AN IMPROVED PRIMAL SOLUTION FOR THE TRANSPORTATION PROBLEMS IN OPERATIONAL RESEARCH

NESARATNAM EDWIN LINOSH

(148379T)

MASTER OF SCIENCE IN OPERATIONAL RESEARCH

DEPARTMENT OF MATHEMATICS

UNIVERSITY OF MORATUWA

SRI LANKA

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NESARATNAM EDWIN LINOSH

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SRI LANKA

July 2018

DECLARATION

I do hereby declare that the work reported in this project report/thesis was exclusively carried out by me under the supervision of Prof. W. B. Daundasekera and Mr. T. M. J. A. Cooray. It describes the results of my own independent research except where due reference has been made in the text. No part of this project report/thesis has been submitted earlier or concurrently for the same or any other degree.

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Signature of the Candidate
N. Edwin Linosh (148379T)

I declare that the above candidate has carried out this research for master thesis under my supervision.

…………………… Date:………………

Signature of the Supervisor
Prof. W. B. Daundasekera
Professor
Department of Mathematics
University of Peradeniya
Sri Lanka

I declare that the above candidate has carried out this research for master thesis under my supervision.

………………………………… Date:…………………

Signature of the Co-Supervisor
Mr. T. M. J. A. Cooray
Senior Lecturer
Department of Mathematics
University of Moratuwa
Sri Lanka

ABSTRACT

Organizations providing goods and services are mainly focusing on cost minimization within their organizations as it is a vital factor for their existence. In common, scheduling activities with less conflict within organizations is vital for their survival. In many organizations, transportation scheduling plays a major role in cost minimization. In particular, transporting goods from manufacturing plants to identified destinations with minimum transportation cost is knows as transportation scheduling or transportation problem.

The objective of the transportation problem is to satisfy the destination requirements with minimum cost while satisfying the operating production capacity. Transportation problem is categorized as a Linear Programming problem. Generally, the Simplex method is the widely used method to solve Linear Programming problems. But, Simplex method is not the most efficient method to solve the transportation problem due to its special structure. Therefore, the most of the time effective and numerical efficient way to solve the transportation problem is Transportation Algorithm (TA) designed from the basic principles of Simplex method.

The Transportation Algorithm consists of two major steps: obtaining the Initial Basic Feasible Solution (IBFS) and finding optimal solution using the IBFS. A better IBFS always reduces the number of iterations and computational time in finding the optimum solution. There are existing standard methods which are available to find the IBFS, but have failed to find an effective IBFS for the most of the transportation problems. To overcome this failure, in this research a modified heuristic approach is proposed to find a more promising IBFS.

In the proposed method, the cumulative difference representation is used instead of cost matrix in order to make the assignments. This technique leads to assign most of the assignments at minimum cost. The cumulative difference representation represents the additional excess cumulative costs throughout the row and column for each possible cost of transportation. The IBFS found by the newly proposed method converges to the optimal solution faster than the standard methods considering the time consumed as well as less number of iterations to achieve it. The proposed method has proved to be in finding better IBFS for all the 70 transportation problems discussed in this study. The IBFS of 41 problems of selected 70 transportation problems them self are the optimal solutions. Further, for the rest of the 29 problems, the difference between IBFS and the optimal solution is only less than five percentage. Therefore, it can be concluded that the newly proposed method to find IBFS is robust in providing an improved primal solution compared to the existing standard methods.

Keywords: Transportation problem, Optimal Solution, IBFS, Cumulative Difference.

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CHAPTER 1

INTRODUCTION

1.1 The Transportation Problem

Business consultants and industrial experts are always focusing on cost minimization alternatives which lead to the expansion of the firms with greater turnover. One of the most important aspects of cost minimization is an effective transportation schedule.

The transportation problem is primarily concerned with the optimal (cost effective) way to deliver goods manufactured at production plants located at different places Known as supply origins to warehouses or customers known as demand destinations. The objective in a transportation problem is to fully satisfy the destination requirements within the operating production capacity constraints at the minimum possible transportation cost. Whenever there is a physical movement of goods from the point of manufacturer to the end consumer through a variety of channels of distribution (wholesalers, retailers, distributors etc.), there is a need to minimize the cost of transportation so as to increase profit on sales.

