q	0.308
1000	2.43	4.18	1.730	12q	0.348
∞	0.00	0.00	2.000	0	0.400

A.2. **A.2.3. Shell panel failure (mode 3)**

From equation 9.15

$$\alpha_u = 0.7 \left(1200 \frac{8 \times 200}{(1000)^2} - 1\right) \left(\frac{2.5 \times 50 \times 50}{1000 \times 8}\right) = \underline{0.503}$$

Assuming 0.7% area of concrete as the area of steel in the shell panel, in two mutually perpendicular directions

$$C_c = 0.503 \times 200 \times 8 + \frac{0.7}{100} \times 8 \times 2600 = 952 \text{ kg/cm.}$$

$$C_t = \frac{0.7}{100} \times 8 \times 2600 \qquad = 146 \text{ kg/cm.}$$

From equation 9.10.

$$q_u = (952 + 146) \times \frac{200}{(1000)^2} \qquad = \underline{0.22 \text{ kg/cm}^2}$$

If the area of steel in the panel is assumed as 1.5%

$$C_c = 0.503 \times 200 \times 8 + \frac{1.5}{100} \times 8 \times 2600 = 1118 \text{ kg/cm}$$

$$C_t = \frac{1.5}{100} \times 8 \times 2600 \qquad = 312 \text{ kg/cm}$$

$$q_u = (1118 + 312) \frac{200}{(1000)^2} \qquad = \underline{0.286 \text{ kg/cm}^2}.$$

The curves governing the modes of failures obtained in the above calculations are plotted in Fig.9.4 as graphs showing variation of ultimate strength with area of tie.

A.2.4. Column head failure (mode 4)

The ties and supporting columns of the shell are assumed as infinitely rigid. Assume the size of column as 160 cm x 160 cm in plan. The yield lines are formed as shown in Fig.(9.3). The total moment capacity of the shell along this failure line can be approximately taken as the component of the moment capacities of the two beams normal to the yield line plus the moment capacity of the panel portion of the failure line. The sections of one beam and the portion of shell panel effective are shown in Fig.(A.5).

$$M_{uc} = \frac{2}{\sqrt{2}} (M_{u,beam}) + M_{u,panel}$$

$$= \frac{2}{\sqrt{2}} (43.7 \times 10^5) + 3.6 \times 10^5 = 65.4 \times 10^5 \text{ kg-cm.}$$

By equation 9.18

$$q_u = \frac{\sqrt{2} \times 65.4 \times 10^5 (200 + e)}{(1000)^3 \, e} = \frac{0.00924 \, (200 + e)}{e}$$

q_u for various values of 'e' are tabulated in Table A.4.

TABLE A.4. ULTIMATE STRENGTH FOR COLUMN-HEAD FAILURE

e(cm)	1	2	3	5	10
q_u(kg/cm^2)	1.86	0.934	0.630	0.378	0.194
e(cm)	15	[illegible]	30	40	50
q_u(kg/cm^2)	[illegible]	[illegible]	0.071	0.05[illegible]	0.04[illegible]

Reprinted from "The Indian Concrete Journal," January 1971

An experimental study of the behaviour of hyperbolic paraboloid shell roofs under uniformly distributed loads

APPENDIX-II

Dr P. C. Varghese and A. C. Mathai

The paper presents the results of model studies concerning the influence of rise to span ratio and edge beam size on the behaviour of square hyperbolic paraboloid shell roofs of the umbrella type, bounded by straight edges. It shows that both these factors have notable influence on the behaviour of the shell at uncracked, cracked and ultimate load stages.

The use of hyperbolic paraboloid shell roof as shown in *Fig* 1 is common these days due to their pleasing appearance, inherent high strength, and requiring only simple shuttering. According to the membrane theory which is usually used for general design purpose, the shell proper is in a state of pure shear under a uniformly distributed load. This causes equal principal tensions and compressions. The reinforcement is provided to withstand the principal tension, the steel being preferably placed along the principal directions, but generally placed from practical considerations in two mutually perpendicular directions of the straight generators. The principal compression produced will determine the minimum safe thickness of the shell.

The elements of the supporting structure of the shell, namely, the edge and ridge beams, are usually designed for the combined effect of bending due to self-weight and the accumulated compression (or tension) transferred along the centre-line of the shell to these members, the element being considered as a column or a tie.

The boundary conditions provided are not consistent with membrane behaviour of the shell and hence the behaviour of the shells under realistic boundary conditions are quite different from the predictions by the membrane theory. A rigorous theoretical investigation of the problem with the existing methods of analysis is difficult, and may well be not realistic for a material like concrete. The model tests being conducted at the Indian Institute of Technology, Madras, are aimed at investigating the behaviour of the shell under realistic boundary conditions by experimental methods. The present paper reports the first stage of the studies.

