IDENTIFICATION OF ELECTROMECHANICAL OSCILLATIONS USING SMALL - SIGNAL STABILITY ASSESSMENT:
A CASE STUDY OF LAKVIJAYA POWER STATION

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Degree of Master of Science

Department of Electrical Engineering

University of Moratuwa
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Thesis/Dissertation submitted in partial fulfilment of the requirements for the degree
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DECLARATION

I declare that this is my own work and this thesis does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any other University or institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

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The above candidate has carried out research for the Master thesis under my supervision.

Signature of the supervisor:

Dr. W. D. Prasad Date:

DEDICATION

This work is dedicated to my beloved parents, my darling wife, Shanika Yashodha and my two daughters.

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Abstract

Lakvijaya power plant is the biggest power plant in Sri Lanka contributing more than 50% of power annually to the national grid. The main objective of this thesis is to study of the electromechanical oscillations of Lakvijaya Power Plant using small-signal stability assessment.

Small signal stability problem refers to the stability problems caused by small disturbances. For small-signal stability studies, the power system can be represented as a linearized state space model. An algorithm is developed in MATLAB using detailed models of synchronous generators, transmission lines and the associated controls. The eigenvalues and eigenvectors of the dynamic state matrix are obtained to study small signal stability. The mode shape calculation is used to identify the dominant oscillations in the system.

Keywords: Eigen-value calculation, Small Signal Stability Analysis, Mode Shapes, Participation Factors, Thermal Plant

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List of Abbreviations

Abbreviation	Description
CEB	Ceylon Electricity Board
LVPS	Lakvijaya Power Station
VS	Voltage Source
Equ	Equation

1 INTRODUCTION

1.1 Power System Stability

Power system operates in synchronous mode in steady-state and can subjected to a wide range of disturbances due to following occurrences [1].

- Loads and generation changes
- Network changes
- Faults and outages of equipment

Hence the stability of the power system will be affected. Power system stability is the ability of an electric power system to remain in state for operating equilibrium after being subjected to a physical disturbance[2]. Simply, loss of synchronism due to system instability when it is subject to a perturbation.

The power system stability of a power system can be classified into few types depend on nature of the disturbance, it can be divided into the two types as shown below[3] [4]:

1. Large-disturbance stability (or stability subsequent to event type disturbances such as a failure in a transmission line).

2. Small-disturbance stability (or stability subsequent to small magnitude disturbances such as load variations)

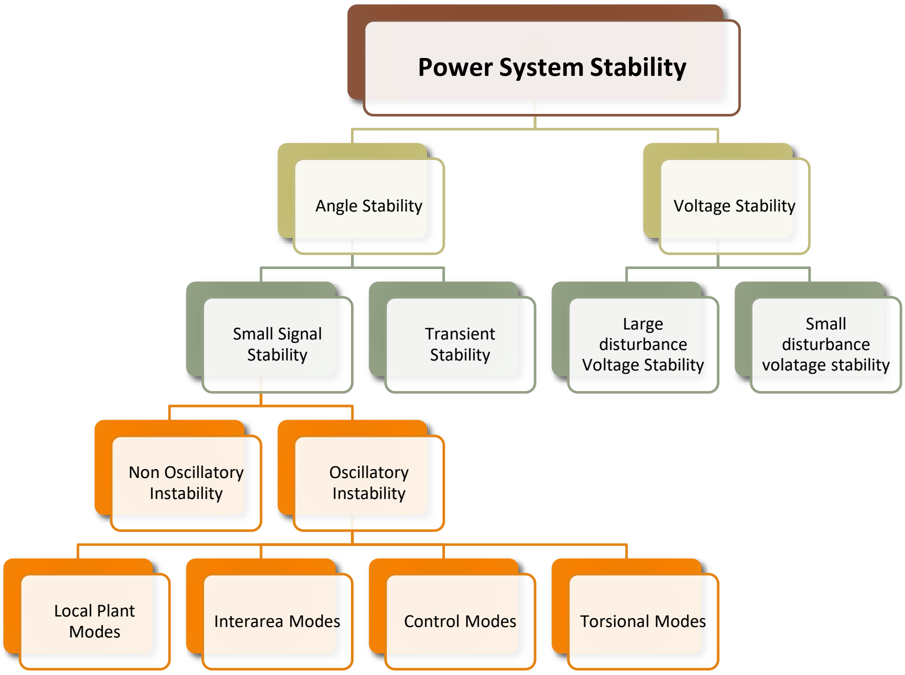

Figure 1-1 Classification of Power System Stability

1.1.1 Transient Stability of a Power System

Transient stability associates the response to large disturbances, due to sudden load remove, load addition, line faults, switching operations or loss due to excitation. Transient stability is a fast phenomenon that is usually visible within seconds to retain synchronism following a disturbance sustaining for a reasonably long period.

1.1.2 Small Signal Stability of a Power System

The steady-state stability of a power system involves the ability of the system to stay in equilibrium when small perturbation impact in an operating point [2][5]. The equilibrium is to be maintained both before and after the disturbances.

Primary concern is to maintain synchronization of the machine and voltage stability of the system. Rotor angle is the main component when the system is keeping synchronism. Rotor angle is the angle between rotor axis and the stator axis. The all machine's rotor angles in the system should be in equilibrium to maintain the system stable. Thus rotor angle mainly affects to the stability of the system. There is non-linear relationship between rotor angle and electric power. The system's damping characteristics is mainly affected by the changes in rotor speed after a disturbance. Therefore oscillatory instability can happen in the system. Here small signal stability analysis is the system study under small disturbances.

These disturbances may produce oscillations due to insufficient damping. Oscillations problem may lead the majority of instabilities and more complex because of different oscillation modes. Since small signal stability analysis are around an equilibrium point, it will allows to transform complex dynamics of the system in to linear equations (simplified linear model).

1.2 Brief introduction Sri Lankan power system & important to Lakvijaya power plant

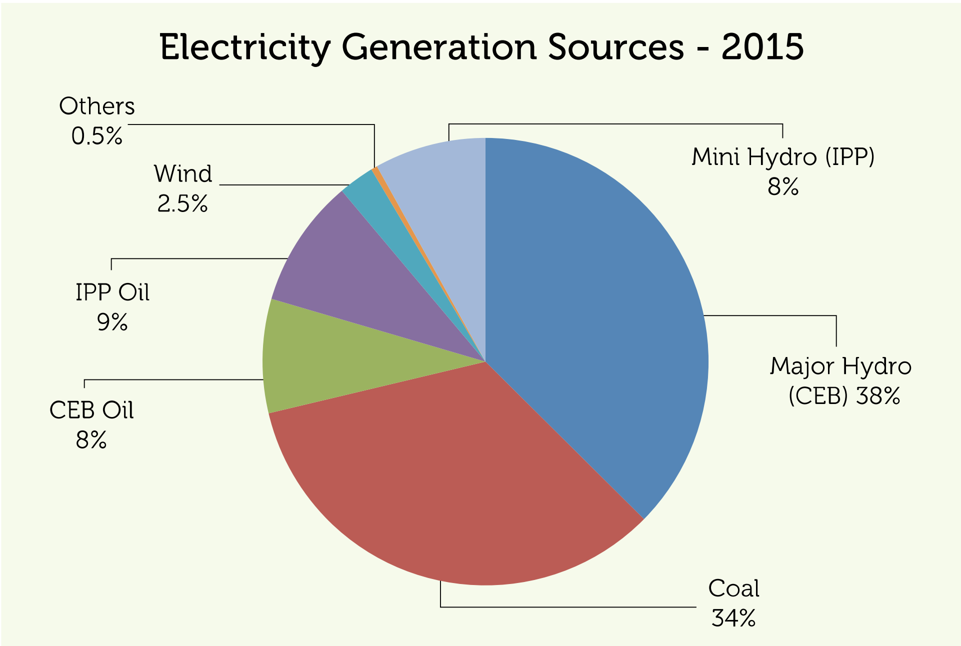

Figure 1-2: Electricity Generation Sources 2015 in Sri Lanka

Sri Lanka is the country which has 100% electrification in south - Asia while 535 kWh/person of average per capita electricity is consumed. The system energy losses which have been gradually decreasing since 2000 stood at 10.4% by the end of the year 2015. Sri Lankan power generation is consist of 4018MW (according to 2016 data) installed capacity while 1691MW hydro plants, 2115MW thermal plants & 176MW solar, biomass & wind plants are contributing to the system. In rainy season, country electricity production depends on hydro power station while Thermal plants, mainly one & only coal plant is supply power in dry season [6].

Lakvijaya power station, the 900MW one & only coal power station is supply nearly >50% of power during dry season.

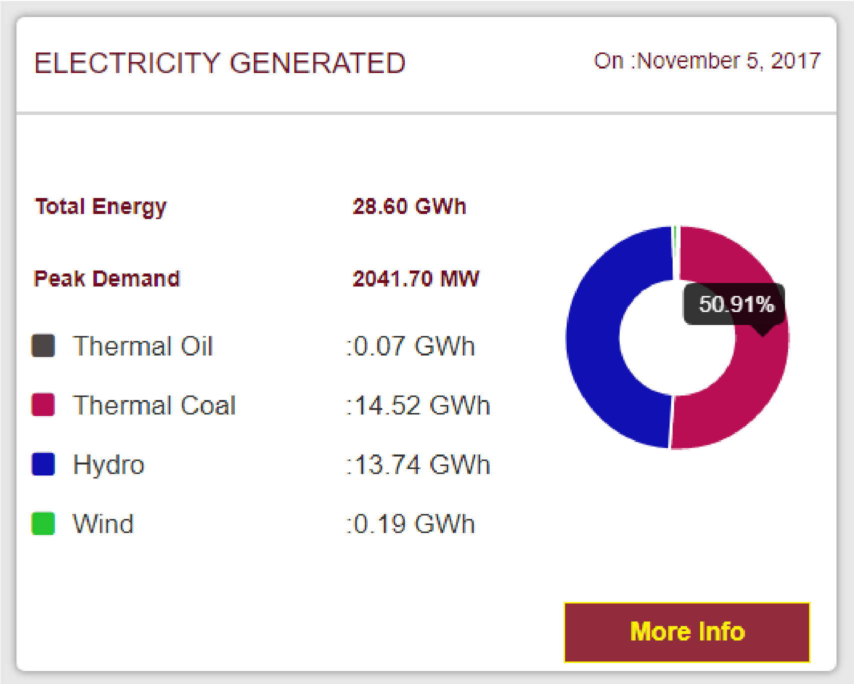

Figure 1-3 : Electricity Generation of a day in dry season 2015

The Lakvijaya power plant consists of 3x 300MW subcritical plants which are connected to 220kV transmission system from new Chilaw substation & Anuradhapura substation. Though Sri Lankan main load centre is Colombo city, the plant is located in 127km away from the load centre. Since 900MW of power generation is continuously supplied from long away which is 35% of the maximum demand, it is required to maintain uninterrupted power supply to the national grid which may leads to blackout condition if all plants get tripped due to external or internal fault.

In October 2017 nationwide 2.5 hours' power cut was implemented after all three units of the Lakvijaya coal power plant broke down due to external fault for 5 days. Therefore this, Lakvijaya Power Station is one of critical power plant which is supplied considerable share of power generation.

1.3 Objectives

Stability of the power system under small magnitude disturbances can be studied using small signal stability assessment. For small signal stability studies, the power system can be represented as a linearized state space model at an operating point [2].

The dominant oscillations of the system can be identified using the eigenvalues of the state matrix (A) and the mode shape and participation factor calculations.

The main goal of the study is to Conducting a small signal stability analysis to Lakvijaya coal power station and identify the oscillations.

1.4 Thesis Outline

The thesis is organized as follows,

Chapter 2 illustrates the methodology which is used for small signal stability analysis.

Chapter 3 describes model-based analysis. The mathematical model used in model-based analysis is discussed in this chapter, including the system.

Chapter 4 focuses on validate the mathematical model use in MATLAB.

Chapter 5 gives the Conclustion.

2 Modelling of power system for small signal stability assessment

2.1 Synchronous Generator Model

Following assumptions are considered when equations for synchronous machine was developed.[2]

a. The stator windings are sinusoidally distributed along the air-gap as far as the mutual effect with the rotor are concerned.
b. The stator slots cause no appreciable variation of the rotor inductance with rotor position
c. Magnetic hysteresis is negligible
d. Magnetic saturation effects are negligible

Assumptions (a), (b), and (c) are reasonable. Final solution will be come by comparing calculated performance based on above assumptions with the actual performance. For a convenient analysis, assumption (d) is made.

The following Figure 2-1 shows the circuits used for synchronous machine in analysis. The stator contains three-phase armature windings which is carrying alternating currents. Feld and amortisseur windings are contained in the rotor circuit. DC source is connected with the field winding. For purposes of analysis, the currents in the amortisseur may be assumed to flow in two sets of closed circuits (flow through d-axis & q-axis). Large number of circuits used to represent amortisseur effects in machine design analysis. Limited number of circuits are used in system analysis. In system stability analysis, it is necessary to represent more than 2 to 4 rotor circuits in each axis. For simplicity of the analysis, one amortisseur circuit is assumed here (see figure 2-1) [7] [8] [9].

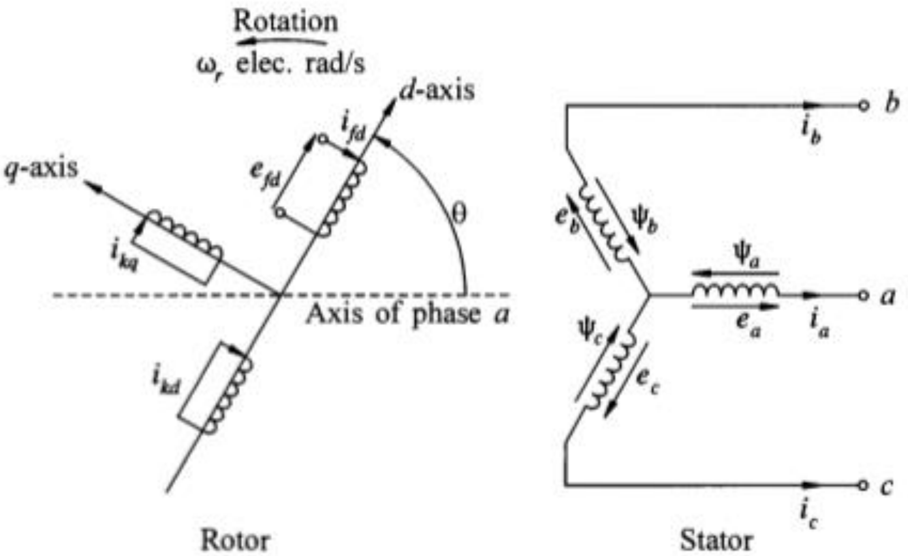

Figure 2-1: Stator and rotor circuits of a synchronous machine

Where;

a ,b ,c	:	Stator phase windings
f_d	:	Field winding
K_d	:	d-axis amortisseur circuit
K_q	:	q-axis amortisseur circuit
K	=	1, 2, …..n; n = no. of amortisseur circuits
θ	=	Angle by which d-axis leads the magnetic axis of phase a winding, electrical rad
ω_r	=	Rotor angular velocity, electrical rad/s

2.1.1 Differential Equation

All the generators of the system were modelled using the 6th order model. This generator model represents the dynamic behaviour of the generator rotor, filed winding, one damper winding along the d-axis and two damper windings along the q-axis. Figure 2-2 & Figure 2-3 show the d-axis and q-axis equivalent circuits of this generator model. Equation (2.1) - Eq. (2.6) show the corresponding differential equations.