The transportation problem can be used to solve many real world problems such as logistics and supply chain management, courier and cargo transportations, physical distribution of products from a company postal delivery, etc

Initially, the problem was formulated as a mathematical model by French Mathematician G. Monge in 1781. A Russian mathematician L. V. Kantorovich made some major advancement in this field during and after the World War-II to solve the post war problems. In 1941, Hitchcock proposed a mathematical model to solve the transportation and logistics problem in the real world scenario. It is in fact a linear programming model, where the objective function is to minimize the transportation cost subject to the supply and demand constraints.

The transportation schedule with the suppliers to customers can be depicted as a network given in Figure 1.1 below:

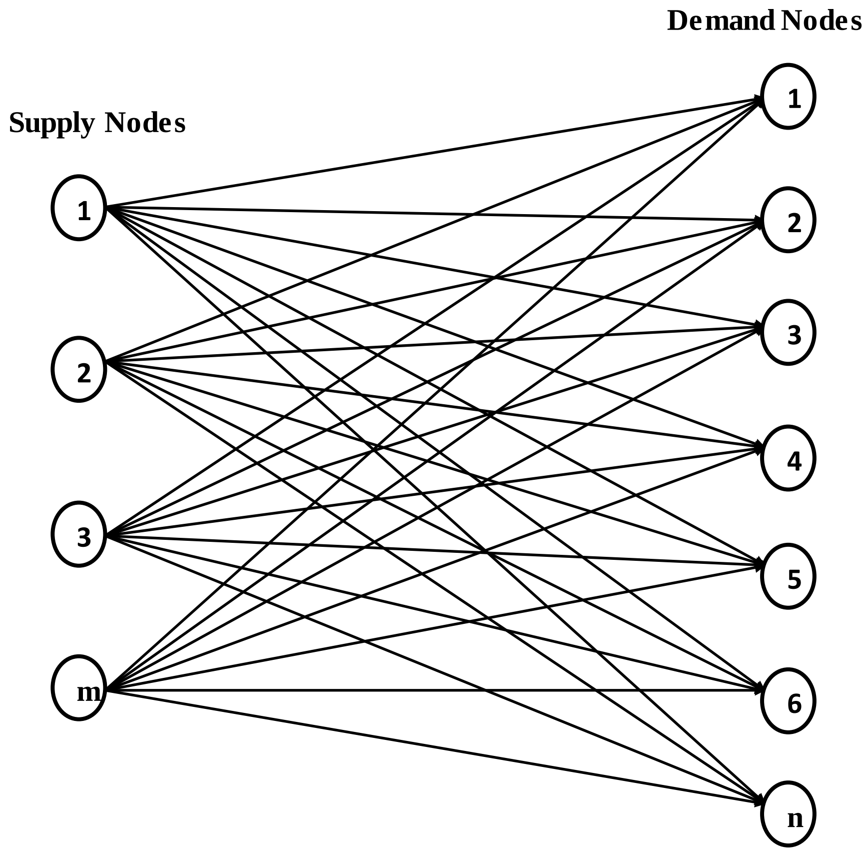

Figure 1.1: Network diagram of the transportation problem

Objective of the mathematical model is to determine the number goods to be transported from suppliers to customers with the minimum cost while satisfying the demands of the customers within the production capacities.

1.2 Mathematical Model of the Transportation Problem for Single Commodity

Assumptions

- Items can be transported from multiple vendors to multiple buyers.
- Demands and supplies are pre-deterministic and constants.
- Single commodity transportation.

Notations

m - Total number of suppliers

n - Total number of buyers

S_i - Supply quantity (availability) in units from i^{th} supplier

D_j - The demand (quantity required) in units of the j^{th} buyer

C_{ij} - The unit transportation cost from i^{th} supplier to j^{th} buyer

X_{ij} - Number of units to be transported from i^{th} supplier to j^{th} buyer in attaining the minimal total cost.

$$Minimize\ \ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij}$$

Subject to

$$\sum_{j=1}^{n} X_{ij} = S_i \ ;\ i = 1,2, \dots \dots . . m$$

$$\sum_{i=1}^{m} X_{ij} = D_j \ ;\ j = 1,2, \dots \dots . . , n$$

$$\sum_{i=1}^{m} S_i \geq=\leq \sum_{j=1}^{n} D_j$$

$$w\square ere\ X_{ij} \geq 0 \ , \forall ij$$

In the above model, if the total supply is exactly the total demand, then the problem is referred to as a balanced transportation problem. Otherwise, it is referred to as an unbalanced transportation problem.