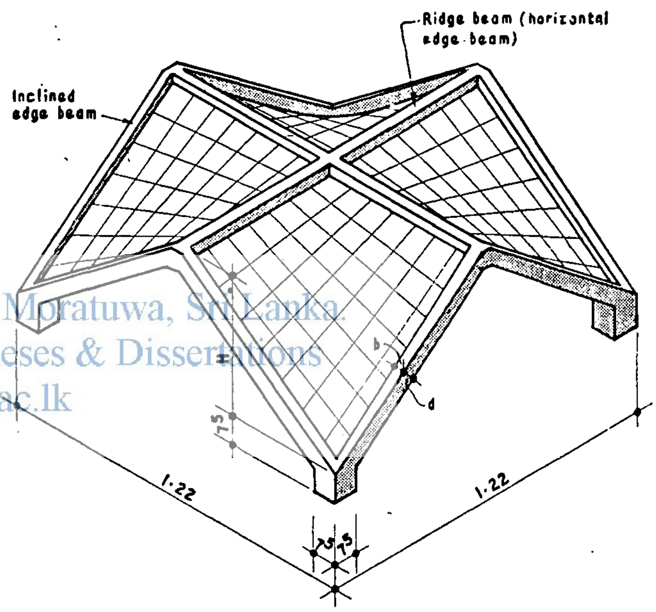


Fig 1 General shape of hyperbolic paraboloid test models

Models

All models were of reinforced microconcrete 122 cm by 122 cm in plan inclusive of edge beams. The roof was formed by an assembly of four basic quadrants of rectangular hyperbolic paraboloids intersecting along horizontal ridges, *Fig* 1. The shell proper was 12·5 mm thick, except near the column where the thickness was gradually increased to 38 mm. The shell reinforcement consisted of 10 gauge (3·2 mm diameter) mild steel wires at 5 cm centre to centre both ways along the straight line generators parallel to the edges, *Fig* 2.

Four different rise to span ratios and three different sizes of edge beams for each rise (in all twelve models) were tested in the study. In addition, two slabs with stiffened edge beams were also tested for comparison. *Table* 1 gives the details of the models tested. The properties of the microconcrete and the steel used are given in *Table* 2. The shell panels, edge beams and supporting columns were cast monolithically. The shuttering was removed four days after casting, and curing was carried out under wet gunny bags for twenty-one days.

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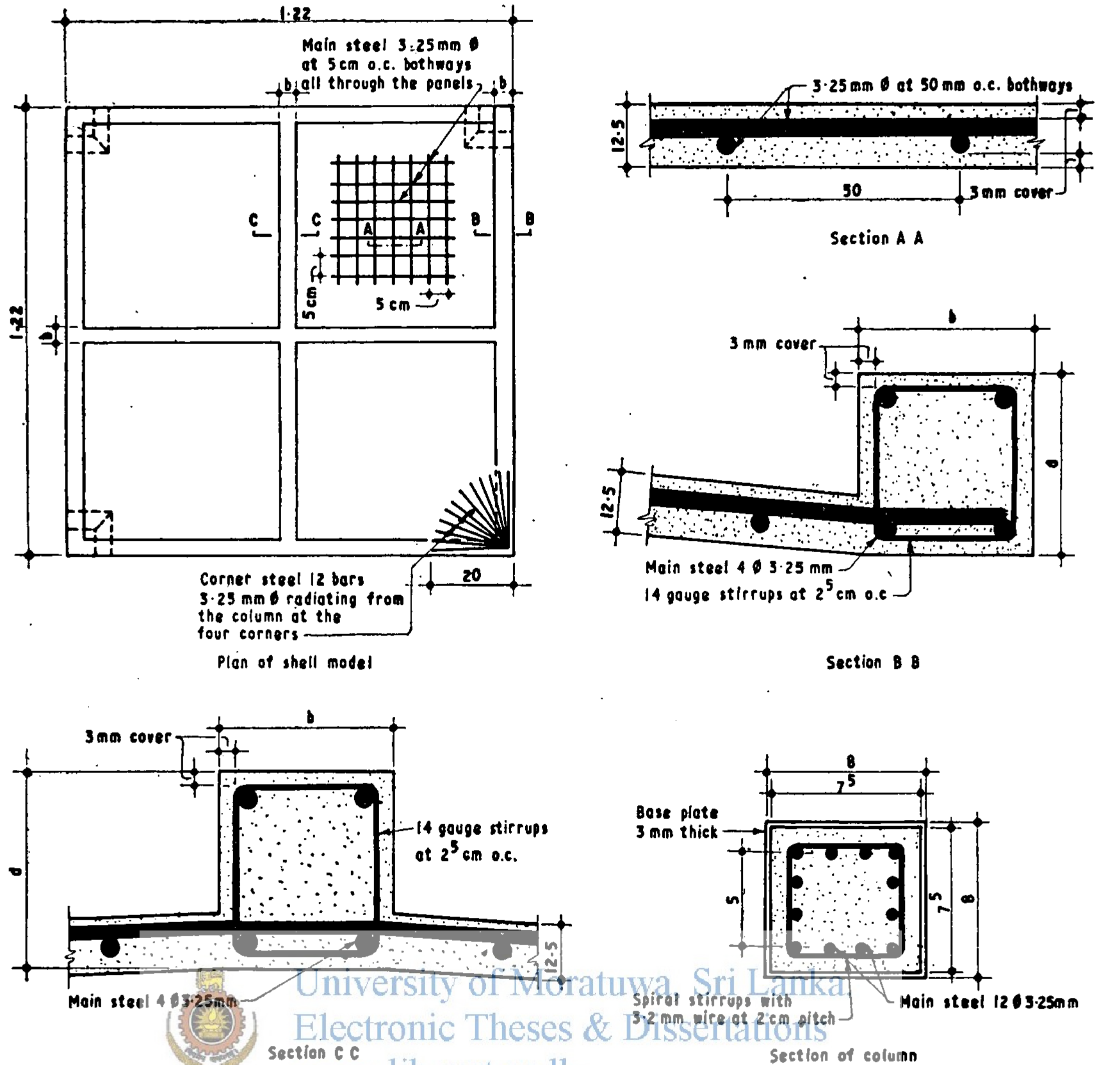