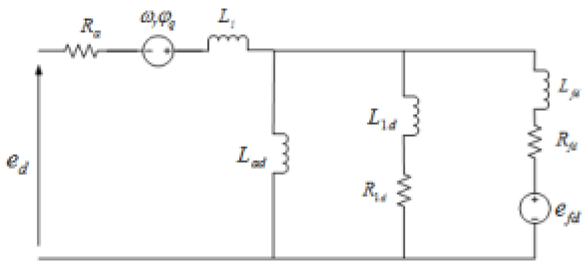

Figure 2-3: d - axis equivalent circuit

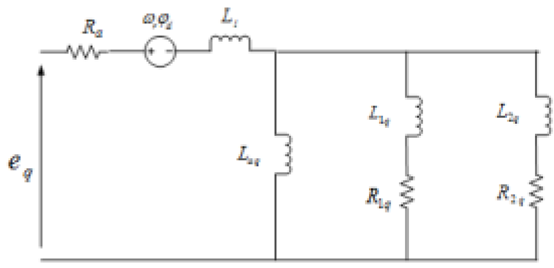

Figure 2-3: q - axis equivalent circuit

.

$$\Delta\dot{\omega} = \frac{1}{2H}\{\bar{T} - (\psi_{ad} i_q - \psi_{aq} i_d) - K_D \Delta\omega_r\} \quad (2.1)$$

$$\Delta\dot{\delta} = \omega_0 \omega_r \quad (2.2)$$

$$\Delta\dot{\varphi}_{fd} = \frac{\omega_0 R_{fd}}{L_{adu}} E_{fd} - \frac{\omega_0 R_{fd}}{L_{fd}} \varphi_{fd} + \frac{\omega_0 R_{fd}}{L_{fd}} \left\{ L''_{ads} \left(-i_d + \frac{\varphi_{fd}}{L_{fd}} + \frac{\varphi_{1d}}{L_{1d}} \right) \right\} \quad (2.3)$$

$$\Delta\dot{\varphi}_{1d} = \omega_0 \left\{ -\frac{R_{1d}}{L_{1d}} \varphi_{1d} + \frac{R_{1d}}{L_{1d}} L''_{ads} \left(-i_d + \frac{\varphi_{fd}}{L_{fd}} + \frac{\varphi_{1d}}{L_{1d}} \right) \right\} \quad (2.4)$$

$$\Delta\dot{\varphi}_{1q} = \omega_0 \left\{ -\frac{R_{1q}}{L_{1q}} \varphi_{1q} + \frac{R_{1q}}{L_{1q}} L''_{aqs} \left(-i_q + \frac{\varphi_{1q}}{L_{1q}} + \frac{\varphi_{2q}}{L_{2q}} \right) \right\} \quad (2.5)$$

$$\Delta\dot{\varphi}_{2q} = \omega_0 \left\{ -\frac{R_{2q}}{L_{2q}} \varphi_{2q} + \frac{R_{2q}}{L_{2q}} L''_{aqs} \left(-i_q + \frac{\varphi_{1q}}{L_{1q}} + \frac{\varphi_{2q}}{L_{2q}} \right) \right\} \quad (2.6)$$

Where;

$$\varphi_{ad} = L''_{ads}\left(-i_d + \frac{\varphi_{fd}}{L_{fd}} + \frac{\varphi_{1d}}{L_{1d}}\right) \qquad (2.7)$$

$$\varphi_{ad} = L''_{ads}\left(-i_d + \frac{\varphi_{fd}}{L_{fd}} + \frac{\varphi_{1d}}{L_{1d}}\right) \qquad (2.8)$$

$$L''_{ads} = \frac{1}{\frac{1}{L_{ads}} + \frac{1}{L_{fd}} + \frac{1}{L_{1d}}} \qquad (2.9)$$

$$L''_{aqs} = \frac{1}{\frac{1}{L_{aqs}} + \frac{1}{L_{1q}} + \frac{1}{L_{2q}}} \qquad (2.10)$$

2.1.2 Algebraic Equations

The voltage and the current at the stator terminal of a synchronous generator is written using a set of algebraic equations.

$$e_q = -R_a i_q + \omega_r \varphi_d = -R_a i_q + X''_d i_d + E''_q \qquad (2.11)$$

$$e_q = -R_a i_q + \omega_r \varphi_d = -R_a i_q + X''_d i_d + E''_q \qquad (2.12)$$

Where;

$$E''_d = -\omega_r L''_{aq}\left[\frac{\psi_{1q}}{L_{1q}} + \frac{\psi_{2q}}{L_{2q}}\right] \qquad (2.13)$$

$$E''_d = -\omega_r L''_{aq}\left[\frac{\psi_{1q}}{L_{1q}} + \frac{\psi_{2q}}{L_{2q}}\right] \qquad (2.14)$$

If the sub-transient saliency is neglected, $X''_d \approx X''_q = X''$. Hence, Eq. (2.11) & Eq. (2.12) can be arranged in matrix form as:

$$\begin{bmatrix} e_q \\ e_d \end{bmatrix} = \begin{bmatrix} -R_a & -X'' \\ X'' & -R_a \end{bmatrix} \begin{bmatrix} i_q \\ i_d \end{bmatrix} + \begin{bmatrix} E''_q \\ E''_d \end{bmatrix} \tag{2.15}$$

$$\begin{bmatrix} i_q \\ i_d \end{bmatrix} = \frac{1}{d} \begin{bmatrix} -R_a & X'' \\ -X'' & -R_a \end{bmatrix} \begin{bmatrix} e_q \\ e_d \end{bmatrix} + \frac{1}{d} \begin{bmatrix} -R_a & X'' \\ -X'' & -R_a \end{bmatrix} \begin{bmatrix} E''_q \\ E''_d \end{bmatrix} \tag{2.16}$$

Where, $\boldsymbol{d} = {R_a}^{\mathbf{2}} + X''^{\mathbf{2}}$

Thus, the stator voltage and the current of an individual generator is written with respect to its d and q axes. When deriving the mathematical model of a multi - machine power system, these equations are transformed into a common reference frame known as the "R − I" frame shown in Figure 2-4 . The relationship between the quantities in the d − q frame and the R − I frame is given by Eq. (2.17) and Eq. (2.18).

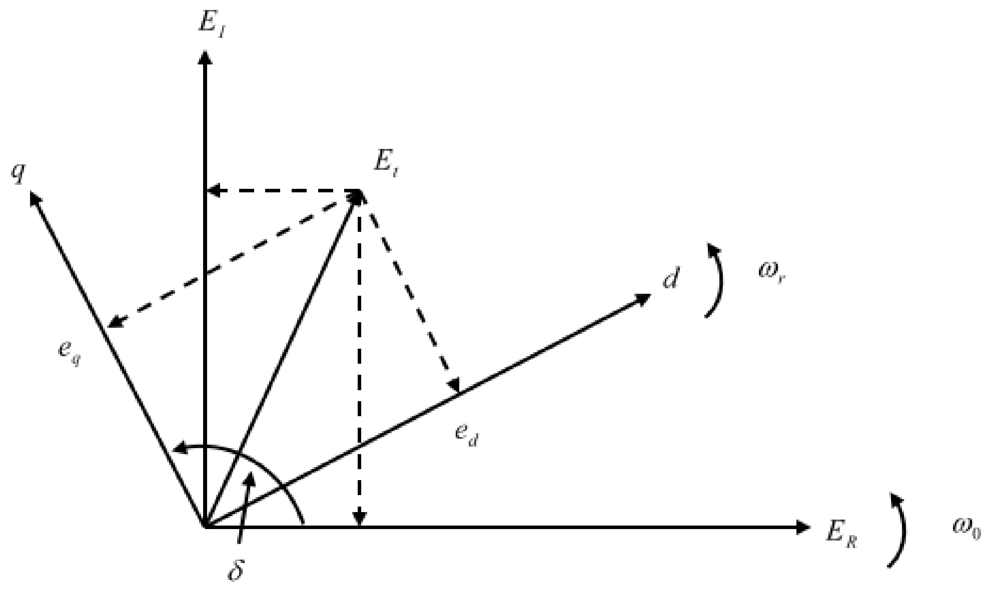

Figure 2-4: Transformation from individual machine d−q frame to common reference frame

$$\begin{bmatrix} e_q \\ e_d \end{bmatrix} = \begin{bmatrix} \cos\delta & \sin\delta \\ \sin\delta & -\cos\delta \end{bmatrix} \begin{bmatrix} E_R \\ E_I \end{bmatrix} \tag{2.17}$$

$$\begin{bmatrix} i_q \\ i_d \end{bmatrix} = \begin{bmatrix} \cos\delta & \sin\delta \\ \sin\delta & -\cos\delta \end{bmatrix} \begin{bmatrix} I_R \\ I_I \end{bmatrix} \tag{2.18}$$

Now, Eq. (2.16) can be transformed into $R - I$ frame as given in Eq. (2.19).

$$\begin{bmatrix} I_R \\ I_I \end{bmatrix} = \frac{\mathbf{1}}{\mathbf{d}} \begin{bmatrix} -R_a & X'' \\ -X'' & -R_a \end{bmatrix} \begin{bmatrix} E_R \\ E_I \end{bmatrix} - \frac{\mathbf{1}}{\mathbf{d}} \begin{bmatrix} \left(-R_a E_q'' + X'' E_d''\right)\cos\delta - \left(X'' E_q'' + R_a E_d''\right)\sin\delta \\ \left(-R_a E_q'' + X'' E_d''\right)\sin\delta + \left(X'' E_q'' + R_a E_d''\right)\cos\delta \end{bmatrix} \tag{2.19}$$

2.1.3 Load model

All the loads were modelled using the constant admittance load model, which allows to include the equivalent admittance of the load into the network admittance matrix. Constant excitation & Constant governor systems are used to simplify the system for analysis.

3 Development of Small Signal Stability Model

The objectives of this chapter is provides the detailed procedure of deriving a small signal stability assessment program.

3.1 Linearization of synchronous machine dynamic equations

The dynamic behaviour of a 6th order synchronous generator is given by the set of first order differential Eq.(2.1) - Eq.(2.6) and the algebraic equations given in Eq.(2.16) and. Eq.(2.18) gives the relationship between the synchronous generator current in the d−q frame and the R−I reference frame. First, linearize Eq. (2.18) around an operating point to derive the following equation.

$$\begin{bmatrix}\Delta i_q\\ \Delta i_d\end{bmatrix} = \begin{bmatrix}(-I_R\sin\delta + I_I\cos\delta)\,\Delta\delta\\ (I_R\cos\delta + I_I\sin\delta)\,\Delta\delta\end{bmatrix} + \begin{bmatrix}\cos\delta & \sin\delta\\ -\sin\delta & -\cos\delta\end{bmatrix}\begin{bmatrix}\Delta I_R\\ \Delta I_I\end{bmatrix} \tag{3.1}$$

Linearizing the synchronous generator equations given in Eq.(2.1) - Eq.(2.6) and substituting from Eq.(3.1), the linearized form of a synchronous generator dynamic equations can be derived as shown in following equations.

$$\begin{aligned}\Delta\dot{\omega} = \; & -\frac{K_D}{2H}\Delta\omega + \frac{1}{2H}\{(\psi_{ad} + L''_{aq}i_d)i_d + (\psi_{aq} + L''_{ad}i_q)i_q\}\Delta\omega - \\ & \frac{1}{2H}\frac{L''_{ads}}{L_{fd}}i_q\Delta\psi_{fd} - \frac{1}{2H}\frac{L''_{ads}}{L_{1d}}i_q\Delta\psi_{1d} + \frac{1}{2H}\frac{L''_{aqs}}{L_{1q}}i_q\Delta\psi_{1q} \\ & + \frac{1}{2H}\frac{L''_{aqs}}{L_{2q}}i_d\Delta\psi_{2q} + \frac{1}{2H}\Delta\bar{T}_m + \\ & + \frac{1}{2H}\{-(\psi_{ad} + L''_{aq}i_d)\cos\delta + (\psi_{aq} + L''_{ad}i_q)\sin\delta\}\Delta I_R \\ & + \frac{1}{2H}\{-(\psi_{ad} + L''_{aq}i_d)\sin\delta \\ & - (\psi_{aq} + L''_{ad}i_q)\cos\delta\}\Delta I_I\end{aligned} \tag{3.2}$$

$$\Delta\dot{\delta} = \omega_0\,\Delta\omega_r \tag{3.3}$$

$$\begin{aligned}\Delta\dot{\psi}_{fd} = \; & -\frac{\omega_0 R_{fd}}{L_{fd}}L''_{ads}i_q\Delta\delta - \frac{\omega_0 R_{fd}}{L_{fd}}\left[\frac{L''_{ads}}{L_{fd}} - 1\right]\Delta\psi_{fd} + \frac{\omega_0 R_{fd}}{L_{fd}}\frac{L''_{ads}}{L_{1d}}\Delta\psi_{1d} \\ & + \frac{\omega_0 R_{fd}}{L_{ads}}\Delta E_{fd} - \frac{\omega_0 R_{fd}}{L_{fd}}L''_{ads}\sin\delta\,\Delta I_R \\ & + \frac{\omega_0 R_{fd}}{L_{fd}}L''_{ads}\cos\delta\,\Delta I_I\end{aligned} \tag{3.4}$$

$$\Delta\dot{\psi}_{1d} = -\frac{\omega_0 R_{1d}}{L_{1d}} L''_{ads} i_q \Delta\delta + \frac{\omega_0 R_{1d}}{L_{1d}} \frac{L''_{ads}}{L_{fd}} \Delta\psi_{fd} + \frac{\omega_0 R_{fd}}{L_{1d}} \left[\frac{L''_{ads}}{L_{1d}} - 1\right] \Delta\psi_{1d} - \frac{\omega_0 R_{1d}}{L_{1d}} L''_{ads} \sin\delta \, \Delta I_R + \frac{\omega_0 R_{1d}}{L_{1d}} L''_{ads} \cos\delta \, \Delta I_I \tag{3.5}$$

$$\Delta\dot{\psi}_{1q} = -\frac{\omega_0 R_{1q}}{L_{1q}} L''_{aqs} i_d \Delta\delta + \frac{\omega_0 R_{1d}}{L_{1q}} \left[\frac{L''_{aqs}}{L_{1q}} - 1\right] \Delta\psi_{1q} + \frac{\omega_0 R_{1q}}{L_{1q}} \frac{L''_{aqs}}{L_{2q}} \Delta\psi_{2q} - \frac{\omega_0 R_{1q}}{L_{1q}} L''_{aqs} \cos\delta \, \Delta I_R - \frac{\omega_0 R_{1q}}{L_{1q}} L''_{aqs} \cos\delta \, \Delta I_I \tag{3.6}$$

$$\Delta\dot{\psi}_{2q} = \frac{\omega_0 R_{2q}}{L_{2q}} L''_{aqs} i_d \Delta\delta + \frac{\omega_0 R_{2d}}{L_{2q}} \frac{L''_{aqs}}{L_{1q}} \Delta\psi_{1q} - \frac{\omega_0 R_{2q}}{L_{2q}} \left[\frac{L''_{aqs}}{L_{2q}} - 1\right] \Delta\psi_{2q} - \frac{\omega_0 R_{2q}}{L_{2q}} L''_{aqs} \cos\delta \, \Delta I_R - \frac{\omega_0 R_{2q}}{L_{2q}} L''_{aqs} \cos\delta \, \Delta I_I \tag{3.7}$$