1.3 Method of Solution to the Transportation Problem

The transportation problem can be viewed as a linear programming problem, where it can be solved using Simplex Algorithm, but due to the special structure of the model, the simplex algorithm is not the most efficient way to solve the transportation problem. The most accepted way to solve the transportation problem is the Transportation Algorithm (TA). The Transportation Algorithm utilizes the special structure of the transportation model to deduce the numerical computations and thereby improving the computational time.

The flow chart of the Transportation Algorithm is depicted in the Figure 1.2.

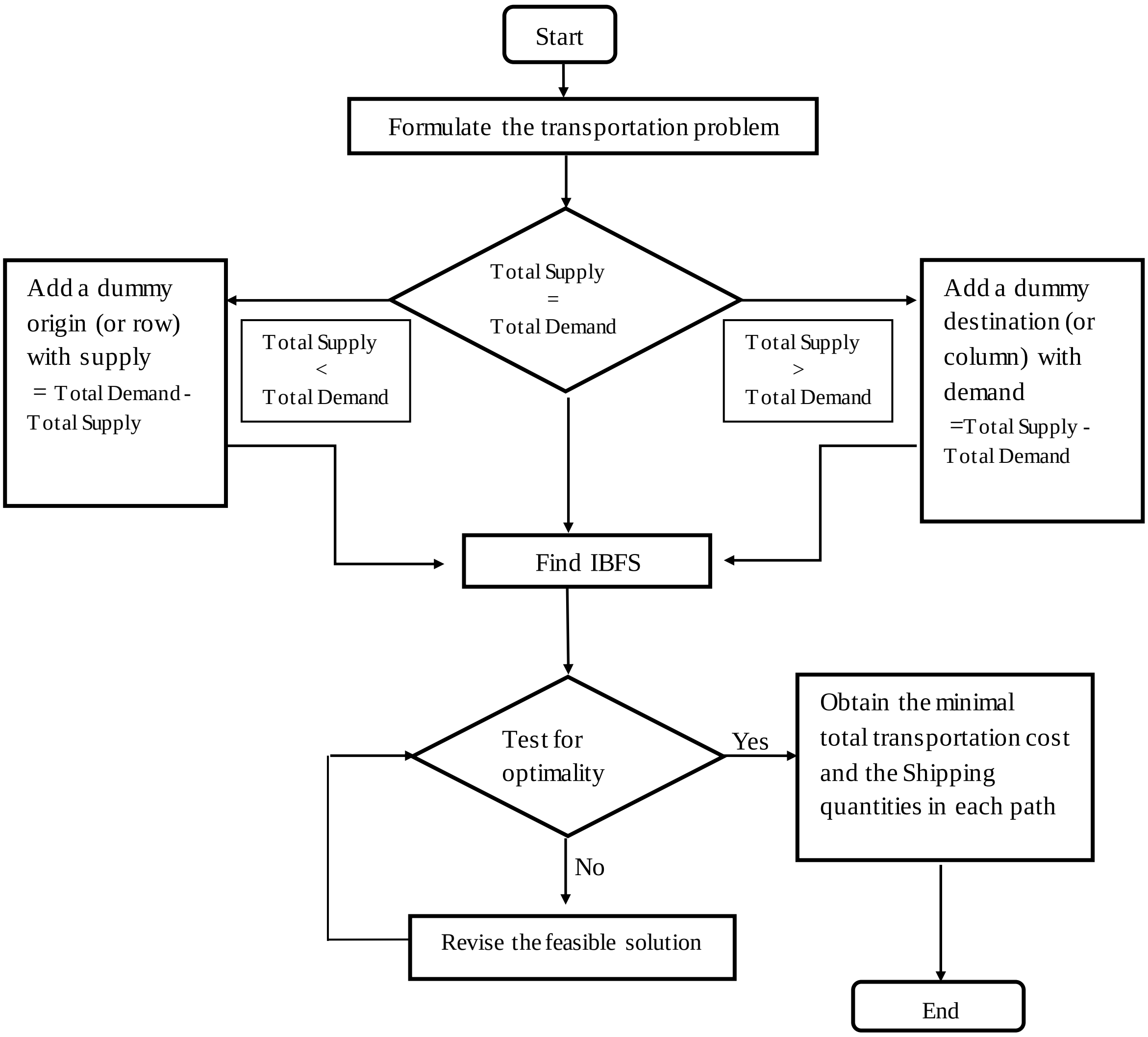

Figure 1.2: Solution procedure of the transportation problem

1.3.1 Transportation Algorithm (TA)

The Transportation Algorithm consists of two main steps:

- Obtaining the Initial Basic Feasible Solution (IBFS)
- Obtaining the optimal solution using the IBFS.