Fig 2 Details of reinforcement

TABLE 1 Designation and dimensions of models

Overall size ($2a \times 2a$) } = 122 cm × 122 cm   Thickness of shell panel } = 12·5 mm

Series	*Designation of model*	*Rise (H), cm*	$\frac{H}{a}$ *ratio*	*Width of edge beam, cm*	*Depth of edge beam, cm*	*Cross-sectional area of tie, cm²*
	D1–1	30·50	0·500	3·0	3·0	3·0
D1	D1–2	30·50	0·500	3·0	3·0	3·0
	D1–3	30·50	0·500	3·0	3·0	3·0
	D2–1	15·25	0·250	2·5	2·5	3·0
D2	D2–2	15·25	0·250	3·8	3·8	3·0
	D2–3	15·25	0·250	3·8	5·7	3·0
	S1–1	10·16	0·167	2·5	2·5	3·0
S1	S1–2	10·16	0·167	3·8	3·8	3·0
	S1–3	10·16	0·167	3·8	5·7	3·0
	S2–1	6·10	0·100	2·5	2·5	4.0
S2	S2–2	6·10	0·100	3·8	3·8	4·0
	S2–3	6·10	0·100	3·8	5·7	4·0
	FS–1	0	0	2·5	2·5	—
FS	FS–2	0	0	3·8	5·7	—

TABLE 2 Properties of microconcrete and steel

	Microconcrete					*Steel*				
Model	*Age at test, days*	*Average cylinder strength, kg/cm²*	*Average split tensile strength, kg/cm²*	*Average modulus of rupture, kg/cm²*	*Secant modulus at 120 kg/cm², kg/cm²*	*Average diameter of main steel, mm*	*Yield stress, kg/cm²*	*Ultimate stress, kg/cm²*	*Modulus of elasticity, kg/cm²*	*Elongation on 2 cm gauge length, per cent*
D1–2	420	291	37	52	$2 \cdot 73 \times 10^5$	3·25	2780	3930	$1 \cdot 95 \times 10^6$	35
D2–1	49	256	24	53	$1 \cdot 90 \times 10^5$	3·28	2300	3320	$1 \cdot 95 \times 10^6$	40
D2–2	42	192	15	45	$1 \cdot 72 \times 10^5$	3·28	2300	3320	$1 \cdot 95 \times 10^6$	40
D2–3	39	211	23	41	$1 \cdot 50 \times 10^5$	3·30	2780	3930	$1 \cdot 95 \times 10^6$	35
S1–1	46	273	24	45	$2 \cdot 16 \times 10^5$	3·23	2150	3300	$2 \cdot 09 \times 10^6$	25
S1–2	47	192	23	40	$1 \cdot 36 \times 10^5$	3·28	2300	3320	$1 \cdot 95 \times 10^6$	40
S1–3	45	307	29	51	$2 \cdot 14 \times 10^5$	3·28	2300	3320	$1 \cdot 95 \times 10^6$	40
S2–1	37	212	20	41	$1 \cdot 52 \times 10^5$	3·30	2780	3930	$1 \cdot 95 \times 10^6$	35
S2–2	33	238	19	43	$1 \cdot 78 \times 10^5$	3·30	2780	3930	$1 \cdot 95 \times 10^6$	35
S2–3	28	160	15	35	$1 \cdot 43 \times 10^5$	3·30	2780	3930	$1 \cdot 95 \times 10^6$	35
FS–1	28	210	19	30	$1 \cdot 58 \times 10^5$	3·30	2780	3930	$1 \cdot 95 \times 10^6$	35
FS–2	28	302	24	38	$2 \cdot 26 \times 10^5$	3·30	2780	3930	$1 \cdot 95 \times 10^6$	35

Test set-up

A general view of the self-straining loading frame using hydraulic jacks connected to a distributor and main pump is shown in *Fig* 4. The roof models were supported on steel ball supports. Mild steel tension ties on the four sides of the model connecting all the four columns were also provided at the level of the intersection of the inclined edge beams with the supporting columns. Horizontal reactions at the supports were measured during the test by means of resistance gauges fixed to the steel ties.

The uniformly distributed load on the roof was simulated by sixteen uniformly spaced concentrated loads applied through timber loading blocks. These loads were applied through four hydraulic jacks, each of 15 ton capacity, connected to the same pressure line, (a fifth jack connected to the pressure line being used to check the jack pressure by a proving ring).

The estimated load was applied in 12 to 16 equal increments, and after each increment the deflections, surface strains and the force in the tie member were measured. The vertical deflection of the shell panel and edge beams were measured at selected points in one quadrant of the shell, with additional dial gauges to check these readings in other quadrants also. The surface strains in concrete at top and bottom were measured in one panel of the shell and on the edge beams with a 2-in Demec gauge. All tests were taken to ultimate failure of the structure which was due to failure of shell only, of edge beam only, or of both together.