Eq.(3.1) to Eq.(3.7) give the linearized form of the synchronous generator dynamic equations in the following form.

$$\mathbf{\Delta\dot{X}_1} = \mathbf{A_1 \Delta X_1} + \mathbf{B_1 \Delta U_1} + \mathbf{E_1 \Delta I_1} \tag{3.8}$$

Here $\Delta\dot{X}_1$ is represent the equation for one generator.

$$\mathbf{\Delta X_1} = \begin{bmatrix} \Delta\omega \\ \Delta\delta \\ \Delta\psi_{fd} \\ \Delta\psi_{1d} \\ \Delta\psi_{1q} \\ \Delta\psi_{2q} \end{bmatrix} \qquad \mathbf{A_1} = [\,]_{6x6}$$

This $\mathbf{A_1}$ matrix is 6x6 in size for one generator as the system has 3 generators as showing in Fig.(3-1). Therefore the $\mathbf{A_1}$ Matrix ($\mathbf{A_{sys}}$) will be size of 18x18.

$$\mathbf{A_{sys}} = \begin{bmatrix} A_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & A_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & A_3 \end{bmatrix}$$,where A_1, A_2 & A_3 are 3 generators A matrices

Linearizing Eq.(2.19) around an operating point gives the change in machine current injections in the R − I reference frame.

$$\begin{bmatrix}\Delta I_R\\ \Delta I_I\end{bmatrix} = \frac{1}{d}\begin{bmatrix}-R_a & X''\\ -X'' & -R_a\end{bmatrix}\begin{bmatrix}\Delta E_R\\ \Delta E_I\end{bmatrix} + \begin{bmatrix} m_1\Delta\delta + m_2\Delta\psi_{fd} + m_3\Delta\psi_{1d} + m_4\Delta\psi_{1q} + m_5\Delta\psi_{2q}\\ n_1\Delta\delta + n_2\Delta\psi_{fd} + n_3\Delta\psi_{1d} + n_4\Delta\psi_{1q} + n_5\Delta\psi_{2q}\end{bmatrix} \tag{3.9}$$

Where;

$$m_1 = -\frac{1}{d}\left(R_a E_q'' - X'' E_d''\right)\sin\delta - \left(R_a E_d'' + X'' E_q''\right)\cos\delta$$

$$m_2 = \frac{1}{d}(R_a\cos\delta + X''\sin\delta)\frac{L_{ads}''}{L_{fd}}$$

$$m_3 = \frac{1}{d}(R_a\cos\delta + X''\sin\delta)\frac{L_{ads}''}{L_{1d}}$$

$$m_4 = -\frac{1}{d}(R_a\sin\delta - X''\cos\delta)\frac{L_{aqs}''}{L_{1q}}$$

$$m_5 = -\frac{1}{d}(R_a\sin\delta - X''\cos\delta)\frac{L_{aqs}''}{L_{2q}}$$

$$n_1 = -\frac{1}{d}\left(R_a E_q'' + X'' E_d''\right)\cos\delta - \left(R_a E_d'' + X'' E_q''\right)\sin\delta$$

$$n_2 = \frac{1}{d}(R_a\sin\delta - X''\cos\delta)\frac{L_{ads}''}{L_{fd}}$$

$$n_3 = \frac{1}{d}(R_a\sin\delta - X''\cos\delta)\frac{L_{ads}''}{L_{1d}}$$

$$n_4 = \frac{1}{d}(R_a\cos\delta + X''\sin\delta)\frac{L_{aqs}''}{L_{1q}}$$

$$n_5 = \frac{1}{d}(R_a\cos\delta + X''\sin\delta)\frac{L_{aqs}''}{L_{2q}}$$

Thus, Eq. (3.9) is in the following form.

$$\Delta I = C_1 \Delta X_1 + D_1 \Delta V \tag{3.10}$$

Accordingly, each synchronous generator contributes to the overall state vector of the system by adding six state variables; [$\Delta\omega$, $\Delta\delta$, $\Delta\psi_{fd}$, $\Delta\psi_{1d}$, $\Delta\psi_{1q}$, $\Delta\psi_{2q}$].

Eq. (3.8) and Eq.(3.9) can be combined to determine the individual generator state-space representation in the general form. A_d, B_d, C_d, D_d & E_d are representing single generator.

$$\Delta \dot{X_d} = A_d \Delta X + B_d \Delta U + E_d \Delta I_d \tag{3.11}$$

$$\Delta I_d = C_d \Delta X + D_d \Delta V \tag{3.12}$$

$$\Delta \dot{X_d} = (A_d + E_d C_d) \Delta X_d + B_d \Delta U + E_d D_d \Delta V \tag{3.13}$$

$$\Delta \dot{X_d} = A_m \Delta X_d + B_m \Delta U + E_m \Delta V \tag{3.14}$$

Matrix sizes are as follows,

$$A_m = [\quad]_{6x6}, B_m = [\quad]_{6x2} , E_m = [\quad]_{2x6}$$

When Eq.(3.14) is substituted by Eq.(3.2) to Eq.(3.9), the matrices sizes are as follows, and can be used derive the overall state-space representation of the system.

$$\Delta \dot{X} = A\,\Delta X + B\,\Delta U + E\,\Delta I \tag{3.15}$$

$$\Delta I = C\,\Delta X + D\,\Delta V \tag{3.16}$$

3.2 Overall system state space representation

The stator terminal is the interface between each synchronous generator and the outside transmission network. For small signal stability assessment of the power system, the transmission network is represented as a set of linear equations. The considered system has 3 generators & 4 Buses.

$$\begin{bmatrix} I_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & \cdots & \cdots & Y_{14} \\ Y_{21} & Y_{22} & \cdots & \cdots & \cdots & Y_{24} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ Y_{41} & Y_{42} & \cdots & \cdots & \cdots & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ \vdots \\ \vdots \\ V_4 \end{bmatrix} \qquad (3.17)$$

Therefore Eq.(3.17) can be arranged as follows,

$$\begin{bmatrix} [\Delta I_D] \\ [\Delta I_L] \end{bmatrix} = \begin{bmatrix} [Y_{11}] & [Y_{12}] \\ [Y_{21}] & [Y_{22}] \end{bmatrix} \begin{bmatrix} [\Delta V_D] \\ [\Delta V_L] \end{bmatrix} \qquad (3.18)$$

Where, the subscripts D and L corresponds to generators and loads respectively. Since Model load as constant admittance, $\Delta I_L = 0$, the relationship between the synchronous generator stator voltages and currents can be expressed using the equivalent network admittance matrix as;

$$[\Delta I_D] = [Y_{eq}][\Delta V_D] \qquad (3.19)$$

Where, $[Y_{eq}] = [Y_{11}] - [Y_{12}][Y_{22}]^{-1}[Y_{21}]$

$[Y_{eq}]$ is converted to 6x6 matrix as follows,

$\Delta I_I \,/\, \Delta I_R \longrightarrow$ I = Y V $\longleftarrow \Delta V_I \,/\, \Delta V_R$

Elements in $[Y_{eq}]$ are type of a + jb & lets consider on element.

$$\Delta I_{1I} + j\Delta I_{1R} = (a + jb)(\Delta V_{1I} + \Delta V_{1R}) \longrightarrow \begin{bmatrix} \Delta I_{1I} \\ \Delta I_{1R} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} \Delta V_{1I} \\ \Delta V_{1R} \end{bmatrix}$$

Thus,

$$Y_{eq} = \begin{matrix} a_{11} & -b_{11} & a_{12} & -b_{12} & a_{13} & -b_{13} \\ b_{11} & a_{11} & b_{12} & a_{12} & b_{13} & a_{13} \\ a_{21} & -b_{21} & a_{22} & -b_{22} & a_{23} & -b_{23} \\ b_{21} & a_{21} & b_{22} & a_{22} & b_{23} & a_{23} \\ a_{31} & -b_{31} & a_{32} & -b_{32} & a_{33} & -b_{33} \\ b_{31} & a_{31} & b_{32} & a_{32} & b_{33} & a_{33} \end{matrix}$$

Combining Eq.(3.16) with Eq.(3.19),

$$\Delta V = (Y_{eq} - D)^{-1} C \, \Delta X \qquad (3.20)$$

Substituting Eq.(3.20) in to Eq.(3.15) and Eq.(3.16) derives the overall system state space representation as,

$$\Delta \dot{X} = A_{sys} \Delta X + B \Delta U \qquad (3.21)$$

Where; $A_{sys} = A + E\left(Y_{eq} - D\right)^{-1} C$

3.3 Eigenvalue Analysis

The main aim of small signal stability analysis is to find the variation of properties of operation parameter which are independent from disturbance intensity. Thus eigenvalue analysis gives quantitative information of different stability modes. Eigen value analysis gives efficient algorithms with excellent convergence properties and precious calculation for small signal stability analysis[10].

The system can be analysed by using eigenvalues and eigenvectors with the linearized power system model in matrix from following equations. The input to the system made to zero to inspect the free response of the system.

$$\Delta \mathrm{X} = [\mathrm{A}] \, \mathrm{X} \qquad (3.22)$$

Where x is a state vector and A is the state matrix of size n x n;

Laplace transform is used to analyse the state equation. The derived equation in 's' domain is as follows,(Eq.(3.23))

$$\det(\mathrm{s}[\mathrm{I}] - [\mathrm{A}]) = 0 \qquad (3.23)$$

The values of 's' which are poles of the system are said to be the eigenvalues of the state matrix A.

This eigenvalues could be either real or complex. There are 'n' no of eigenvalues are exist for an n x n matrix. The following Eq.(3.24) show s the complex conjugates of the real eigenvalue of the state matrix A.

$$\lambda = \sigma + j\omega \qquad (3.24)$$

3.4 Eigenvalues and Stability

If eigenvalues of the system is on left side of the imaginary axis of complex plane, the system is stable. If not it is unstable. If any eigenvalue move over right side of complex plane, system is unstable & modes are said to be unstable.

Therefore here all eigenvalue should be lied on left hand plane to be system stable. The time related characteristics of oscillatory modes of eigenvalue λ which is given by $e^{\lambda t}$ is gives the stability of the system.

A non-oscillatory mode is given by the real eigenvalues and decaying mode is given by a negative eigenvalues.

A conjugate pair complex eigenvalues indicate oscillatory modes of response.

$$\lambda = \sigma + j\omega \tag{3.25}$$

1. The said to be stable if conjugate of pair of complex eigenvalues has negative real parts. This corresponds to an oscillatory mode that decays with time.
2. The said to be unstable if a pair has positive real parts. This is corresponding oscillatory mode grows exponentially with time.
3. If any one of the eigenvalues has a real part, the system will have an undamped oscillatory response.

The real component of an eigenvalue gives the damping, and the imaginary part gives the frequency of oscillation.

Frequency Oscillation (Hz):

$$f = \frac{\omega}{2\pi} \tag{3.26}$$

Damping Ratio:

$$\zeta = \frac{-\sigma}{\sqrt{\sigma^2 + \omega^2}} \tag{3.27}$$

3.5 Eigenvectors

For any eigenvalue λ_i ., the column vector ϕ_i that satisfies is called the right eigenvector for λ_i (Eq.(3.28)).

$$[\mathbf{A}]\, \phi_i = \lambda\, \phi_i \tag{3.28}$$

Where i = 1, 2, 3, …… n

$$\phi_i = \begin{bmatrix} \phi_{1i} \\ \phi_{2i} \\ \vdots \\ \vdots \\ \phi_{ni} \end{bmatrix} \tag{3.29}$$

The distribution of the modes of response of the power system is given by the right eigenvector.

Consistently, there is a row vector ψ_i that satisfies Eq.(3.25) and is called the left eigenvector of [A] associated with the eigenvalue λ_i.

$$\psi_i\, [\mathbf{A}] = \lambda\, \psi_i \tag{3.30}$$

$$\psi_i = \begin{bmatrix} \psi_{1i} \\ \psi_{2i} \\ \vdots \\ \vdots \\ \psi_{mi} \end{bmatrix} \tag{3.31}$$

The left eigenvectors, together with the initial conditions of the system state vector x, determine the magnitudes of the modes.

3.6 Mode Shape

Mode shapers are given by right eigenvector. Simply a mode shape gives the relative activity of the state variable when particular mode is excited. Thus the degrees of activity of the K_{th} state variable x_k in the I_{th} mode is given by the element φ_{ki} if the right eigenvector φ_i.

The magnitude of the elements if φ_i gives the extent of the activities of the n state variables in the I^{th} mode, and the angles of the elements give phase displacement of the state variables with regard to the mode. Thus mode shape can be used to analyse the magnitude and phase displacement of the speed and rotor angle state variables in an oscillatory mode[11].

3.7 Participation Factors

The participation factor is non-dimensional which gives most influenced mode from the identified states.

$$P_i = \begin{bmatrix} P_{1i} \\ P_{2i} \\ \vdots \\ \vdots \\ P_{ni} \end{bmatrix} = \begin{bmatrix} \phi_{1i}\psi_{1i} \\ \phi_{2i}\psi_{2i} \\ \vdots \\ \vdots \\ \phi_{ni}\psi_{mi} \end{bmatrix} \tag{3.32}$$

The element $P_{ki} = \psi_{ki}\ \phi_{ik}$ is called a participation factor.

Simply, the Participation factor P_{ki} gives the measure of relative participation of k^{th} state variable. This will be allowed to identify most influenced mode related to the state variable.

The highest value of participation factor of pair of eigenvalues (mode) related state variable is the more active state in the system at the moment. That mode give the oscillation frequency & damping ratio of the oscillation [3].

3.8 System Model – 900MW Lakvijaya Power Station

3.8.1 General outline

Lakvijaya power station is consist of 3 x 353MVA identical coal power plant which are contributing 3 x 300MW rated power to the SriLankan grid. The generated power is connected to the national grid from New-Chilaw substation and New-Anuradhapura substation via 220kV double circuit transmission lines. Following Figure 3-1 shows the bus arrangement of the plant.

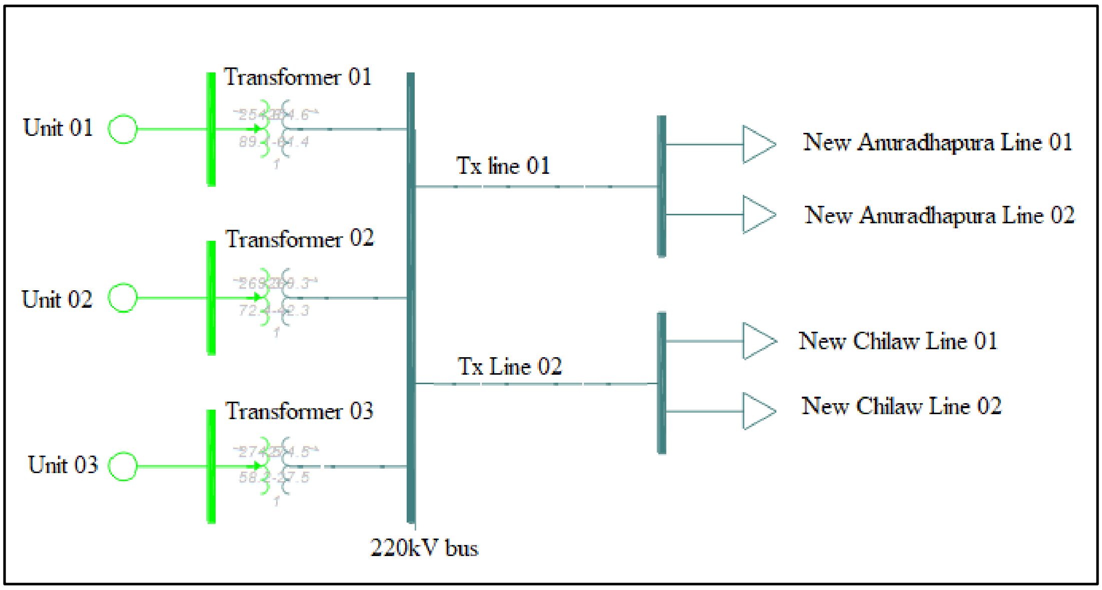

Figure 3-1: Lakvijaya Power Station

The considered network for calculation is shown in Figure 3-2.