1.3.2 The Initial Basic Feasible Solution (IBFS)

Ever since the transportation model was formulated, the biggest concern among the OR scientists was to find a more promising technique to obtain an initial basic feasible solution. In the past, many researchers proposed numerical methods to achieve this. Among them North-West Corner Rule, Least Cost Method and Vogel's Approximation Method are the most acceptable methods. These three benchmark methods are briefly explained below.

North- West Corner Method (NWCM)

As far as North-West Corner Rule is concerned the allocation process starts from the upper left hand corner (North-West Corner) cell of the transportation table. Maximum feasible quantity is allocated by considering the demand and supply. If the demand of the column is fulfilled, then move horizontally to the next cell in the next column, if the supply of the row is fulfilled, then move vertically to the next cell in the next row, if the supply and demand are fulfilled simultaneously move to the next cell diagonally. Continue the allocation process until the supply and demand conditions are fulfilled.

Least Cost Method (LCM)

In Least Cost method, the allocation begins with the cell which corresponds to a minimum transportation cost. Allocate the maximum feasible quantity in the cell corresponding to the minimum cost. The allocation process is continued on the next available minimum cost cell until the supply and demand conditions are fulfilled.

Vogel's Approximation Method (VAM)

In Vogel's Approximation method, the allocation is made to the minimum transportation cost cell in a row or column corresponding to maximum penalty. The row or the column penalty is estimated by finding the difference between the smallest and the next smallest elements of a particular row or column. The allocation process is continued to minimum cost cells corresponding to a maximum penalties until the supply and demand conditions are fulfilled.

1.3.3 Optimal Solution

After finding the IBFS, it is necessary to check whether the current Basic Feasible solution is optimum or not. Testing for optimality and revising sub optimal solutions involve analysis of each unused cell to determine the potential for reducing the total transportation cost of the solution. This is accomplished by transferring one unit into an empty cell and noting its impact on costs. If costs are increased, that implies that using the cell would increase total costs. If costs remain the same, that implies the existence of an alternative option with the same total cost as the current plan. However, if analysis reveals a decrease in the cost, the implication is that an improved solution is possible. The test for optimality requires that every unused cell be evaluated for potential improvement. Either Stepping-Stone or Modified Distribution (MODI) which are described below can be used to obtain the optimal solution.

The Stepping-Stone Method

In the stepping-stone method, cell evaluation proceeds by borrowing one unit from a occupied cell and using it to assess the impact of shifting units into the empty cell. The name stepping-stone derives from early descriptions of the method that likened the procedures to crossing a shallow pond by stepping from stone to stone. Here, the occupied cells are the "stones"; shifting units into empty cells require borrowing units from occupied cells. To maintain the balance of supply and demand for every row and column, a shift of one unit into an empty cell requires a series of shifts from other occupied cells.

The Modified Distribution (MODI) Method

This is an alternative method for evaluating empty cells is the MODI method. It involves computing row and column index numbers that can be used for cell evaluation. In many aspects, it is simpler than the stepping-stone method because it avoids having to trace cell evaluation paths. Nevertheless, the cell evaluations it produces are identical to those obtained using the stepping-stone method. However, if a solution is not optimal, one stepping-stone path must be traced to obtain an improved solution.

1.4 Background of the Study

IBFS has a great impact on reducing the computational time to reach the optimal solution to a transportation problem. The impact is even greater when the problem is in large scale. The existing standard methods to find IBSF have advantages and disadvantage as well. None of these methods is capable of finding a better IBFS for all the transportation problems. Similarly, available methods in literature for finding IBFS also give a good IBFS only for a small range of problems.

Therefore, there is a necessity for a more computationally efficient method to find an improved IBFS for large scale of transportation problems.

