

**EMPIRICAL FORMULAS TO ESTIMATE THE ORDER  
OF 2-D FIR FAN FILTERS**

R.H.N.S. Jayathissa

168462K

Degree of Master of Science

Department of Electronics and Telecommunication Engineering

University of Moratuwa

Sri Lanka

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## **DECLARATION**

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## ABSTRACT

Two-dimensional (2-D) finite impulse response (FIR) fan filters belong to a special class of 2-D filters which has the capability of directional filtering. They are used in many applications such as geological and seismological data processing, and array signal processing. In this dissertation, three accurate formulas are proposed to estimate the order of 2-D FIR fan filters designed using the windowing technique in conjunction with the 2-D separable Kaiser window. Maximum passband ripple  $A_p$ , minimum stopband attenuation  $A_a$ , half of the fan angle  $\theta$ , passband width  $B$ , transition width  $T$  and Kaiser window parameter  $\beta$  are used as the main parameters in the derivation of formulas.

Here, three estimation formulas are proposed for three different values of transition width, that is  $T=0.01\pi$  rad/sample,  $0.05\pi$  rad/sample and  $0.1\pi$  rad/sample by employing two steps. In the first step, a set of filters with different specifications are designed in order to experientially determine the Kaiser Window parameter  $\beta$  and the minimum order of the filter required to satisfy the given specifications. In the second step, the formulas for the Kaiser window parameter  $\beta$  and the minimum order of the filter are empirically derived through multiple linear regression using the data obtained in the first step.

In numerical evaluation, statistical means of the absolute error between the estimated and true values of the Kaiser window parameter  $\beta$  and the minimum filter order are calculated. It is found that mean of absolute error of estimated  $\beta$  and true  $\beta$  is less than one. Also, for minimum filter order it is slightly greater. Mean of absolute error of estimated order is varied from 2-14 of true filter order which are 13.85 for  $0.01\pi$  rad/sample, 3.11 for  $0.05\pi$  rad/sample and 2.33 for  $0.1\pi$  rad/sample. The proposed formulas provide very good accuracy for widely employed 2-D fan filter specifications. By using these formulas, a 2-D fan filter can be designed without trial and error to determine the Kaiser window parameter  $\beta$  and the minimum filter order saving significant time in the design process.

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## **LIST OF ABBREVIATIONS**

1-D	One-Dimensional
2-D	Two-Dimensional
FIR	Finite Impulse Response
IIR	Infinite Impulse Response