3.8.2 Network

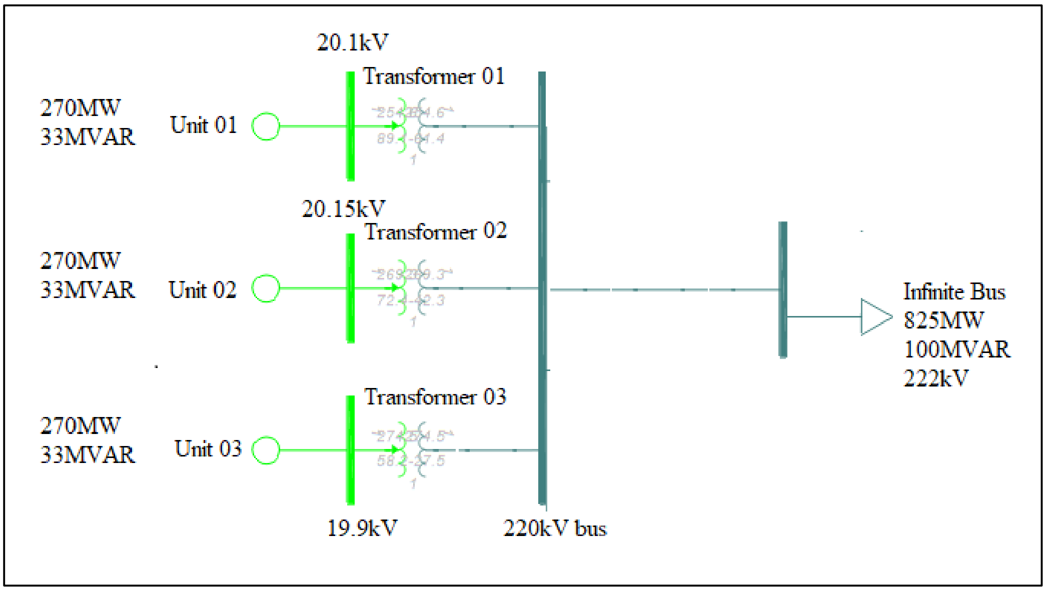

Figure 3-2: Network configuration

Regarding the modelling of the generators' dynamic characteristics in MATLAB there are a number of parameters that must be inputted. These include,

- Machine inertia
- Machine sub-transient reactance
- Machine armature resistance

All generator data can be found Appendix A.

The power system steady state and dynamic model represent in Eq.(2.1) to Eq.(3.32) were calculated using MATLAB. The results are as follows,

The A_1 as stated in equation E24 as follows,

The admittance matrix size of 4x4 as in Eq.(3.17) as follows,

$$Y = \begin{bmatrix} -2.3364i & 0 & 0 & 7.1428i \\ 0 & -2.3364i & 0 & 7.1428i \\ 0 & 0 & -2.3364i & 7.1428i \\ 7.1428i & 7.1428i & 7.1428i & 2.3809i \end{bmatrix} \tag{3.33}$$

The A_{sys} of overall system state space representation Eq.(3.21) as follows,

-0.17139	-218.086	-0.46419	-0.70513	0.015703	0.022329	0	-8.55647	-0.0001	-0.00016	0.001467	0.010845	0	-8.50198	-0.0001	-0.00016	0.001467	0.010845
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1.59682	-1.62289	1.574721	0.000271	0.002007	0	-0.7402	-9E-06	-1.4E-05	0.000127	0.000938	0	-0.73549	-9E-06	-1.4E-05	0.000127	0.000938
0	-32.5928	-21.16	-22.143	0.005541	0.040957	0	-15.1088	-0.00018	-0.00028	0.002591	0.019151	0	-15.0126	-0.00018	-0.00028	0.002591	0.019151
0	-0.01246	-2.4E-05	-3.6E-05	-0.07487	-47.013	0	-0.00048	-5.8E-09	-8.8E-09	8.19E-08	6.05E-07	0	-0.00047	-5.8E-09	-8.8E-09	8.19E-08	6.05E-07
0	-13.8839	-0.02626	-0.03989	11.07024	-12.6583	0	-0.53212	-6.5E-06	-9.8E-06	9.12E-05	0.000674	0	-0.52873	-6.5E-06	-9.8E-06	9.12E-05	0.000674
0	-8.15195	-0.0001	-0.00016	0.001379	0.010194	-0.17139	-200.917	-0.45549	-0.69191	0.014734	0.020938	0	-7.99167	-9.8E-05	-0.00015	0.001379	0.010195
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	-0.75026	-9.5E-06	-1.4E-05	0.000127	0.000938	0	-1.57626	-1.62289	1.574721	0.000271	0.002007	0	-0.73551	-9E-06	-1.4E-05	0.000127	0.000938
0	-15.314	-0.00019	-0.00029	0.002591	0.019151	0	-32.1736	-21.16	-22.143	0.005541	0.040957	0	-15.0129	-0.00018	-0.00028	0.002591	0.019151
0	-0.00046	-5.8E-09	-8.8E-09	7.8E-08	5.77E-07	0	-0.01164	-2.4E-05	-3.6E-05	-0.07487	-47.013	0	-0.00045	-5.5E-09	-8.4E-09	7.8E-08	5.77E-07
0	-0.51388	-6.5E-06	-9.8E-06	8.69E-05	0.000643	0	-12.965	-0.02626	-0.03989	11.07024	-12.6583	0	-0.50378	-6.2E-06	-9.3E-06	8.69E-05	0.000643
0	-8.06899	-0.0001	-0.00015	0.001365	0.010091	0	-7.96104	-9.7E-05	-0.00015	0.001365	0.010091	-0.17139	-197.499	-0.45239	-0.68721	0.014582	0.02072
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	-0.75026	-9.5E-06	-1.4E-05	0.000127	0.000938	0	-0.74022	-9E-06	-1.4E-05	0.000127	0.000938	0	-1.56628	-1.62289	1.574721	0.000271	0.002007
0	-15.3141	-0.00019	-0.00029	0.002591	0.019151	0	-15.1092	-0.00018	-0.00028	0.002591	0.019151	0	-31.97	-21.16	-22.143	0.005541	0.040957
0	-0.00046	-5.8E-09	-8.8E-09	7.77E-08	5.75E-07	0	-0.00045	-5.5E-09	-8.4E-09	7.77E-08	5.75E-07	0	-0.01151	-2.4E-05	-3.6E-05	-0.07487	-47.013
0	-0.51191	-6.5E-06	-9.8E-06	8.66E-05	0.00064	0	-0.50506	-6.2E-06	-9.3E-06	8.66E-05	0.00064	0	-12.8262	-0.02626	-0.03989	11.07024	-12.6583

Figure 3-3: A_{sys} of overall system state space representation

The eigenvalues for the system A_{sys} from the matrix in Figure 3-4 is shown in follows,

```
 -0.11509 +    14.975i
 -0.11509 -    14.975i
-0.098303 +    14.118i
-0.098303 -    14.118i
-0.095076 +    13.803i
-0.095076 -    13.803i
   -20.29 +         0i
  -20.343 +         0i
  -20.343 +         0i
  -6.3647 +    21.929i
  -6.3647 -    21.929i
  -6.3664 +    21.929i
  -6.3664 -    21.929i
  -6.3664 +    21.929i
  -6.3664 -    21.929i
  -3.4138 +         0i
  -3.4047 +         0i
  -3.4048 +         0i
```

Figure 3-4: Eigenvalues for the system

There are six couple eigenvalues in the system as showing in figure 3-4 and all the eigenvalues are in the left side of the coordinate system as showing in figure 3-5.

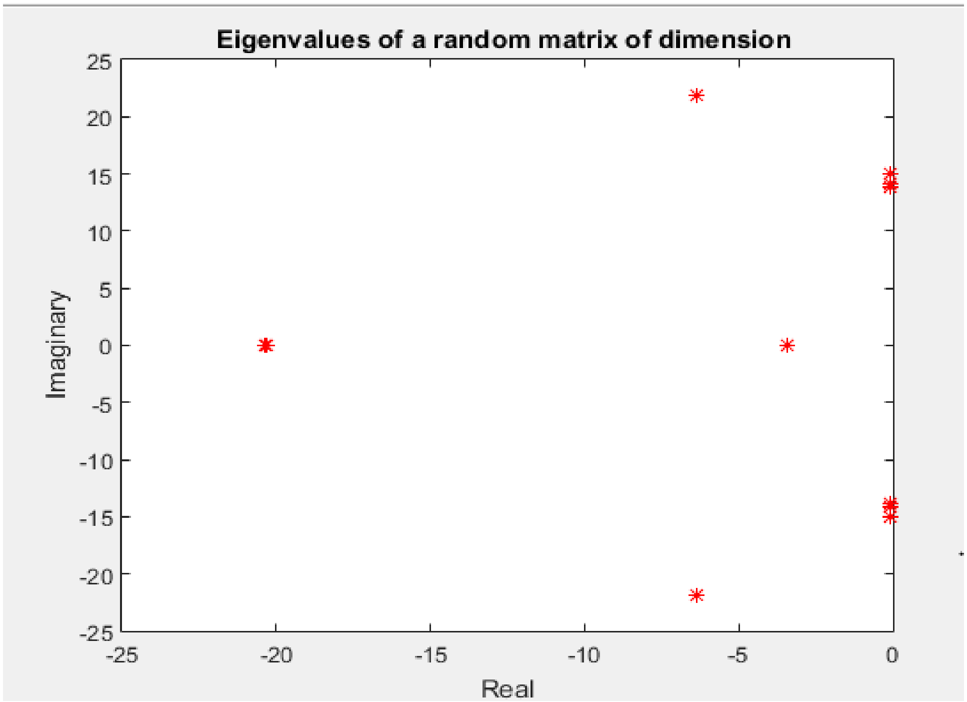

Figure 3-5: Eigenvalues

Summary of Oscillation modes of the system are as follows,

	Eigenvalues	Oscillation frequency Hz	Oscillation modes
λ_1, λ_2	- 0.11509 ± 14.975i	2.3833	Mode 1
λ_3, λ_4	- 0.098303 ± 14.118i	2.2469	Mode 2
λ_5, λ_6	- 0.095076 ± 13.803i	2.2011	Mode 3
λ_7	20.29 + 0i	-	-
λ_8	20.343 + 0i	-	-
λ_9	20.343+ 0i	-	-
$\lambda_{10}, \lambda_{11}$	- 6.3647 ± 21.929i	3.4901	Mode 4
$\lambda_{12}, \lambda_{13}$	- 6.3664 ± 21.929i	3.4901	Mode 5
$\lambda_{14}, \lambda_{15}$	- 6.3664 ± 21.929i	3.4901	Mode 6
λ_{16}	-19.114 + 0i	-	-
λ_{17}	-19.114 + 0i	-	-
λ_{18}	-19.114 + 0i	-	-

Table 3-1 : Summary of the system

Here the Table 3-1 shows,

There are six oscillation modes in total and is stable after small disturbance because the real parts of all the eigenvalues are negative.