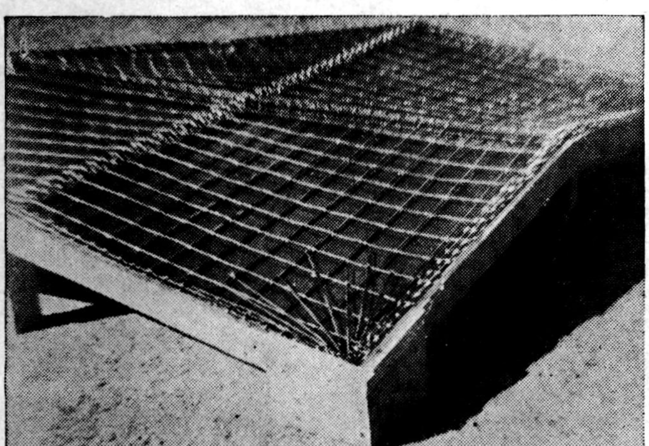

Fig 3 Formwork with reinforcement in position

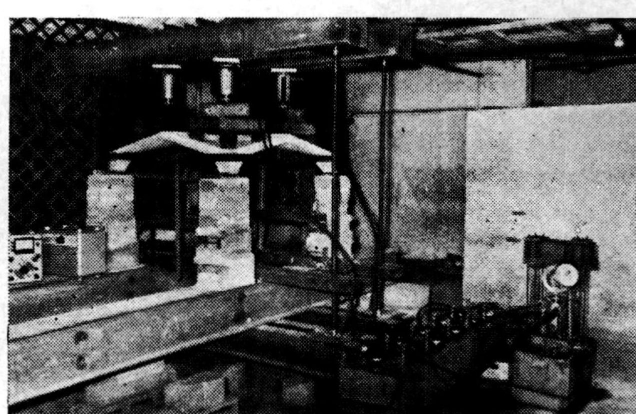

Fig 4 General view of the test set-up

Discussion of test results

General pattern of cracking : The pattern of cracks to be expected on the basis of the membrane theory will comprise a number of cracks in the diagonal direction perpendicular to the sagging parabola. The crack patterns obtained in the tests are not substantially different from this expected pattern, if the cracks due to local effects are eliminated. Typical crack patterns are shown in *Fig* 5. One of the places where cracking appeared early is the high corner, *i.e.*, the corner near the intersection between the ridge and the edge beams. Cracks were observed at the bottom surface of the shell, and later central diagonal cracks developed on the top surface of the shell. In addition, with stiffer edge beams, cracks were found to develop around the top surface of the panels, near their junction with the edge beams, as a result of the negative moment (tension on top) in this region.

Comparing the crack patterns of the shell and the plate, a striking difference is observed between the crack patterns of the stiffened flat slabs, and shells of even as low a rise as 0·05 of the span. The principal directions of cracking in the slab and the shells with similar supports are almost normal to each other. This shows clearly the substantial difference between the modes of transfer of the loads.

Cracking loads : A comparison of the observed cracking loads with the cracking loads predicted on the basis of the membrane theory is given in *Table* 3. The observed cracking loads are in general lower than the predicted, due to the presence of bending moments either of a general or of a local nature in the shells.

Failure of test models and failure patterns: The failure pattern of the models under test can in general be classified into three types :

(*i*) failure of local nature

(*ii*) failure of edge beams and subsequent failure of panels

(*iii*) independent failure of the shell panels.

Each of these types of failures is discussed below :

(*i*) Failure of local nature—The most important of the local failures was noted over the supporting columns. In considering the ideal shear to act along the edge beam, the supports have to be taken along the line of the resultant force. Generally, however, there will be bending at these junctions. In addition, the vertical supports provided here have to carry the vertical loads applied, and hence their capacity to carry these loads and the resistance of the local areas against punching shear have to be evaluated. *Fig* 5(d) shows one of the early models that failed at the supports. Later models were sufficiently strengthened at these junctions against such failures. Practical construction with hinged supports has been described by Anton Tedesko[1].

(*ii*) Failure of edge beams and subsequent failure of shells—The flatter shells, (*i.e.*, all models of the S2 series), failed by the crushing of the mortar in the edge beam. The failure of these edge beams induces high moments in the thin shell panels leading to collapse. Failure can occur for the horizontal ridge beam, or for the inclined edge beams. The size of the edge beams is of great importance in shallow shells in which the axial forces and bending moments in the beams are very high. A model with an $\frac{H}{a}$-ratio of 0·25 (model D2-1) could carry a load of 8·5 tonnes with no distress to the edge beams, 2·5 cm × 2·5 cm in size, whereas the 3·8-cm × 5·7-cm edge beam of a model with an $\frac{H}{a}$-ratio 0·1 (S2-3) failed under a total load of only 4·25 tonnes. These tests clearly reveal that with shallower shells stronger edge beams are necessary, and that local failure of these beams before failure of the shell proper is to be expected.

(*iii*) Independent failure of shell panels—This is the most tolerable type of failure since the edge beams which are secondary members are not allowed to fail before the shell panels. The failures of the models of the D2 series are representative of this type of failure. In a few cases, the edge beam being placed over the shell,

TABLE 3 Comparison of cracking loads

Model	*Cracking load by membrane theory, tonnes*	*Load at commencement of cracks in bottom surface, tonnes*	*Load at commencement of high corner cracks, tonnes*	*Load at commencement of central diagonal cracks, tonnes*	*Ratio of load at commencement of central diagonal cracks to cracking load by membrane theory*	*Ratio of load at first crack to cracking load by membrane theory*
D1–2	12·50	4·50	5·50	5·00	0·40	0·36
D2–1	4·30	2·50	3·00	3·00	0·70	0·58
D2–2	2·95	2·50	2·50	3·00	1·02	0·85
D2–3	4·15	2·00	2·00	2·50	0·60	0·48
S1–1	2·86	1·25	1·75	1·75	0·61	0·44
S1–2	2·78	1·00	1·75	3·00	1·08	0·36
S1–3	3·36	1·25	1·50	2·00	0·60	0·37
S2–1	1·47	0·75	—	2·00	1·36	0·51
S2–2	1·42	1·00	1·25	2·00	1·40	0·71
S2–3	1·18	1·75	1·50	2·50	1·61	1·27
FS–1	—	0·30	—	—	—	—
FS–2	—	0·40	—	—	—	—