1.5 Scope of the Thesis

In this study, after clearly analysing the available three standard methods and other alternative methods available in the literature for finding IBFS, we propose a new heuristic approach to find a better IBFS for a wider range of transpotation problems.

1.6 Contents of the Thesis

This thesis is mainly categorized into five chapters. Chapter 2 deals with literature review. Analysis of the existing methodologies along with newly proposed method is presented in Chapter 3. Chapter 4 presents the analysis of test results of the new method and the discussion on that. Finally, the conclusion with the future research directions is drawn in Chapter 5.

CHAPTER 2

LITERATURE REVIEW

2. 1 Introduction

This chapter discusses the aspects of transportation problem: mainly the origin of transportation problem, past episodes of the transportation problem, methodologies on solving the transportation problem and moreover the literatures related to Initial Basic Feasible Solution (IBFS).

2.2 Transportation Problem

The transportation problem was initially formalized by the French mathematician Monge in 1781. Major advances were made in the field during World War II by the Soviet/Russian mathematician and economist named Leonid Kantorovich. Consequently, the problem as it is now sometimes known as the Kantorovich transportation problem. In 1939 Kantorovich published a paper on continuous version of the problem. Many scientific disciplines have contributed towards analysing problems associated with the transportation problem, including operation research, economics, engineering, Geographic Information Science and geography. It is explored extensively in the mathematical programming and engineering literatures. Sometimes referred to as the facility location and allocation problem, the Transportation optimization problem can be modelled as a large-scale mixed integer linear programming problem. The origin of transportation was first presented in 1941 by Hitchcock, in a study titled “The Distribution of a Product from Several sources to numerous Localities”. This study is considered to be the first important contribution to the solution of transportation problems. In 1947 Koopmans presented an independent study, not related to Hitchcock’s, and called “Optimum Utilization of the Transportation System’’. These two contributions helped in the further development of transportation methods which involve a number of shipping sources and a number of destinations. The transportation problem received this name because many of its applications involve determining how to optimally transport goods.

However in 1951, Dantzig applied the concept of Linear Programming to solve the Transportation models.

Ample number of transportation models were introduced and the simplest of them was first presented by Hitchcock in 1941. In 1951, Dantzing placed the transportation problems in a framework of linear programming and then solved it by using the Simplex method. Later in 1954, Charnes and Cooper found that the Simplex method is not very suitable for the transportation problem, especially for a large scale transportation problem due to the special structure of its mathematical model. Hence the "Stepping Stone Method" was developed by them as an alternative to the Simplex method, in which, an optimal solution to the transportation can be obtained from an Initial Basics Feasible Solution.

Further, as a result of continuously applying the Simplex method to transportation problems, Dantzig introduced the "Modified Distribution Method" (MODI) as an alternative method to the Stepping Stone method to the transportation problems from the IBFS.

There are various heuristic methods available to obtain an initial basic feasible solution. Among them the standard classical methods of them are such as North West Corner Rule , Least Cost Method, and Vogel's Approximation Method . Goyal improved Vogel's Approximation Method for solving the unbalanced transportation problems, while Ramakrishnan discussed some improvements to Goyal's Modified Vogel's Approximation Method for unbalanced transportation problem. Sultan and Goyal studied initial basic feasible solution and resolution to degeneracy in transportation problems.

2.3 Related Methodologies on Initial Basic Feasible Solution (IBFS)

Recently there are several related studies have been conducted to find an improved IBFS to solve the transportation problem. Most of these approaches have some ability to reach improved IBFS as compared to that of standard methods.

In 2012, Hlayel and Alia have developed a method known as Best Candidate Method (BCM). Initially, the method determines the best combination of candidates for each row and column to minimize the total cost from the cost matrix. Then, allocates the supply and demand to the lowest value among the selected candidates and repeats the process of allocation until the supplies and demands are fulfilled. The authors claim that the BCM approach has significantly reduced the computational complexity in transportation problem compared to other existing methods.

In 2014, Mollah *et al* proposed a method in which in addition to the cost entries of the cost matrix, two additional disibution values were computed by finding the difference from the row maximum value and column maximun value for each of the corresponding cost values. Hence, using the distribution values, the row distribution and the column distribution indicators are calculated by finding the difference between largest and second largest distribution values of each corresponding rows and columns and allocation is made to the largest distribution values until fulfilling the supply and demand condition. The authors have shown that in some selected problems, a better initial basic feasible solution is obtained compared to the standard methods].