Following figure shows the participation factors of the system,

0.00017655+0.046959i	0.00017655-0.046959i	-0.00012365-0.019139i	-0.00012365+0.019139i	-1.9653e-06-0.00028491i	-1.9653e-06+0.00028491i	-0.00028014	-0.00023455	-2.64E-05	7.0755e-06+1.9118e-05i	7.0755e-06-1.9118e-05i	-1.8238e-07-1.553e-05i	-1.8238e-07+1.553e-05i	1.9519e-09+7.7079e-08i	1.9519e-09-7.7079e-08i	3.38E-05	2.47E-05	1.94E-08
-0.00036087-0.046957i	-0.00036087+0.046957i	0.00015783+0.019138i	0.00015783-0.019138i	2.3526e-06+0.0002849i	2.3526e-06-0.0002849i	-0.00027778	-0.00023257	-2.62E-05	-1.6088e-05-1.245e-05i	-1.6088e-05+1.245e-05i	8.3588e-06+1.3052e-05i	8.3588e-06-1.3052e-05i	-4.2374e-08-6.4225e-08i	-4.2374e-08+6.4225e-08i	3.21E-05	2.34E-05	1.85E-08
-7.1363e-06-2.0915e-05i	-7.1363e-06+2.0915e-05i	-1.8762e-07-5.7778e-08i	-1.8762e-07+5.7778e-08i	-2.7044e-08-5.5957e-08i	-2.7044e-08+5.5957e-08i	-0.01236	-0.034774	-0.0039466	-7.3368e-08-5.7299e-08i	-7.3368e-08+5.7299e-08i	3.223e-08+2.4378e-08i	3.223e-08-2.4378e-08i	-1.523e-10-1.0808e-10i	-1.523e-10+1.0808e-10i	0.20513	0.45228	0.00034526
0.00015219+2.0265e-05i	0.00015219-2.0265e-05i	9.1974e-07-8.7527e-07i	9.1974e-07+8.7527e-07i	3.895e-07+3.1226e-08i	3.895e-07-3.1226e-08i	0.12928	0.36576	0.041511	8.3004e-07-5.7464e-07i	8.3004e-07+5.7464e-07i	-3.5643e-07+2.5498e-07i	-3.5643e-07-2.5498e-07i	1.6107e-09-1.2278e-09i	1.6107e-09+1.2278e-09i	-0.019878	-0.04318	-3.30E-05
9.253e-06-4.6191e-06i	9.253e-06+4.6191e-06i	-8.563e-07+1.1116e-06i	-8.563e-07-1.1116e-06i	-1.7735e-10+1.3333e-08i	-1.7735e-10-1.3333e-08i	2.37E-07	2.02E-07	2.27E-08	0.031585-0.1101i	0.031585+0.1101i	-0.053972+0.18809i	-0.053972-0.18809i	0.00023834-0.00097452i	0.00023834+0.00097452i	6.27E-09	3.96E-09	3.12E-12
-2.4012e-06+4.4182e-06i	-2.4012e-06-4.4182e-06i	-4.5507e-08-1.0785e-06i	-4.5507e-08+1.0785e-06i	-4.3841e-09-1.3037e-08i	-4.3841e-09+1.3037e-08i	-2.59E-07	-2.21E-07	-2.49E-08	-0.031577+0.11009i	-0.031577-0.11009i	0.053964-0.18809i	0.053964+0.18809i	-0.0002383+0.0009745i	-0.0002383-0.0009745i	-2.79E-10	-1.75E-10	-1.38E-13
2.4444e-05+0.010049i	2.4444e-05-0.010049i	-0.00018053-0.034872i	-0.00018053+0.034872i	-0.00013922-0.024721i	-0.00013922+0.024721i	-0.00028605	-0.00015317	-0.00011543	5.3207e-06+1.4088e-05i	5.3207e-06-1.4088e-05i	-7.7274e-08-5.16e-06i	-7.7274e-08+5.16e-06i	2.6586e-07+9.6435e-06i	2.6586e-07-9.6435e-06i	4.01E-05	6.71E-06	1.96E-05
-6.3889e-05-0.010049i	-6.3889e-05+0.010049i	0.00024281+0.034871i	0.00024281-0.034871i	0.00017283+0.02472i	0.00017283-0.02472i	-0.00028364	-0.00015188	-0.00011446	-1.1946e-05-9.1181e-06i	-1.1946e-05+9.1181e-06i	2.7914e-06+4.3277e-06i	2.7914e-06-4.3277e-06i	-5.3198e-06-8.0239e-06i	-5.3198e-06+8.0239e-06i	3.81E-05	6.38E-06	1.86E-05
-2.7289e-06-7.6516e-06i	-2.7289e-06+7.6516e-06i	4.7933e-06+1.1851e-05i	4.7933e-06-1.1851e-05i	1.5793e-06+3.8917e-06i	1.5793e-06-3.8917e-06i	-0.012266	-0.02209	-0.01677	-6.5955e-08-4.8392e-08i	-6.5955e-08+4.8392e-08i	1.3838e-08+1.011e-08i	1.3838e-08-1.011e-08i	-2.4764e-08-1.7009e-08i	-2.4764e-08+1.7009e-08i	0.22367	0.11353	0.32043
5.6045e-05+6.6795e-06i	5.6045e-05-6.6795e-06i	-8.1993e-05-1.0438e-05i	-8.1993e-05+1.0438e-05i	-2.6129e-05-3.8067e-06i	-2.6129e-05+3.8067e-06i	0.1283	0.23234	0.1764	7.139e-07-5.2665e-07i	7.139e-07+5.2665e-07i	-1.4935e-07+1.1063e-07i	-1.4935e-07-1.1063e-07i	2.5604e-07-2.0148e-07i	2.5604e-07+2.0148e-07i	-0.021675	-0.010839	-0.030598
2.9403e-06-1.1157e-06i	2.9403e-06+1.1157e-06i	-4.3174e-06+2.3802e-06i	-4.3174e-06-2.3802e-06i	-1.8408e-06+1.3882e-06i	-1.8408e-06-1.3882e-06i	2.46E-07	1.33E-07	1.01E-07	0.029192-0.090287i	0.029192+0.090287i	-0.023143+0.071563i	-0.023143-0.071563i	0.039242-0.14004i	0.039242+0.14004i	5.63E-09	8.08E-10	2.36E-09
-8.8401e-07+1.0699e-06i	-8.8401e-07-1.0699e-06i	8.6143e-07-2.3026e-06i	8.6143e-07+2.3026e-06i	2.1729e-07-1.3493e-06i	2.1729e-07+1.3493e-06i	-2.69E-07	-1.47E-07	-1.11E-07	-0.029186+0.090283i	-0.029186-0.090283i	0.02314-0.071562i	0.02314+0.071562i	-0.039237+0.14004i	-0.039237-0.14004i	-2.50E-10	-3.58E-11	-1.04E-10
1.8877e-05+0.0081217i	1.8877e-05-0.0081217i	-7.9745e-05-0.015721i	-7.9745e-05+0.015721i	-0.00025694-0.046474i	-0.00025694+0.046474i	-0.00028372	-1.05E-05	-0.00026022	5.0682e-06+1.3399e-05i	5.0682e-06-1.3399e-05i	-4.697e-08-3.0843e-06i	-4.697e-08+3.0843e-06i	3.1936e-07+1.1512e-05i	3.1936e-07-1.1512e-05i	4.10E-05	5.64E-06	2.10E-05
-5.0757e-05-0.0081213i	-5.0757e-05+0.0081213i	0.00010782+0.015721i	0.00010782-0.015721i	0.00032011+0.046473i	0.00032011-0.046473i	-0.00028132	-1.04E-05	-0.00025802	-1.1368e-05-8.6679e-06i	-1.1368e-05+8.6679e-06i	1.6692e-06+2.5864e-06i	1.6692e-06-2.5864e-06i	-6.352e-06-9.5772e-06i	-6.352e-06+9.5772e-06i	3.90E-05	5.35E-06	2.00E-05
-2.3953e-06-6.6835e-06i	-2.3953e-06+6.6835e-06i	2.6306e-06+6.4343e-06i	2.6306e-06-6.4343e-06i	4.4879e-06+1.0696e-05i	4.4879e-06-1.0696e-05i	-0.012098	-0.0015089	-0.037598	-6.483e-08-4.6977e-08i	-6.483e-08+4.6977e-08i	8.6277e-09+6.2627e-09i	8.6277e-09-6.2627e-09i	-3.0868e-08-2.1071e-08i	-3.0868e-08+2.1071e-08i	0.225	0.093744	0.33888
4.899e-05+5.7617e-06i	4.899e-05-5.7617e-06i	-4.4606e-05-5.5085e-06i	-4.4606e-05+5.5085e-06i	-7.2282e-05-9.6637e-06i	-7.2282e-05+9.6637e-06i	0.12655	0.015871	0.39546	6.9561e-07-5.1957e-07i	6.9561e-07+5.1957e-07i	-9.269e-08+6.9106e-08i	-9.269e-08-6.9106e-08i	3.178e-07-2.5155e-07i	3.178e-07+2.5155e-07i	-0.021803	-0.0089499	-0.03236
2.5534e-06-9.2932e-07i	2.5534e-06+9.2932e-07i	-2.2163e-06+1.1134e-06i	-2.2163e-06-1.1134e-06i	-4.1917e-06+2.7168e-06i	-4.1917e-06-2.7168e-06i	2.46E-07	9.24E-09	2.29E-07	0.028946-0.087656i	0.028946+0.087656i	-0.014455+0.043766i	-0.014455-0.043766i	0.049099-0.17111i	0.049099+0.17111i	5.53E-09	6.49E-10	2.43E-09
-7.8126e-07+8.9146e-07i	-7.8126e-07-8.9146e-07i	4.7885e-07-1.0765e-06i	4.7885e-07+1.0765e-06i	6.428e-07-2.638e-06i	6.428e-07+2.638e-06i	-2.69E-07	-1.02E-08	-2.51E-07	-0.02894+0.087652i	-0.02894-0.087652i	0.014454-0.043765i	0.014454+0.043765i	-0.049093+0.17111i	-0.049093-0.17111i	-2.45E-10	-2.88E-11	-1.07E-10

Figure 3-6: Matrix of Participation Factors

3.9 Sensitivity Analysis

Participation factors of the system are used to identify the sensitivity of the eigenvalue to the corresponding element in the state vectors.

Following Table 3-2 shows the normalized participation factors related to all state vectors.

	λ_1, λ_2	λ_3, λ_4	λ_5, λ_6	$\lambda_{10}, \lambda_{11}$	$\lambda_{12}, \lambda_{13}$	$\lambda_{14}, \lambda_{15}$
$\Delta\dot{\omega}_1$	**1.0000000**	0.5488427	0.0061305	0.0001780	0.0000794	0.0000004
$\Delta\dot{\delta}_1$	0.9999775	0.5488367	0.0061305	0.0001776	0.0000792	0.0000004
$\Delta\dot{\psi}_{fd,1}$	0.0004706	0.0000056	0.0000013	0.0000008	0.0000002	0.0000000
$\Delta\dot{\psi}_{1d,1}$	0.0032696	0.0000364	0.0000084	0.0000088	0.0000022	0.0000000
$\Delta\dot{\psi}_{1q,1}$	0.0002202	0.0000402	0.0000003	**1.0000000**	**1.0000000**	0.0056357
$\Delta\dot{\psi}_{2q,1}$	0.0001071	0.0000310	0.0000003	0.9999333	0.9999785	0.0056356
$\Delta\dot{\omega}_2$	0.2140007	**1.0000000**	0.5319298	0.0001315	0.0000264	0.0000542
$\Delta\dot{\delta}_2$	0.2139959	0.9999892	0.5319253	0.0001312	0.0000263	0.0000541
$\Delta\dot{\psi}_{fd,2}$	0.0001730	0.0003666	0.0000904	0.0000007	0.0000001	0.0000002
$\Delta\dot{\psi}_{1d,2}$	0.0012019	0.0023702	0.0005682	0.0000077	0.0000009	0.0000018
$\Delta\dot{\psi}_{1q,2}$	0.0000670	0.0001414	0.0000496	0.8284530	0.3843606	0.8169821
$\Delta\dot{\psi}_{2q,2}$	0.0000296	0.0000705	0.0000294	0.8283990	0.3843529	0.8169672
$\Delta\dot{\omega}_3$	0.1729524	0.4508324	**1.0000000**	0.0001251	0.0000158	0.0000647
$\Delta\dot{\delta}_3$	0.1729485	0.4508275	0.9999916	0.0001248	0.0000157	0.0000646
$\Delta\dot{\psi}_{fd,3}$	0.0001512	0.0001993	0.0002496	0.0000007	0.0000001	0.0000002
$\Delta\dot{\psi}_{1d,3}$	0.0010504	0.0012888	0.0015691	0.0000076	0.0000006	0.0000023
$\Delta\dot{\psi}_{1q,3}$	0.0000579	0.0000711	0.0001075	0.8059504	0.2355436	**1.0000000**
$\Delta\dot{\psi}_{2q,3}$	0.0000252	0.0000338	0.0000584	0.8058980	0.2355390	0.9999821

Table 3-2: Normalized Participation factors at full load

Here participation factors measure the relation between oscillation modes and the state variables. The 'BOLD' participation factors mean that the oscillation modes are highly sensitive to the corresponding state variables.

There (λ_1, $\lambda_{2)}$, (λ_3, λ_4), & (λ_5, λ_6) are the identified as modes in the system which are directly connect with rotor.

4 Validation of small signal stability model

The complete system is developed in MATLAB and simulated with the PSCAD software. The system is model with PSCAD as showing in following Figure 4-1.

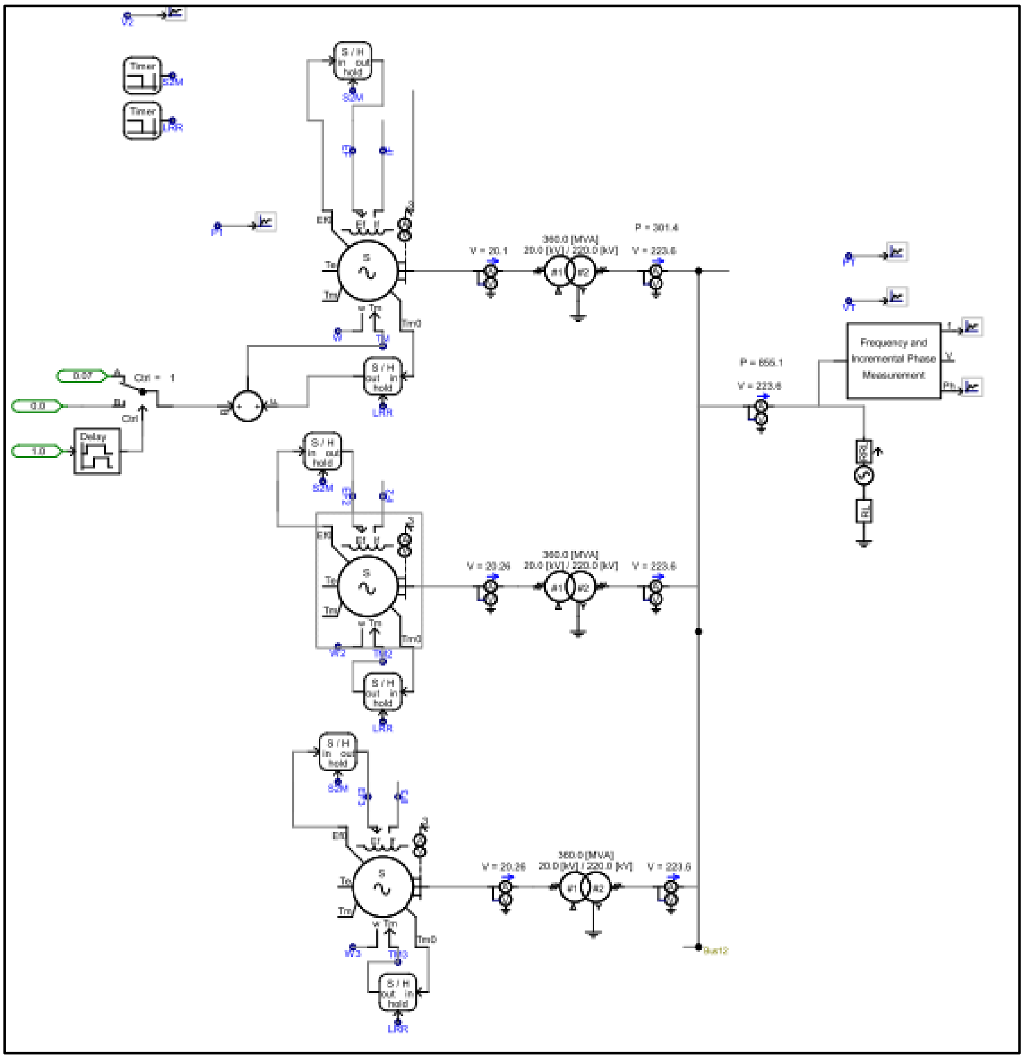

Figure 4-1: PSCAD model of the system

A voltage source was connected to 220kV bus to represent the infinite bus. When system was simulated as shown as above with voltage source configured with the relevant data. The Synchronous machines were generated equal 276MW as showing in following figures (Figure 4-5, Figure 4-3 and Figure 4-4).