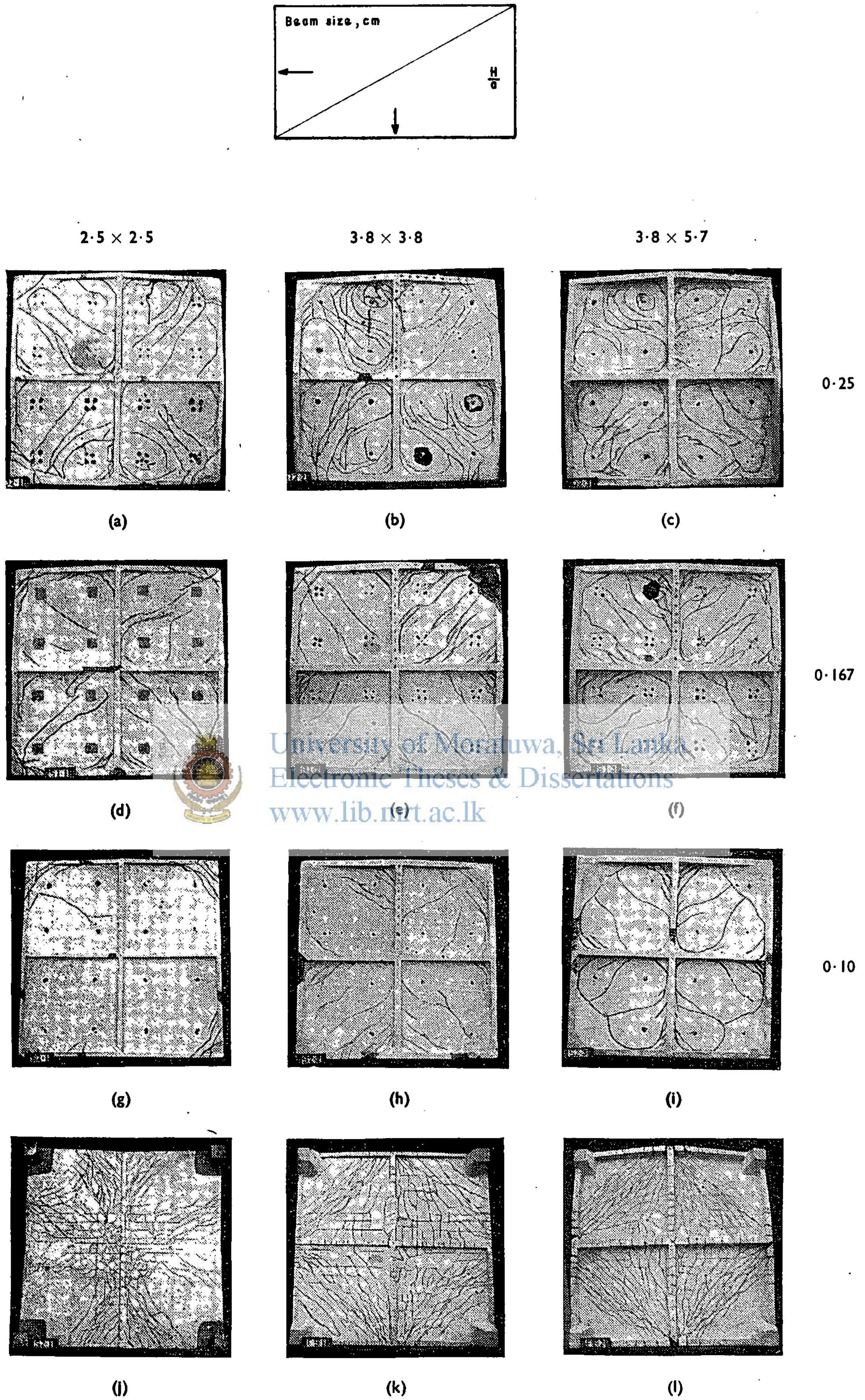

Fig 5 Typical crack patterns showing influence of variations in rise, and edge beam size: (a) to (i) top surface of shells, and (j) to (l) bottom surface of shell S2-1 and flat slabs

failure of bond between the panel and the edge beams was also observed, indicating that in practical construction there should be sufficient binding provided to enable the transfer of the loads. In D2 type shells with $\frac{H}{a}$ ratio of 0·25, independent failure of panels can be produced even with edge beams of small size, suggesting the importance of edge beam size and the rise-span ratio of the shell.

Failure loads : Since some of the final failures were of local nature, it will be misleading to compare failure loads. However, the failure loads obtained are given in *Table* 4. Comparison of failure loads of models FS-2 (1·76 tonnes) and S2-3 (4·25 tonnes) show that even a low rise-span ratio substantially increases the load-carrying capacity of the shell as compared with a slab.

Deflections : The deflection at the centre of the shell is taken as representative of the load-deflection response of the shell. The central deflection for the various models is shown in *Fig* 6. An increase in stiffness was noticed for the deeper shells at higher loads as observed by Dayaratnam and Scrivener in their experimental investigations[2,3]. It was also observed that cracking of the shell did not reduce the stiffness of these deep shells. *Fig* 7 shows the variation of the central deflection with rise-span ratio for three selected loads. It is seen that the deflection in hyperbolic paraboloid shells of usual proportions is less than one-seventh of the deflection in similar stiffened slabs. The deflected shapes of the horizontal ridge beams for different $\frac{H}{a}$ –ratios are compared in *Fig* 8. It is observed that an increase in the size of the edge beams results in a decrease in deflection, *Fig* 9.