In 2015, Khan *et al.* derived a method to solve transportation problem with a different approach. The method first formulate the cost matrix into opportunity cost matrix and then it uses the total opportunity cost, which is formulated by finding the row and column wise summation and assigning the units for the minimum opportunity cost element of the respective row or column, where the summation of the column wise or row wise opportunity cost are maximum. This Total Opportunity Cost-SUM method shows better efficiency than the standard methods and other Total Opportunity Cost Approach according to the authors' comparison. By referring to the counter examples, the author has illustrated that in certain instances Cost-Sum approach obtained the optimal solution transportation problem or else gives an IBFS which is closer to the optimal solution.

In 2015, Sood and Jain proposed a method with the maximum difference to solve the transportation problem. The method finds the difference between the minimum and the largest element in each column and row. Then, by analyzing the column-wise and row-wise difference, the authors suggest to assign the supply to the smallest element of the row or column which shows the maximum difference. The comparison of Northwest Corner Rule, Least Cost method and Vogel's Approximation methods with this maximum difference method shows that the maximum difference method proposed by the authors provide an optimal solution in selected all four problems.

In 2015, Monalisha Pattnaik proposed another alternative approach known as Monalisha's method. In that method, initially constructs the row reduced matrix by decreasing the row-wise-minimum element from the cost matrix. Then, construct the column reduce matrix by decreasing the column-wise-minimum element from the row reduce matrix. Finally, it applies the Vogel's approximation procedures to the reduced matrix to obtain the initial feasible solution to the transportation problem. In comparison with the existing standard methods, Monalisha's method shows that for certain some transportation problems, it gives better initial basic feasible solutions.

In 2015, A Modified Vogel's Approximation Method was developed by Ulla *et al.* to solve large scale transportation problems. The authors find the row wise and column wise differences by subtracting the largest elements from each row and column respectively and formulate a reduced matrix by adding the column-wise and row wise difference together. Indicators are calculated by finding the difference between largest and second largest element and then assign the supply to the largest elements along with the largest indicator. Method gives the direct optimal solution compared with other standard methods in five selected problems.

In 2016, Mollah *et al* developed a method which gives a better initial feasible solution using allocation table method. The cost matrix has been converted into allocation table by keeping the minimum odd cost and subtracting the minimum odd cost from the odd cost cells of the transportation matrix. Then, they perform the allocation to the minimum odd cost values starting from minimum demand or supply. The authors claim that this is a better method than the TOC-SUM method and the standard methods by comparing the solution of Allocation Table Method for some bench marked problems.

In 2016, Reena and Bhathawala proposed a new approach to find optimal solution. In this approach, the smallest cost elements from each row and column are identified in the transportation cost matrix. Subsequently subtract the identified smallest cost from each row and column and then compare the supply and demand, whichever is minimum is then allocate the minimum of the supply or demand. Repeat the process until the supply and demand conditions are satisfied. The authors claimed that for some selected transportation problems the Initial Basic Faceable Solution obtain from this method itself is the optimal solution.

In 2016, Amaravathy *et al* proposed a method named MDMA. In which each cost element is divided by the maximum element and the assignment is made for the minimum value based on supply and demand constraints. In this process the cost values are repeatedly divided by the maximum element and the assignment is taken place in the minimum values until the entire supply and demand conditions are satisfied. The authors assume by providing evidence that this method gives the optimal solution only for selected problems.

In 2017, Kalam and Hossain introduced a new method for solving transportation problem based on the average penalty technique, where the average penalty of each row and column is estimated by finding the average difference between maximum and minimum elements of each row and column. The allocation is done to the minimum cost value corresponds to the row or column which contains the largest average penalty. The allocation process is continued at the available remaining largest penalty row or column until the supply and demand conditions are fulfilled. The authors claim that their method gives a better IBFS for certain transportation problems as compared to the North-West Corner Method, Least Cost Method and Vogel's Approximation Method.

CHAPTER 3

METHODOLOGY

In this study, a new heuristic algorithm was designed to find an improved Initial Basic Feasible Solution (IBFS) to the transportation problem. At the outset, a clear analysis is made based on the three most pervasively used standard methods to find IBFS, namely North-West Corner Rule, Least Cost method and Vogel's Approximation method. The alternative methods proposed to find the IBFS are also analysed to add more information to the existing pool of knowledge.