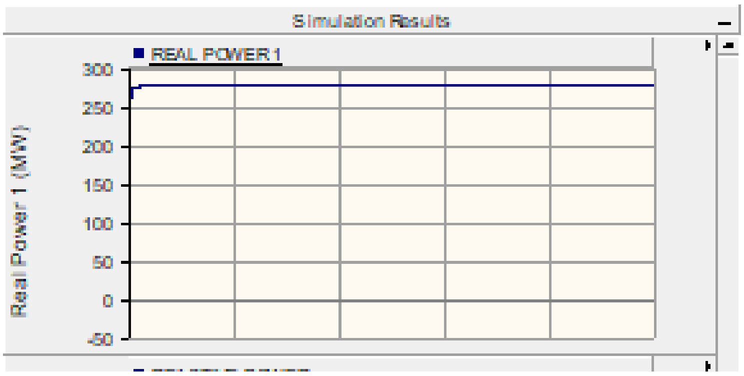

Figure 4-2: Unit 01 Real power vs time

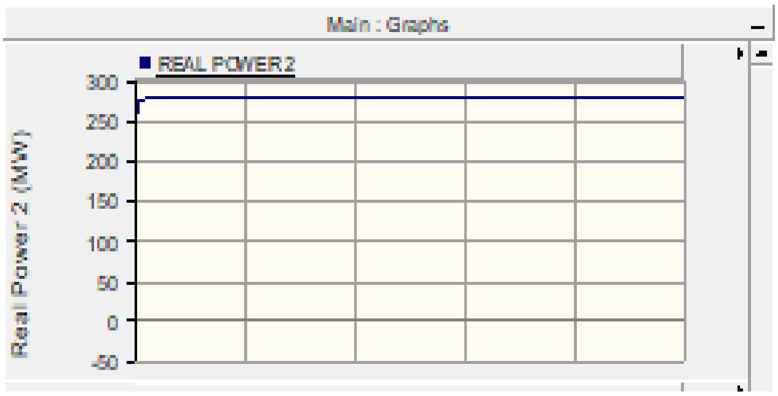

Figure 4-3: Unit 02 Real power vs time

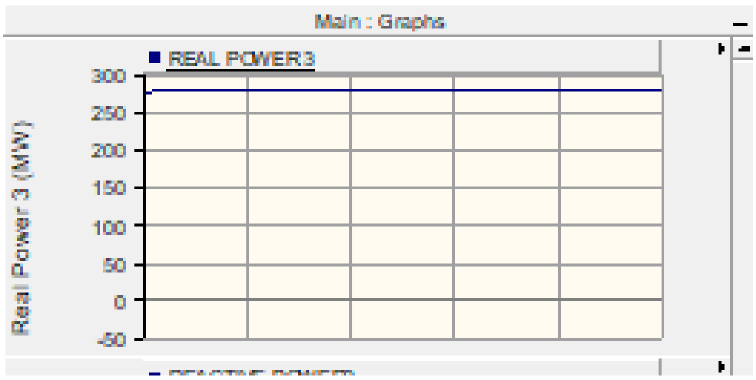

Figure 4-4 : Unit 03 Real power vs time

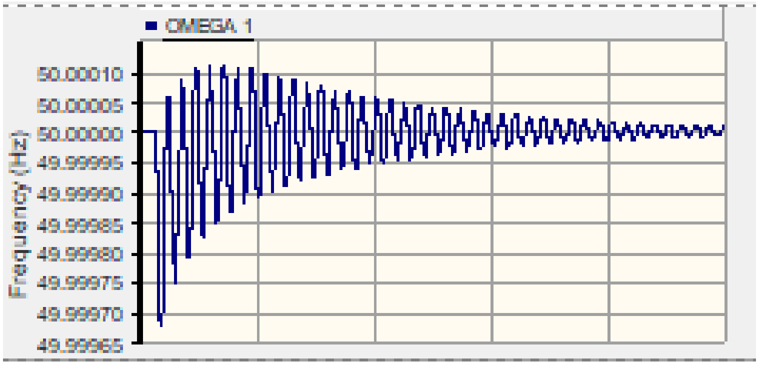

Figure 4-6: Unit 01 Frequency vs time

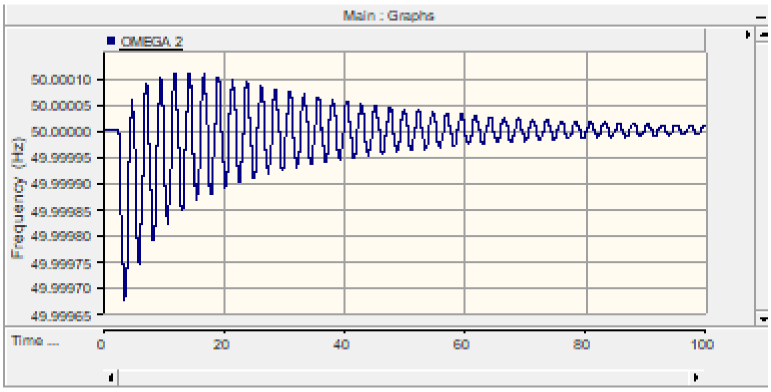

Figure 4-7: Unit 02 Frequency vs time

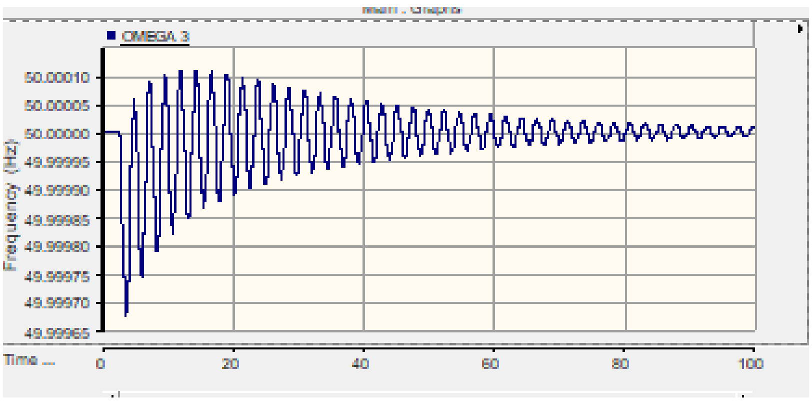

Figure 4-8: Unit 03 Frequency vs time

A 10% step disturbance was given to the generator 01 (Unit 01) mechanical torque at 25th second in system in PSCAD simulation. System is responded as following figure 4-9.

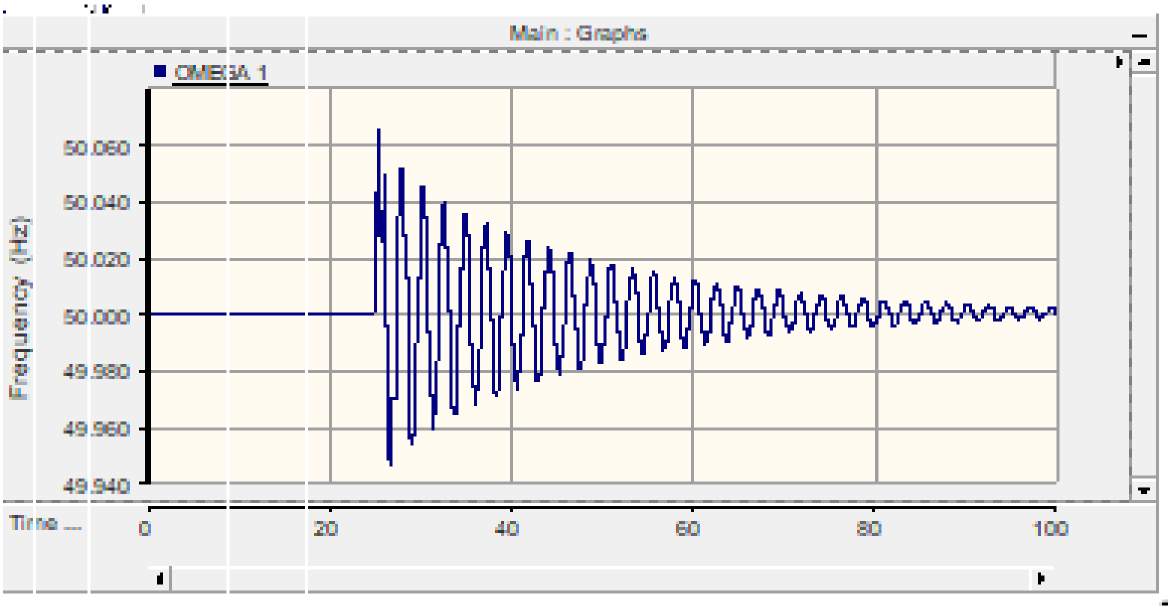

Figure 4-9 : Unit 01 frequency response when disturbance is applied

Frequency of the system = (76.0442 – 73.7964)Hz = 2.2478Hz

This frequeny is much similar to the calculated oscilation frquecny. It can say that PSCAD model is match with the develped system in the MATLAB.

5 CONCLUSIONS AND FUTURE DIRECTIONS

This thesis presented a dynamic model for the Lakvijaya Power Station was developed to investigate the development of low frequency oscillations in the Power Plant. The Power Plant model was enveloped in MATLAB and testing was done to determine the system stability. This simulation was done with reference to the load flow occurred on the 28th May, 2017 as the day-time peak demand. Two different actual scenarios (When plant is at full load & minimum load) were simulated in this model to validate the MATLAB code with PSCAD model.

The system developed in the MATLAB software performed load flow of power plant, calculation of the initial values of the power plant model and the construction of the system state matrix. The Eigenvalues of A_{sys}, Participation factors, and mode shape were calculated from the MATLAB code. No of Oscillation modes were identified from the Eigenvalues. The relation between the oscillation modes and state variables were identified from the participation factors.

Here the infinite bus is considered at main bus end at the power station for simplify the analysis. If the infinite bus consideration can be taken from another places like Chilaw end, Anuradhapura end or somewhere in 132kV line, extensive details about the oscillations can be obtained.

The oscillations related to excitation system can be obtained when excitation parameters are included to the system matrix. But system matrix size (A_{sys}) will be increased & calculation will be more complex.

6 Reference

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7 Appendix

7.1 Appendix A - Generator Data

Main Capability Data

DC resistance of Stator Winding Phase (75^0C)	0.000228 Ω
Induction of Rotor Winding	0.87 H
DC resistance of Rotor Winding (75^0C)	0.125 Ω
Direct Axis Synchronous Reactance X_d	183.6 %
Quadrature Axis Synchronous Reactance X_q	179 %
Direct Axis Transient Reactance X_d’	20 %
Quadrature Axis Transient Reactance X_q’	33.3 %
Direct Axis Sub-transient Reactance X_d”	15.5 %
Quadrature Axis Sub-transient Reactance X_q”	15.2 %
Positive phase resistance	0.323 %
Negative phase resistance	2.622 %
Zero phase resistance	0.262 %
Stator winding leakage reactance X_e	12.4 %
Rotor winding leakage reactance X_f	11 %
Number of coil in a stator slot	2
Number of windings per phase	9
Over Load Capacity	1.87 %

Table 7-1 : Main Capability Data

7.2 Appendix B - Case Study

When all plants are running at minimum load

When all units are running at minimum load (160MW). The PSCAD model outputs are showing in bellow figures.(Figure 7-3, Figure 7-4 and Figure 7-5)

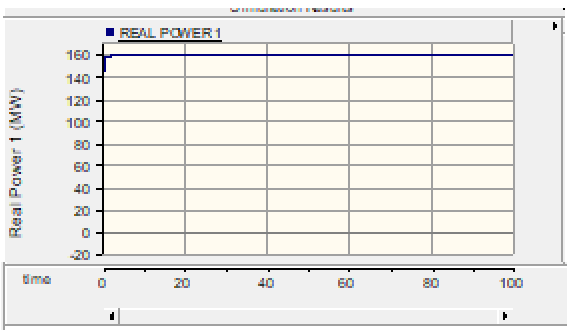

Figure 7-1: Unit 01 Power vs time

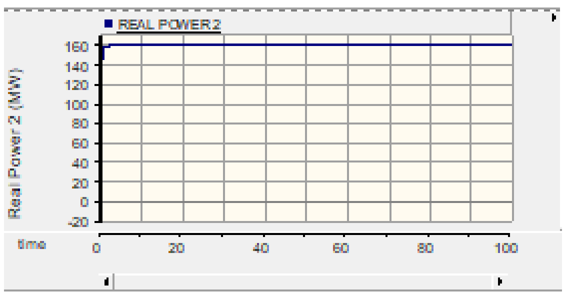

Figure 7-2: Unit 02 Power vs time

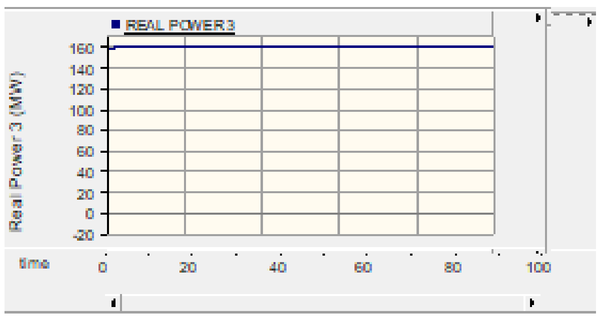

Figure 7-3: Unit 03 Power vs time

Eigenvalues of the system are as follows,

```
-0.11346 +    10.637i
-0.11346 -    10.637i
-0.094343 +   10.091i
-0.094343 -   10.091i
-0.094594 +    9.776i
-0.094594 -    9.776i
-20.286 +          0i
-20.343 +          0i
-20.343 +          0i
-6.366 +      21.929i
-6.366 -      21.929i
-6.3667 +     21.928i
-6.3667 -     21.928i
-6.3667 +     21.928i
-6.3667 -     21.928i
-3.422 +           0i
-3.4071 +          0i
-3.4073 +          0i
```

Figure 7-4: Eigenvalues of the system at 160MW

All the eigenvalues in figure 7-6 are located in the left side of the coordinated system and are in stable mode.

Summary of Oscillation modes of the system are as follows,

	Eigenvalues	Oscillation frequency Hz	Oscillation modes
λ_1, λ_2	- 0.11346 ± 10.637i	1.651	Mode 1
λ_3, λ_4	- 0.094343 ± 10.091i	1.605	Mode 2
λ_5, λ_6	- 0.094594± 9.776i	1.55	Mode 3
$\lambda_{10}, \lambda_{11}$	- 6.366 ± 21.929i	3.4901	Mode 4
$\lambda_{12}, \lambda_{13}$	- 6.3667 ± 21.929i	3.4901	Mode 5
$\lambda_{14}, \lambda_{15}$	- 6.3667 ± 21.929i	3.4901	Mode 6

Normalized participation factors of the system is shown in Table 7-1.

	λ_1, λ_2	λ_3, λ_4	λ_5, λ_6	$\lambda_{10}, \lambda_{11}$	$\lambda_{12}, \lambda_{13}$	$\lambda_{14}, \lambda_{15}$
$\Delta\dot{\omega}_1$	**1.0000E+00**	6.5473E-01	4.4257E-02	3.5685E-05	1.6043E-05	1.6905E-06
$\Delta\dot{\delta}_1$	9.9996E-01	6.5472E-01	4.4257E-02	3.5612E-05	1.6010E-05	1.6870E-06
$\Delta\dot{\psi}_{fd,1}$	1.1525E-03	1.8373E-04	1.4381E-05	6.2689E-07	1.6729E-07	1.7280E-08
$\Delta\dot{\psi}_{1d,1}$	5.1360E-03	7.6173E-04	5.7005E-05	6.7985E-06	1.8142E-06	1.8740E-07
$\Delta\dot{\psi}_{1q,1}$	6.1558E-05	1.6333E-05	4.8683E-07	**1.0000E+00**	**1.0000E+00**	1.0657E-01
$\Delta\dot{\psi}_{2q,1}$	2.0621E-05	8.4780E-06	4.5516E-07	9.9999E-01	1.0000E+00	1.0657E-01
$\Delta\dot{\omega}_2$	5.2195E-01	**1.0000E+00**	1.2696E-01	3.2924E-05	1.0461E-05	5.9554E-06
$\Delta\dot{\delta}_2$	5.2193E-01	9.9999E-01	1.2695E-01	3.2856E-05	1.0440E-05	5.9430E-06
$\Delta\dot{\psi}_{fd,2}$	7.4904E-04	5.9030E-04	2.2915E-06	6.1238E-07	1.1652E-07	6.5031E-08
$\Delta\dot{\psi}_{1d,2}$	3.3379E-03	2.4473E-03	9.0865E-06	6.6412E-06	1.2636E-06	7.0524E-07
$\Delta\dot{\psi}_{1q,2}$	3.7575E-05	3.3461E-05	2.0638E-06	9.6416E-01	6.8853E-01	3.9650E-01
$\Delta\dot{\psi}_{2q,2}$	1.1686E-05	1.3982E-05	1.3798E-06	9.6415E-01	6.8853E-01	3.9649E-01
$\Delta\dot{\omega}_3$	2.1937E-01	3.4672E-02	**1.0000E+00**	2.7867E-05	5.1915E-07	1.3344E-05
$\Delta\dot{\delta}_3$	2.1936E-01	3.4671E-02	9.9998E-01	2.7810E-05	5.1807E-07	1.3317E-05
$\Delta\dot{\psi}_{fd,3}$	4.4059E-04	4.1891E-05	6.5218E-04	5.8580E-07	6.6364E-09	1.6723E-07
$\Delta\dot{\psi}_{1d,3}$	1.9634E-03	1.7367E-04	2.5854E-03	6.3529E-06	7.1969E-08	1.8135E-06
$\Delta\dot{\psi}_{1q,3}$	2.0058E-05	1.7420E-06	3.0675E-05	8.9253E-01	3.8441E-02	**1.0000E+00**
$\Delta\dot{\psi}_{2q,3}$	5.6573E-06	5.6215E-07	1.2379E-05	8.9252E-01	3.8441E-02	1.0000E+00

Table 7-2: Normalized Participation Factors at minimum load

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It is noticed that Oscillation modes are related to same state variable as same as the loads at full load (276MW).