Stresses in shell panel : Membrane theory predicts a state of uniform compression and uniform tension of equal magnitude along the hogging and sagging parabolas respectively under a uniformly distributed load. The observed strain on the shell, however, indicates that in addition to direct forces, bending moments too are present along these directions even though they cannot be quantitatively determined.

Behaviour of edge beams : (*i*) Horizontal edge beams (ridge beams)—According to the membrane theory, the horizontal edge beams should be in compression which will vary linearly from zero at the outer end to a maximum at the centre. In addition, the shear is transferred below the beam axis causing a bending with tension at the top of the beam. Measurements, however, show considerable bending of these beams with tension at the bottom. A comparison of the maximum values

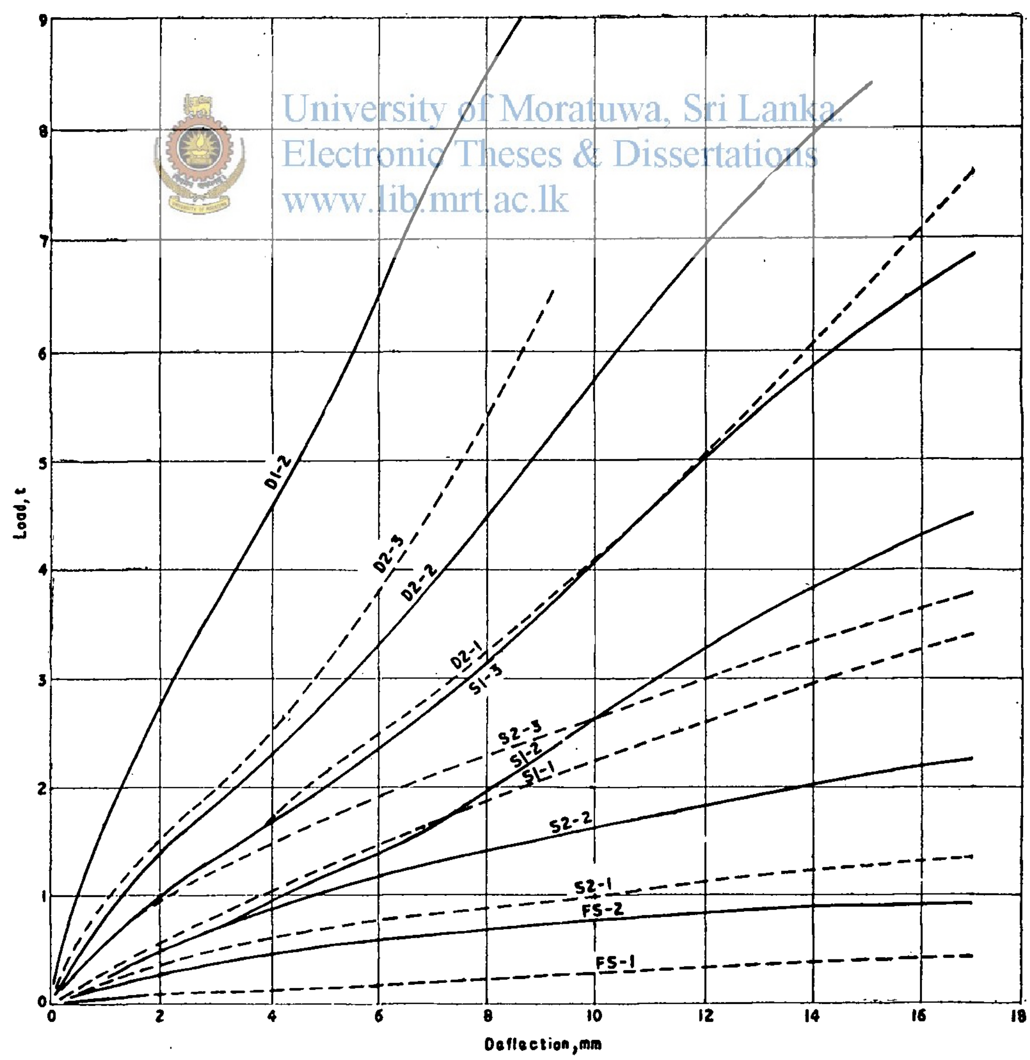

Fig 6 Load versus deflection of centre of shell

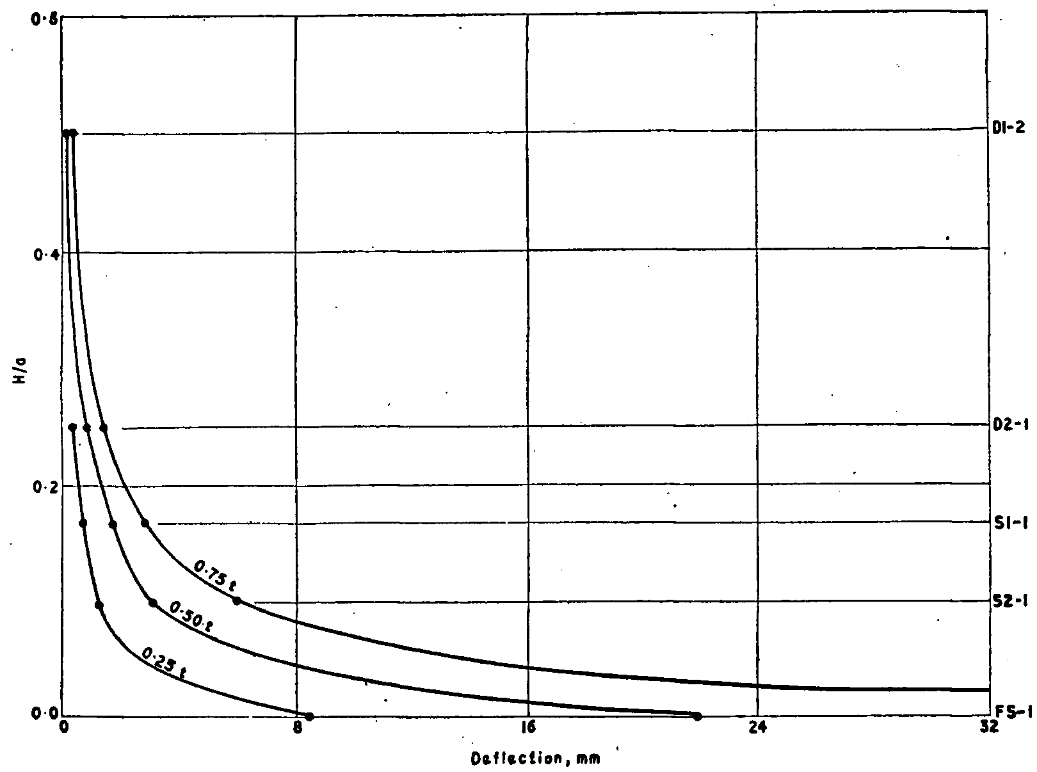

Fig 7 Variation of deflection with $\frac{H}{a}$ ratio