3.1 Analysis of the Three Standard Methods

This section contains the step by step procedures of the three standard methods to find IBFS along with a numerical example to illustrate the process. Also, the major shortcomings of these methods are pointed out which causes a deviation between the IBFS of these methods and the optimal solution.

3.1.1 North- West Corner Rule (NWCR)

The following are the steps to be followed in a sequential manner so as to arrive at the IBFS by using NWCR:

Step 1: Select the upper left-hand corner cell of the transportation table and allocate as many units as possible equal to the minimum between available supply and demand.

Step 2: Adjust the supply and demand numbers in the respective rows and columns.

Step 3: If the demand for the first cell is satisfied, then move horizontally to the next cell in the second column.

Step 4: If the supply for the first row is exhausted, then move down to the first cell in the second row.

Step 5: If for any cell, supply equals demand, and then the next allocation can be made in cell either in the next row or column.

Step 6: Continue the process until all supply and demand values are exhausted.

Among the existing techniques to obtain IBFS, the North-West Corner Rule is one of the easiest techniques to apply. It consumes minimal number of calculation to obtain the IBSF. The technique is designed in such a way to consider supply and demand constraints only and does not incorporate transportation cost minimization into consideration. As a consequence, the obtained IBFS deviates substantially from the optimal solution.

3.1.2 Least Cost Method (LCM)

LCM can be depicted the stepwise form as given below:

Step1: Select the cell with the least unit transportation cost and allocate as many units as possible to that cell.

Step2: If the minimum cost exists in several cells, select a cell arbitrarily and assign the possible number of goods. Then consider the remaining cells of the same unit transportation cost.

Step3: Select a cell with the next higher unit transportation cost and continue the process till all requirements are met.

In Least Cost Method, the IBFS is found by considering the available minimum costs for assignments at each instant. But this method does not pay attention on all consecutive assignments to the best possible minimum costs. Therefore, this method does not guarantee that all the assignments are made into the best possible minimum costs. In some cases, this method leads to an unavailability of a minimum cost for any row or column due to a previous assignment. This shortcoming of the Least Cost Method may lead to an IBFS which may deviate from the optimal solution.

3.1.3 Vogel's Approximation Method (VAM)

Below explain the VAM in the stepwise form:

Step 1: Determine the difference between the lowest two cells in all rows and columns, including dummies.

Step 2: Identify the row or column with the largest difference.

Step 3: Allocate as much as possible to the lowest-cost cell in the row or column with the highest difference.

Step 4: Stop the process if all row and column requirements are met. If not, go to the next step.

Step 5: Recalculate the differences between the two lowest cells remaining in all rows and columns. Any row and column with zero supply or demand should not be used in calculating further differences. Then go to Step 2

In Vogel's Approximation Method, the IBFS is found based on the difference between the minimum cost and the second minimum cost with respect to each row and column. This technique prevents the immediate next assignment done to a non minimum cost. This method seems to be a better method compared to the other two methods. However, this method also does not ensure for best IBFS for all the transportation problems.

Hence, it can be inferred that these three methods provide IBFS for transportation problems, but these IBFSs may not always be the best for a given transportation problem.

3.2 Analysis of the Related Works from Literature

By referring to the literature discussed in Chapter 2, it was found that in some approaches in finding IBFS, decisions are made based on the original cost matrix of the given transportation problem using different logical reasons compared to the standard methods described above. However, in most of the other proposed approaches, the original cost matrices are converted into different representations and subsequently the decisions are made based on the modified representation matrix.

3.3 Overview of the New Proposed Heuristic Algorithm

In the analysis of standard methods and the approaches available in literature, it was revealed that in most of the cases, initial basic feasible assignments are made based on representation matrix which gives a better IBFS rather than the direct cost matrix. In the transportation problems, the transportation assignments have to be made according to the supply and demand. While doing an assignment to a particular cell it is to be considered that all the next consecutive assignments being made according to the possible minimum cost on that row or column. In this context, Cumulative Difference Representation is introduced which is capable enough to represent the additional excess cumulative costs throughout the row and column for each cost if the particular cost is not assigned. Hereafter, throughout this report the newly proposed method is referred as Cumulative Difference method.