7.2.1 Validation

When 10% disturbance is given to the unit 01, following figures 7-2 show the response.

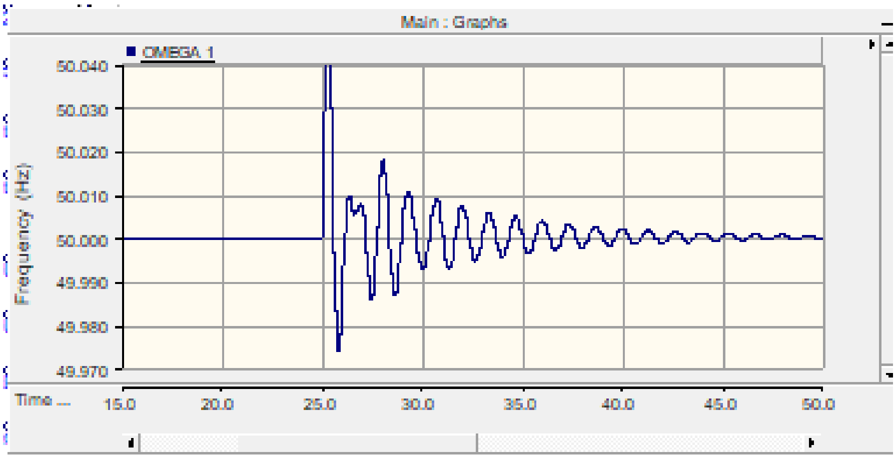

Table 7-3: Unit 01 response - frequency vs time

Frequency of the system = (35.941 – 34.608) Hz = 1.333 Hz

This frequeny is much similar to the calculated oscilation frquecny.