of the axial forces and the bending moments when the shells are subjected to a load of one tonne is given in *Table* 5. The maximum values of the axial forces are as low as one half to one sixth of the value predicted by the membrane theory. Similarly the bending moments are appreciable especially for the shallower shells, their values increasing with increased stiffness of the edge beams. It may be noted, however, that for beams

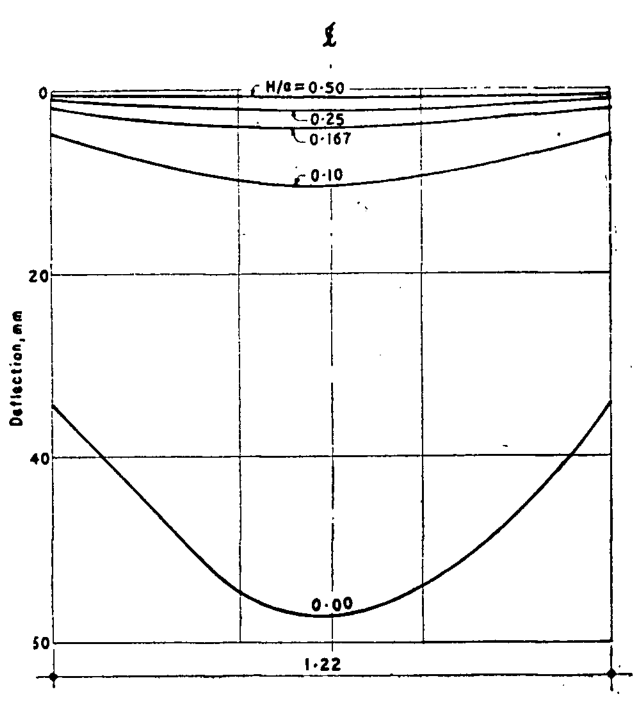

Fig 8 Deflected shapes of horizontal edge beams for various $\frac{H}{a}$ ratios; load on shell 1 tonne

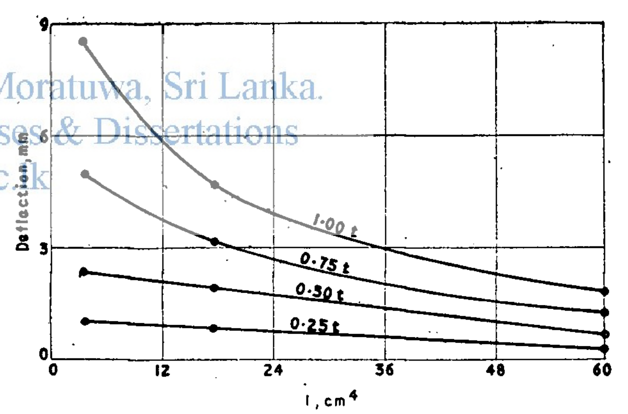

Fig 9 Variation of deflection at centre of shell versus moment of inertia of ridge beams

provided above the neutral surface the eccentricity from the shell reduces the bending stresses due to the self-weight of the shell and the edge beam.

(*ii*) Inclined edge beams—The maximum bending moments in the inclined edge beams for a load of one tonne on the shell are given in *Table* 6. The bending moments decrease with increase in rise and decrease in stiffness of the edge beams.

