Degenerating Orr-Sommerfeld Eigenmodes and Development of Three-dimensional Perturbations

R. Shanthini

Division of Fluid Mechanics,
Luleå University of Technology,
S-951 87, Luleå, Sweden.

June 1990

Abstract

Degeneracies of temporally damped Orr-Sommerfeld eigenmodes are studied primarily for plane Poiseuille flow. The discrete spectrum of the eigenmodes is shown to possess infinitely many degeneracies, each appearing at a certain combination of k (the modulus of resultant wavenumber) and αR (the streamwise wavenumber times the Reynolds number). The streamwise phase velocities of the degeneracies are close to about $\frac{2}{3}$ of the centreline velocity, and their damping rates decrease with increasing Reynolds number. Interestingly, several degeneracies are operative even at sub-transitional Reynolds number. Degeneracies are found also in water table flow and some of these degeneracies are shown to have the same characteristics and modal structures as those in plane Poiseuille flow.

The responses of the degeneracies in plane Poiseuille flow are investigated through the initial-value problem. The initial perturbation field is chosen to be free of normal vorticity, and its velocity component normal to the walls is described by the generalized Orr-Sommerfeld eigenfunction in the normal direction. The subsequent temporal development of this normal velocity is shown to be that of monotonical decay, in the linear regime. Notable initial growth is exhibited by the streamwise velocity of the two-dimensional perturbation flow in case of the least damped degeneracy. Nevertheless, in all cases of the degeneracies investigated, the two-dimensional perturbation flow looses its kinetic energy with increasing time.

In the case of a three-dimensional perturbation flow, the spanwise variations in the normal velocity induce normal vorticity. The amplitude of the induced vorticity and, hence, that of the streamwise perturbation velocity are shown to grow to significant peak values, before the exponential decay predicted by the linear theory sets in. The amplitude of the induced vorticity is shown to increase also with increasing Reynolds number. The presence of normal vorticity causes the perturbation flow to gain kinetic energy from the basic Poiseuille flow. This gain is so great in cases of the least damped symmetric and antisymmetric degeneracies, that the perturbation flow overcomes its energy loss due to viscous dissipation and exhibits significant initial growth of its kinetic energy, at crucial Reynolds numbers such as the transitional one. These growths, according to the linear theory, do not influence the normal velocity that is monotonically decaying with time. It is shown, however, that these growths are to be followed by nonlinear equations at about the transitional Reynolds number. It is also shown that describing the solution of the nonlinear system as waves travelling in one single oblique direction does not change the monotonous behaviour of the normal velocity.

Key words: degeneracy, double eigenvalue, Orr-Sommerfeld equation, direct resonance, stability, plane Poiseuille flow, water table flow, initial-value problem, three-dimensionality, vortex stretching, induced vorticity, nonlinearity, Reynolds stress, perturbation energy.

"... The education of the individual, in addition to promoting his own innate abilities, would (*should*) attempt to develop in him a sense of responsibility for his fellow-men, in place of the glorification of power and success in our present society..."

- ALBERT EINSTEIN (*Ideas and Opinions*)

To one of my pre-university teachers,

THACHCHANAMOORTHI,

who always insisted that the prime goal of education should be to make a human more humane.

CONTENTS

"... I have learned a number of things, and am educated, now, but I wasn't at first. I was ignorant at first. At first it used to vex me because, with all my watching, I was never smart enough to be around when the water was running up-hill; but now I do not mind it. I have experimented and experimented until now I know it never does run up-hill, except in the dark. I know it does in the dark, because the pool never goes dry; which it would, of course, if the water didn't come back in the night..."

- MARK TWAIN, 1905 (*Eve's Diary*)