7.3 Appendix C : MATLAB Script

```
Xd=(1.836/(2*3.142*50)); Xq=(1.79/(2*3.142*50)); Xxd=(0.2/(2*3.142*50));
Xxq=(0.333/(2*3.142*50)); %convert X in to L
Xxxd=(0.155/(2*3.142*50)); Xxxq=(0.152/(2*3.142*50));  Xl=(0.124/(2*3.142*50));
%X=2pi*f*l >> l=x/(2*pi*f) ,x as l% %%%%%%
Tttd=0.035; Td0=8.47; Ttd0=8.47; Ttq0=94;  Tttq0=0.079;
%%% basic values%
format shortG
Xad=Xd-Xl;      Xaq=Xq-Xl;
Xfd=((Xad*(Xxd-Xl))/(Xad-Xxd+Xl));            %inv(A)*B = A\B
X1d=((1/(Xxxd-Xl))-(1/Xad)-(1/Xfd))\1;        %inv((1/(Xxxd-Xl))-(1/Xad)-(1/Xfd)) =
((1/(Xxxd-Xl))-(1/Xad)-(1/Xfd))\1
X1q=((1/(Xxq-Xl))-(1/Xaq))\1;
X2q=((1/(Xxxq-Xl))-(1/Xaq)-(1/X1q))\1;
R1d=((1/Tttd)*(X1d+(Xad*Xl*Xfd/(Xad*Xl+Xad*Xfd+Xfd*Xl))));
Rfd=((Xad+Xfd)/Ttd0);
Ttt0=((1/R1d)*(X1d+((Xad*Xfd)/(Xad+Xfd))));
Ttd=((1/Rfd)*(Xfd+((Xad*Xl)/(Xad+Xl))));
Xxxad=((Xad*Xfd*X1d)/(Xad*X1d+Xfd*X1d+Xfd*Xad));
Xxxaq=((Xaq*X1q*X2q)/(Xaq*X1q+X1q*X2q+X2q*Xaq));
R1q=((Xaq+X1q)/Ttq0);
R2q=(1/Tttq0)*(X2q+((Xaq*X1q)/(Xaq+X1q)));
Xxad=Xxd-Xl;
Xxaq=Xxq-Xl;
H=4.376; Ra=0.125; Kd=1.5;
%%%End basic values%


%%
%%%Unit 1 generator side%
It_1=((sqrt(((279/(353*0.976))^2) + ((61.78/(353*0.2164))^2)))/(19/20));
Q=0.2182; Et_1=(20.15/20);


omg0_1=1.0;
del_1=atan((Xq*It_1*cos(Q)-Ra*It_1*sin(Q))/(Et_1+Ra*It_1*cos(Q)+Xq*It_1*sin(Q)));
ed_1=Et_1*sin(del_1); eq_1=Et_1*cos(del_1);
id_1=It_1*sin(Q+del_1);
iq_1=It_1*cos(Q+del_1) ;
ifd_1=((eq_1+Ra*iq_1+Xd*id_1)/Xad);
pifd_1=((Xad+Xfd)*ifd_1-(Xad*id_1));
piad_1=Xxad*(-id_1+(pifd_1/Xfd)); piaq=(-1*Xxaq*iq_1);


a11_1=(-1*(Kd/(2*H))) ;
a12_1=(((((piad_1+Xxxaq*id_1)*id_1)+(piaq+Xxxad*iq_1)*iq_1))/(2*H)) ;
a13_1= (-1*(Xxxad*iq_1)/(2*H*Xfd)) ;
a14_1= (-1*(Xxxad*iq_1)/(2*H*X1d)) ;
```

```
a15_1= ((Xxxaq*iq_1)/(2*H*X1q))  ;
a16_1= ((Xxxaq*id_1)/(2*H*X2q)) ;
 b11_1=1/(2*H) ;
 e11_1=(((-1*(piad_1+Xxxaq*id_1)*cos(del_1))+((piaq+Xxxad*iq_1)*sin(del_1)))/(2*H ));
 e12_1=(((-1*(piad_1+Xxxaq*id_1)*sin(del_1))-((piaq+Xxxad*iq_1)*cos(del_1)))/(2*H)) ;

a21_1=1 ; %in pu unit

a32_1=((-1*omg0_1*Rfd*Xxxad*iq_1)/Xfd);
a33_1=((omg0_1*Rfd*((Xxxad/Xfd)-1))/Xfd);
a34_1=(1*(omg0_1*Rfd*Xxxad)/(Xfd*X1d)); %-1*
 b32_1=((omg0_1*Rfd)/(Xad));
 e31_1=((-1*(omg0_1*Rfd*Xxxad*sin(del_1)))/(Xfd));
 e32_1=((omg0_1*Rfd*Xxxad*cos(del_1))/(Xfd));

a42_1=((-1*omg0_1*R1d*Xxxad)/(X1d));
a43_1=(-1*(omg0_1*R1d*Xxxad)/(X1d*Xfd));  %%-1*
a44_1=((omg0_1*R1d*((Xxxad/X1d)-1))/(X1d));
 e41_1=(((-1*omg0_1*R1d*Xxxad)*sin(del_1))/(X1d)) ;
 e42_1=(((omg0_1*R1d*Xxxad)*cos(del_1))/(X1d)) ;

a52_1=((omg0_1*R1q*Xxxaq*id_1)/X1q);
a55_1=((omg0_1*R1q*((Xxxaq/X1q)-1))/(X1q));
a56_1=(-1*(omg0_1*R1d*Xxxaq)/(X1d*X2q)); %-1*
 e51_1=((-1*omg0_1*R1q*Xxxaq*cos(del_1))/X1q)  ;
 e52_1=((-1*omg0_1*R1q*Xxxaq*sin(del_1))/X1q)  ;

a62_1=((omg0_1*R2q*Xxxaq*id_1)/X2q) ;
a65_1=((omg0_1*R2q*Xxxaq)/(X2q*X1q)) ;
a66_1=((omg0_1*R2q*((Xxxaq/X2q)-1))/X2q) ;
 e61_1=((-1*omg0_1*R2q*Xxxaq*cos(del_1))/X2q)  ;
 e62_1=((-1*omg0_1*R2q*Xxxaq*sin(del_1))/X2q)  ;

%%
%%%Unit 2 generator side%
%It_2=0.912;  Q=0.0895; Et_2=0.987;
It_2=((sqrt(((276/(353*0.976))^2) + ((61.24/(353*0.2164))^2)))/(20.017/20));
Q=0.2182; Et_2=20.017/20;

del_2=atan((Xq*It_2*cos(Q)-Ra*It_2*sin(Q))/(Et_2+Ra*It_2*cos(Q)+Xq*It_2*sin(Q)));
ed_2=Et_2*sin(del_2); eq_2=Et_2*cos(del_2);
id_2=It_2*sin(Q+del_2);
iq_2=It_2*cos(Q+del_2) ;
ifd_2=((eq_2+Ra*iq_2+Xd*id_2)/Xad);
pifd_2=((Xad+Xfd)*ifd_2-(Xad*id_2));
```

```
piad_2=Xxad*(-id_2+(pifd_2/Xfd)); piaq=-1*Xxaq*iq_2; omg0_2=1;

a11_2=-1*(Kd/(2*H)) ;
a12_2=((((piad_2+Xxxaq*id_2)*id_2)+(piaq+Xxxad*iq_2)*iq_2)/(2*H)) ;
a13_2= (-1*(Xxxad*iq_2)/(2*H*Xfd)) ;
a14_2= (-1*(Xxxad*iq_2)/(2*H*X1d)) ;
a15_2= ((Xxxaq*iq_2)/(2*H*X1q))  ;
a16_2= ((Xxxaq*id_2)/(2*H*X2q)) ;
  b11_2= 1/(2*H) ;
  e11_2=(((-1*(piad_2+Xxxaq*id_2)*cos(del_2))+((piaq+Xxxad*iq_2)*sin(del_2)))/(2*H))
;
  e12_2=(((-1*(piad_2+Xxxaq*id_2)*sin(del_2))-((piaq+Xxxad*iq_2)*cos(del_2)))/(2*H))
;

a21_2=1 ;

a32_2=((-1*omg0_2*Rfd*Xxxad*iq_2)/Xfd)  ;
a33_2=((omg0_2*Rfd*((Xxxad/Xfd)-1))/Xfd) ;
a34_2=(1*(omg0_2*Rfd*Xxxad)/(Xfd*X1d)) ;
 b32_2=((omg0_2*Rfd)/(Xad)) ;
 e31_2=((-1*(omg0_2*Rfd*Xxxad*sin(del_2)))/(Xfd)) ;
 e32_2=((omg0_2*Rfd*Xxxad*cos(del_2))/(Xfd)) ;

a42_2=((-1*omg0_2*R1d*Xxxad)/(X1d)) ;
a43_2=(-1*(omg0_2*R1d*Xxxad)/(X1d*Xfd)) ;
a44_2=((omg0_2*R1d*((Xxxad/X1d)-1))/(X1d)) ;
 e41_2=(((-1*omg0_2*R1d*Xxxad)*sin(del_2))/(X1d)) ;
 e42_2=(((omg0_2*R1d*Xxxad)*cos(del_2))/(X1d));

a52_2=((omg0_2*R1q*Xxxaq*id_2)/X1q)  ;
a55_2=((omg0_2*R1q*((Xxxaq/X1q)-1))/(X1q)) ;
a56_2=(-1*(omg0_2*R1d*Xxxaq)/(X1d*X2q)) ;
 e51_2=((-1*omg0_2*R1q*Xxxaq*cos(del_2))/X1q)  ;
 e52_2=((-1*omg0_2*R1q*Xxxaq*sin(del_2))/X1q)  ;

a62_2=((omg0_2*R2q*Xxxaq*id_2)/X2q)  ;
a65_2=((omg0_1*R2q*Xxxaq)/(X2q*X1q)) ;
a66_2=((omg0_2*R2q*((Xxxaq/X2q)-1))/X2q) ;
 e61_2=((-1*omg0_2*R2q*Xxxaq*cos(del_2))/X2q) ;
 e62_2=((-1*omg0_2*R2q*Xxxaq*sin(del_2))/X2q) ;

%%
%%%Unit 3 generator side%
%It_3=0.9265;  Q=0.08946;    Ra= 0.00228; Et_3=0.987; %wrong
It_3=((sqrt(((276/(353*0.976))^2) + ((59.27/(353*0.2164))^2)))/(19.9/20));
```

```
Q=0.2182; Et_3=19.9/20;

del_3=atan((Xq*It_3*cos(Q)-Ra*It_3*sin(Q))/(Et_3+Ra*It_3*cos(Q)+Xq*It_3*sin(Q)));
ed_3=Et_3*sin(del_3); eq_3=Et_3*cos(del_3);
id_3=It_3*sin(Q+del_3);
iq_3=It_3*cos(Q+del_3);
ifd_3=((eq_3+Ra*iq_3+Xd*id_3)/Xad);
pifd_3=((Xad+Xfd)*ifd_3-(Xad*id_3));
piad_3=Xxad*(-id_3+(pifd_3/Xfd)); piaq=-1*Xxaq*iq_3;
omg0_3=1;

a11_3=-1*(Kd/(2*H));
a12_3=((((piad_3+Xxxaq*id_3)*id_3)+(piaq+Xxxad*iq_3)*iq_3)/(2*H));
a13_3= (-1*(Xxxad*iq_3)/(2*H*Xfd));
a14_3= (-1*(Xxxad*iq_3)/(2*H*X1d));
a15_3= ((Xxxaq*iq_3)/(2*H*X1q)) ;
a16_3= ((Xxxaq*id_3)/(2*H*X2q));
 b11_3=1/(2*H);
 e11_3=(((-1*(piad_3+Xxxaq*id_3)*cos(del_3))+((piaq+Xxxad*iq_3)*sin(del_3)))/(2*H));
 e12_3=(((-1*(piad_3+Xxxaq*id_3)*sin(del_3))-((piaq+Xxxad*iq_3)*cos(del_3)))/(2*H));

a21_3=1;

a32_3=((-1*omg0_3*Rfd*Xxxad*iq_3)/Xfd) ;
a33_3=((omg0_3*Rfd*((Xxxad/Xfd)-1))/Xfd);
a34_3=(1*(omg0_3*Rfd*Xxxad)/(Xfd*X1d));
b32_3=((omg0_3*Rfd)/(Xad));
 e31_3=((-1*(omg0_3*Rfd*Xxxad*sin(del_3)))/(Xfd));
 e32_3=((omg0_3*Rfd*Xxxad*cos(del_3))/(Xfd));

a42_3=((-1*omg0_3*R1d*Xxxad)/(X1d));
a43_3=(-1*(omg0_3*R1d*Xxxad)/(X1d*Xfd));
a44_3=((omg0_3*R1d*((Xxxad/X1d)-1))/(X1d));
 e41_3=(((-1*omg0_3*R1d*Xxxad)*sin(del_3))/(X1d));
 e42_3=(((omg0_3*R1d*Xxxad)*cos(del_3))/(X1d));

a52_3=((omg0_3*R1q*Xxxaq*id_3)/X1q) ;
a55_3=((omg0_3*R1q*((Xxxaq/X1q)-1))/(X1q));
a56_3=(-1*(omg0_3*R1d*Xxxaq)/(X1d*X2q));
 e51_3=((-1*omg0_3*R1q*Xxxaq*cos(del_3))/X1q) ;
 e52_3=((-1*omg0_3*R1q*Xxxaq*sin(del_3))/X1q);

a62_3=((omg0_3*R2q*Xxxaq*id_3)/X2q) ;
a65_3=((omg0_1*R2q*Xxxaq)/(X2q*X1q)) ;
a66_3=((omg0_3*R2q*((Xxxaq/X2q)-1))/X2q);
```

```
 e61_3=((-1*omg0_3*R2q*Xxxaq*cos(del_3))/X2q) ;
 e62_3=((-1*omg0_3*R2q*Xxxaq*sin(del_3))/X2q) ;


%%
%%%%Line Side%%%
%Xq=1.79;  Xd=1.836;
%Ra= 0.00228;
%del=0.9480;
Xxx=((Xxxq+Xxxd)/2); %Xxx=((Xxxq+Xxxd)/2);%
%Xxxad=0.0301; Xxxaq=0.0279; X1d=0.05236; X1q=0.0796; X2q=0.0443; Xfd=.07953;


%%%Unit 1 Network side
omgr_1=1;    %ifd_1=1.1907;  id_1=0.7962; iq_1=0.4489;
pi1d_1=Xad*(ifd_1-id_1); pi2q_1=(-1*Xaq*iq_1) ;
pi1q_1 = pi2q_1; pifd_1=((Xad+Xfd)*ifd_1-Xad*id_2);
Eeed_1=(-1*omgr_1*Xxxaq*(pi1q_1/X1q)+(pi2q_1/X2q));
Eeeq_1=(omgr_1*Xxxad*(pifd_1/Xfd)+(pi1d_1/X1d));


%----U1 data-------------------%
d=(((Ra)^2)+ ((Xxx)^2));
m1_1=((-1*((Ra*Eeeq_1-Xxx*Eeed_1)*sin(del_1))-((Ra*Eeed_1+Xxx*Eeeq_1)*cos(del_1)))/d)
;
m2_1=(((Ra*cos(del_1)+ Xxx*sin(del_1))*Xxxad)/(Xfd*d));
m3_1=((Ra*cos(del_1)+ Xxx*sin(del_1))*Xxxad)/(X1d*d);
m4_1=(-1*(Ra*sin(del_1)- Xxx*cos(del_1))*Xxxaq)/(X1q*d);
m5_1=(-1*(Ra*sin(del_1)- Xxx*cos(del_1))*Xxxaq)/(X2q*d);


n1_1=((-1*((Ra*Eeeq_1+Xxx*Eeed_1)*cos(del_1))-((Ra*Eeed_1+Xxx*Eeeq_1)*sin(del_1)))/d)
;
n2_1=((Ra*sin(del_1)- Xxx*cos(del_1))*Xxxad)/(Xfd*d);
n3_1=((Ra*sin(del_1)- Xxx*cos(del_1))*Xxxad)/(X1d*d);
n4_1=((Ra*cos(del_1)+ Xxx*sin(del_1))*Xxxaq)/(X1q*d);
n5_1=((Ra*cos(del_1)+ Xxx*sin(del_1))*Xxxaq)/(X2q*d);


%%%Unit 2 Network side


omgr_2=1;   %ifd_2=1.1794; id_2=0.7915; iq_2=0.4531;
pi1d_2=Xad*(ifd_2-id_2); pi2q_2=(-1*Xaq*iq_2) ;
pi1q_2 = pi2q_2; pifd_2=(Xad+Xfd)*ifd_2-Xad*id_2;
Eeed_2=(-1*omgr_2*Xxxaq*(pi1q_2/X1q)+(pi2q_2/X2q));
Eeeq_2=(omgr_2*Xxxad*(pifd_2/Xfd)+(pi1d_2/X1d));


%----U2 data-------------------%
d=(((Ra)^2)+ ((Xxx)^2));
m1_2=(-1*((Ra*Eeeq_2-Xxx*Eeed_2)*sin(del_2))-((Ra*Eeed_2+Xxx*Eeeq_2)*cos(del_2)))/d ;
```

```
m2_2=((Ra*cos(del_2)+ Xxx*sin(del_2))*Xxxad)/(Xfd*d);
m3_2=((Ra*cos(del_2)+ Xxx*sin(del_2))*Xxxad)/(X1d*d);
m4_2=(-1*(Ra*sin(del_2)- Xxx*cos(del_2))*Xxxaq)/(X1q*d);
m5_2=(-1*(Ra*sin(del_2)- Xxx*cos(del_2))*Xxxaq)/(X2q*d);

n1_2=(-1*((Ra*Eeeq_2+Xxx*Eeed_2)*cos(del_2))-((Ra*Eeed_2+Xxx*Eeeq_2)*sin(del_2)))/d ;
n2_2=((Ra*sin(del_2)- Xxx*cos(del_2))*Xxxad)/(Xfd*d);
n3_2=((Ra*sin(del_2)- Xxx*cos(del_2))*Xxxad)/(X1d*d);
n4_2=((Ra*cos(del_2)+ Xxx*sin(del_2))*Xxxaq)/(X1q*d);
n5_2=((Ra*cos(del_2)+ Xxx*sin(del_2))*Xxxaq)/(X2q*d);

%%%Unit 3 Network side
omgr_3=1;          %ifd_3=1.1930; id_3=0.8070; iq_3=0.4552;
pi1d_3=Xad*(ifd_3-id_3); pi2q_3=(-1*Xaq*iq_3) ;
pi1q_3 = pi2q_3; pifd_3=(Xad+Xfd)*ifd_3-Xad*id_3;
Eeed_3=(-1*omgr_3*Xxxaq*(pi1q_3/X1q)+(pi2q_3/X2q));
Eeeq_3=(omgr_3*Xxxad*(pifd_3/Xfd)+(pi1d_3/X1d));

%----U3 data--------------------%
d=(((Ra)^2)+ ((Xxx)^2));
m1_3=(-1*((Ra*Eeeq_3-Xxx*Eeed_3)*sin(del_3))-((Ra*Eeed_3+Xxx*Eeeq_3)*cos(del_3)))/d ;
m2_3=((Ra*cos(del_3)+ Xxx*sin(del_3))*Xxxad)/(Xfd*d);
m3_3=((Ra*cos(del_3)+ Xxx*sin(del_3))*Xxxad)/(X1d*d);
m4_3=(-1*(Ra*sin(del_3)- Xxx*cos(del_3))*Xxxaq)/(X1q*d);
m5_3=(-1*(Ra*sin(del_3)- Xxx*cos(del_3))*Xxxaq)/(X2q*d);

n1_3=(-1*((Ra*Eeeq_3+Xxx*Eeed_3)*cos(del_3))-((Ra*Eeed_3+Xxx*Eeeq_3)*sin(del_3)))/d ;
n2_3=((Ra*sin(del_3)- Xxx*cos(del_3))*Xxxad)/(Xfd*d);
n3_3=((Ra*sin(del_3)- Xxx*cos(del_3))*Xxxad)/(X1d*d);
n4_3=((Ra*cos(del_3)+ Xxx*sin(del_3))*Xxxaq)/(X1q*d);
n5_3=((Ra*cos(del_3)+ Xxx*sin(del_3))*Xxxaq)/(X2q*d);
%%%end of G &  Line side

%%
format shortG
%Unit1 A matrix
A_1
=([[a11_1,a12_1,a13_1,a14_1,a15_1,a16_1];[a21_1,0,0,0,0,0];[0,a32_1,a33_1,a34_1,0,0];
[0,a42_1,a43_1,a44_1,0,0];[0,a52_1,0,0,a55_1,a56_1];[0,a62_1,0,0,a65_1,a66_1]]);
A_2
=([[a11_2,a12_2,a13_2,a14_2,a15_2,a16_2];[a21_2,0,0,0,0,0];[0,a32_2,a33_2,a34_2,0,0];
[0,a42_2,a43_2,a44_2,0,0];[0,a52_2,0,0,a55_2,a56_2];[0,a62_2,0,0,a65_2,a66_2]]);
A_3
=([[a11_3,a12_3,a13_3,a14_3,a15_3,a16_3];[a21_3,0,0,0,0,0];[0,a32_3,a33_3,a34_3,0,0];
[0,a42_3,a43_3,a44_3,0,0];[0,a52_3,0,0,a55_3,a56_3];[0,a62_3,0,0,a65_3,a66_3]]);
```

```
A_A=([0 0 0 0 0 0 ; 0 0 0 0 0 0; 0 0 0 0 0 0; 0 0 0 0 0 0; 0 0 0 0 0 0; 0 0 0 0 0
0]);
AAA1=([A_1 A_A A_A  ;A_A A_2 A_A;A_A A_A A_3]); %%not used for any where just to
report`

B_1 = ([[b11_1,0]; [0,0]; [0,b32_1];[0,0];[0,0];[0,0]]);
B_2 = ([[b11_2,0]; [0,0]; [0,b32_2];[0,0];[0,0];[0,0]]);
B_3 = ([[b11_3,0]; [0,0]; [0,b32_3];[0,0];[0,0];[0,0]]);
BB=zeros(6,2);
Bm=([B_1 BB BB ; BB B_2 BB; BB BB B_3]) ; %%

E_1 =
([[e11_1,e12_1];[0,0];[e31_1,e32_1];[e41_1,e42_1];[e51_1,e52_1];[e61_1,e62_1]]);
E_2 =
([[e11_2,e12_2];[0,0];[e31_2,e32_2];[e41_2,e42_2];[e51_2,e52_2];[e61_2,e62_2]]);
E_3 =
([[e11_3,e12_3];[0,0];[e31_3,e32_3];[e41_3,e42_3];[e51_3,e52_3];[e61_3,e62_3]]);
EE=([0 0; 0 0; 0 0; 0 0; 0 0; 0 0]);
Em=([E_1 EE EE; EE E_2 EE;EE EE E_3]);  %%

D_1= (1/d) *([-Ra -Xxx ; Xxx -Ra]);
D_2=D_1;
D_3=D_1; DD=([0 0 ; 0 0]);
Dm=([D_1 DD DD  ;DD D_2 DD ; DD DD D_3]); %%

C_1=([0 m1_1 m2_1 m3_1 m4_1 m5_1 ; 0 0.1*n1_1 n2_1 n3_1 n4_1 n5_1]);
C_2=([0 m1_2 m2_2 m3_2 m4_2 m5_2 ; 0 0.1*n1_2 n2_2 n3_2 n4_2 n5_2]);
C_3=([0 m1_3 m2_3 m3_3 m4_3 m5_3 ; 0 0.1*n1_3 n2_3 n3_3 n4_3 n5_3]);
CC=([0 0 0 0 0 0 ; 0 0 0 0 0 0]);
Cm=([C_1 CC CC; CC C_2 CC;CC CC C_3]); %%

Am_1=A_1+ (E_1*C_1); % individial A matrix
Am_2=A_2+ (E_2*C_2);
Am_3=A_3+ (E_3*C_3);
AAA=([0 0 0 0 0 0 ; 0 0 0 0 0 0; 0 0 0 0 0 0; 0 0 0 0 0 0; 0 0 0 0 0 0; 0 0 0 0 0
0]);
Am=([Am_1 AAA AAA  ;AAA Am_2 AAA;AAA AAA Am_3]) ;      %

%%
%admitance matrix

y11=complex(0,-2.3364); y14=complex(0,7.1428);
y22=y11;  y24=y14;
y33=y11;  y34=y14;
y41=complex(0,7.1428); y42=y41; y43=y41; y44=complex(0,2.3809);
```

```
Y=([[y11 ,0 ,0, y14 ]; [0, y22 ,0 ,y24 ]; [0 ,0, y33, y34 ] ;[ y41 ,y42, y43, y44
]]);


Y11=([y11 ,0 ,0; 0, y22 ,0; 0 ,0, y33]);   Y12=([y14 ; y24 ;y34]);
Y21=([ y41 , y42, y43]);  Y22=([ y44 ]);


Y_eq = Y11-(Y12*(inv(Y22))*Y21);
P11=Y_eq(1,1); P12=Y_eq(1,2); P13=Y_eq(1,3); P21=Y_eq(2,1); P22=Y_eq(2,2);
P23=Y_eq(2,3); P31=Y_eq(3,1); P32=Y_eq(3,2); P33=Y_eq(3,3);
p11=real(P11); p22=imag(P11);  p13=real(P12); p24=imag(P12); p15=real(P13);
p26=imag(P13); %convert 3x3 into 6x6
p31=real(P21); p42=imag(P21);  p33=real(P22); p44=imag(P22); p35=real(P23);
p46=imag(P23);
p51=real(P31); p62=imag(P31);  p53=real(P32); p64=imag(P32); p55=real(P33);
p66=imag(P33);
%p31 p42 p33 p44 p35 p46   p51 p62 p53 p64 p55 p66
Yeq=[p11 0 p13 0 p15 0; 0 p22 0 p24 0 p26 ; p31 0 p33 0 p35 0  ; 0 p42 0 p44 0 p46 ;
p51 0 p53 0 p55 0  ; 0 p62 0 p64 0 p66 ];




%%
K= (Am+ Em*Cm + (Em*Dm*(inv(Yeq-Dm))*Cm)) ; %%System Matrix (A_sys)
% xlswrite('filename.xlsx',K)  % SAVE MATRIX ON EXCEL
global AA
AA=eig(K) % Eigen values
%  xlswrite('AA.xlsx',K) ;


%plot eigenValues
figure(1)
     plot(real(AA),imag(AA),'r*') %   Plot real and imaginary parts
     xlabel('Real')
     ylabel('Imaginary')
     title('Eigenvalues of a random matrix of dimension ');


global PP
global V
global W %left (W) eigenvectors
% %% find right(V) & left (W) eigenvectors, D- Diagonal matrix of eigenvalues
 [V,D,W] = eig(K);
%


 %PP =  V*W ; % Participatoin factors
 for j=1:18
```

```
     for k=1:18
         PP(j,k)=V(j,k)*W(j,k); % Participatoin factors
         PPU(j,k)=abs(PP(j,k));  %absolute PP
         VV(j,k)=abs(V(j,k));


           PPUnew(k,j)= (PPU(k,j))/(max(PPU(:,j))); %same as PPUnew(:,j)
     end
%            PPUnew(:,j)= (PPU(:,j))/(max(PPU(:,j)));  %(absolute PP)/max(PP)in column
 end


%   xlswrite('ParticipationFac.xlsx',PP) ;


for j=1:18
    for m=1:18
     PPUnew(m,j)= (PPU(m,j))/(max(PPU(:,j)));
    end
end


%   xlswrite('PPU_Normalize.xlsx',PPUnew);
% AA(:,i)=A(:,i)/max(A(:,i));
% %%


%step response
P=K ;
Q=Bm;
R=[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];
D_D=[0 0 0 0 0 0];
X0 = [omg0_1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  0] ;  %initial values


%lsim(A, B, C, D, U, T, X0)     % simulate and plot the response (the output)
A=K; B=Bm(:,1); %extract first column form Bm for plotting
C=R; DDD=0;
t=0:0.01:50;
for k=1:length(t)
    inputnew(1,k)=0.001;
end


  sys=ss(A,B,C,DDD);
  figure(2);
  lsim(ss(A,B,C,DDD),inputnew,t,X0); %plot step response for 10%
   title('Response for 10% step');



%%
```