Horizontal reaction at supports: The values of the tie forces measured by means of electrical strain gauges attached to the tie member show reasonable agreement with those calculated by the membrane theory except in the case of very shallow shells. But this is not an indication of membrane behaviour. Calculations using load transfer by radial shear, instead of the assumed tangential shear in membrane behaviour, will give the

TABLE 4 Failure loads of models

Model	*Failure load, tonnes*	*Cause of failure*
D1–2	9·35	punching at supporting column
D2–1	8·50	failure of shell panel
D2–2	8·00	separation of shell panel from edge beam
D2–3	6·90	separation of shell panel from edge beam
S1–1	4·00	punching at supporting column
S1–2	6·00	punching at supporting column
S1–3	7·25	failure of shell panel
S2–1	2·16	failure of horizontal and inclined edge beams
S2–2	3·25	failure of inclined edge beams
S2–3	4·25	failure of horizontal and inclined edge beams
FS–1	1·20	crushing of concrete on top surface
FS–2	1·76	crushing of concrete on top surface

TABLE 5 Axial force and bending moment in horizontal edge beams (ridge beams) for one-tonne load on shell

Model	*Maximum axial force by membrane theory, kg*	*Maximum axial force developed, kg*	*Ratio of maximum axial force developed to maximum axial force by membrane theory*	*Maximum bending moment, kg cm*	*Eccentricity, i.e., maximum bending moment / maximum axial force developed, cm*
D2–1	1000	215	0·215	250	1·16
D2–2	1000	375	0·375	300	0·80
D2–3	1000	325	0·325	500	1·54
S1–1	1500	300	0·200	210	0·70
S1–2	1500	550	0·370	975	1·77
S1–3	1500	650	0·430	1600	2·46
S2–1	2500	400	0·160	625	1·56
S2–2	2500	650	0·260	1100	1·70
S2–3	2500	700	0·280	1600	2·29

same order of magnitude of forces. The theoretical and experimental values of the forces are given in *Fig* 10. The theoretical curve is a rectangular hyperbola indicating high horizontal reaction for very shallow shells. But since membrane theory does not in the limit reduce to the plate theory, the calculations are not applicable to very low values of rise. For zero rise the value of horizontal reaction should be zero, and the curve should pass through the origin.

Conclusion

It has been observed in these model tests that the rise-span ratio is the most significant parameter affecting the behaviour of hyperbolic paraboloid shells. An increase in rise-span ratio decreases the deflections, the horizontal reaction at the supports, the bending moments and direct forces in the edge beams, and the compressive and tensile stresses in the panels for a given uniformly distributed load.

For a given configuration of the shell the variation of the edge beam size has a considerable effect on the behaviour of the shell as a whole. For a given rise there is an optimum size of edge beam which will ensure ideal behaviour of the shell. The ultimate behaviour of the shells, especially the shallower shells depends very largely on the edge beams. With very small beams the shell fails due to the failure of the edge beams before

TABLE 6 Maximum bending moment in inclined edge beams for one-tonne load on shell

Model	*Maximum bending moment, kg cm*
D2–1	300
D2–2	500
D2-3	850
S2–1	525
S2–2	1200
S2–3	1800

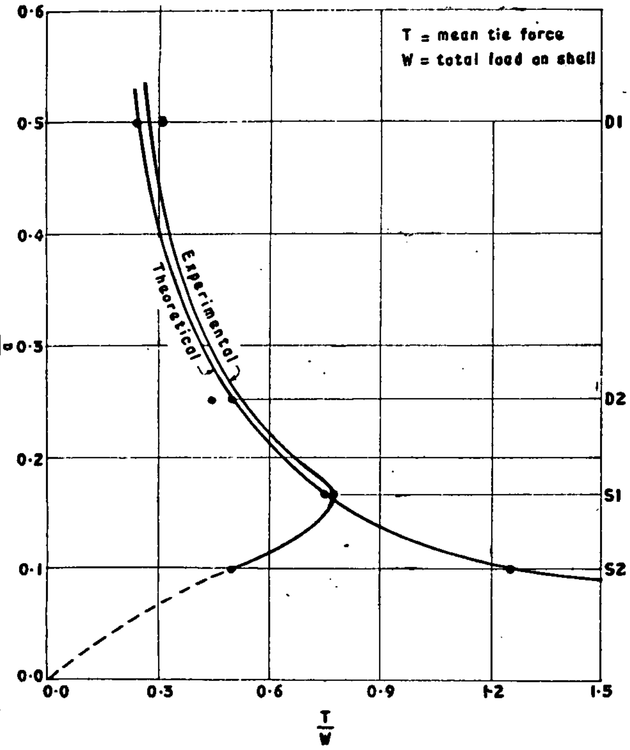

Fig 10 Variation of horizontal thrust with $\frac{H}{a}$ ratio

the failure of the shell proper. With larger edge beams the ultimate failure is due to failure of the shell itself. The size of edge beam required to produce failure of the shell decreases with increase in the rise-span ratio, and in deeper shells the behaviour of the shell is not sensitive to size of edge beams. Increase in size of edge beams increases bending moments in the edge beams and fixing moments in the shell, and so for ideal performance there is an optimum size beyond which it is not desirable to increase the size of edge beams. Whether a shell without edge beams can be built successfully depends on its rise-span ratio.

The membrane theory does not clearly predict the state of stress in the shell. There exist large direct forces and bending moments especially along the edges and near the corners. In all cases cracking of the shell occurred at loads less than that predicted by the membrane theory.

The membrane theory does not account for any variation in the behaviour of the shells with change in size of edge beams, and these properties should always be considered by the designer in proper detailing of the shell. Local failures, as has been discussed already, should also be prevented by proper detailing of the reinforcement.

Acknowledgment

The work described in this paper formed the subject of a thesis[4] submitted for the MTech degree of the Indian Institute of Technology, Madras. The authors wish to record their appreciation and gratitude for the facilities and finances provided by the Institute for carrying out the investigation.

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Printed and published by Lt Col C. L. Proudfoot (Retd.) for The Cement Marketing Co. of India Ltd., at 'Cement House', 121, Maharshi Karve Road, Bombay, and printed at the Times of India Press, Bombay, India.