The algorithm of the Cumulative Difference method is given in the following section.

3.4 Algorithm of the Cumulative Difference Method

Step1: Construct the transportation cost matrix $n \times m$ with the cost C_{ij}, supply and demand. If it is an unbalanced problem, it is to be converted as a balance problem.

Step 2: Construct the Cumulative Difference Representation Matrix (CDRM) from the transportation cost matrix.

The ij^{th} entry of the CDRM can be obtained by cumulatively adding the difference between C_{ij} and the reaming element which are larger than C_{ij} throughout i^{th} row and j^{th} column in the cost matrix

Step3: Find the Cumulative Index of each row and column by subtracting the largest and the second largest values of the corresponding row and column of the Cumulative Difference Representation Matrix (CDRM).

Step 4: Identify the row or column with the largest Cumulative Index. Allocate maximum feasible quantity satisfying the demand and supply to the cell corresponding to the highest value in the largest Cumulative Index row or column. Eliminate the row or column corresponding to the highest value cell if the complete allocation is made for the supply or demand.

Step 5: Continue Step 3 and Step 4 until the allocation of all the supply or demand is fulfilled.

Step 7: The allocation gives the Initial Basic Feasible Solution. The transportation cost is given by multiplying the allocations made in CDRM with the transportation cost matrix.

3.5 Illustration of the Cumulative Difference Method

Table 3.4 exhibits data of the considered transportation problem. It can be observed that the problem consists of three buyers and three suppliers, where the buyers' requirements are given along the corresponding supply. Transportation cost from each supplier to each buyer is given in the 3×3 matrix off the margins of the table 3.4 given below:

Table 3.1: Considered transportation problem

	D1	D2	D3	Supply
S1	4	3	5	**90**
S2	6	5	4	**80**
S3	8	10	7	**100**
Demand	**70**	**120**	**80**	

Step1: Construct the transportation cost matrix $n \times m$ with the cost C_{ij}. This is a balanced transportation problem.

Table 3.2: Transportation cost matrix

	D1	D2	D3
S1	4	3	5
S2	6	5	4
S3	8	10	7

Step2: Construct the Cumulative Difference Representation Matrix (CDRM) from the transportation cost matrix.

The ij^{th} entry of the CDRM can be obtained by cumulatively adding the difference between C_{ij} and the reaming element which are larger than C_{ij} throughout i^{th} row and j^{th} column in the cost matrix.

By referring to the Table 3.3, the upper right corner of the cells exhibit the computational process of row wise cumulative difference and bottom left corner values exhibit the column wise cumulative difference of the transportation cost matrix for the given transportation problem.

Table 3.3: Computational illustration of CDRM

0+1 2+4	1+2 2+7	0+0 0+2
0+0 0+2	0+1 0+5	1+2 1+3
2+0 0+0	0+0 0+0	1+3 0+0

By adding row wise of cumulative difference and column wise cumulative difference the Cumulative Difference Representation Matrix (CDRM) of the transportation cost matrix for the given transportation problem in constructed. The Table 3.4 gives the Cumulative Difference Representation Matrix (CDRM) of the transportation cost matrix for the given transportation problem.

Table 3.4: CDRM of the given transportation problem

7	12	2
2	6	7
2	0	4

Step3: Find the Cumulative Index of each row and each column by subtracting the largest and the second largest values of the corresponding row and column of the CDRM.

Table 3.5: CDRM with Cumulative Index

7	12	2	(5)
2	6	7	(1)
2	0	4	(2)
(5)	(6)	(3)	

As it can be seen from the Table 3.5, the largest value of the 1st row is 12 and the second largest value is 7. Therefore, the Cumulative Index of the 1st row is 5 (12-7). Similarly the Cumulative Indexes of each row can be obtained as shown in the Table 3.5 given above.

Similar approach is used to find the Cumulative Indices of columns. Largest value of the 1st column is 7 and the second largest value is 2. Therefore, the Cumulative Index of the column-1 is 5 (7-2).

Step4: Identify the row or column with the largest Cumulative Index. Allocate maximum feasible quantity satisfying the demand and supply to the cell corresponding to the highest value in the largest Cumulative Index row or column. Eliminate the row or column corresponding to the highest value cell if the complete allocation is made for the supply or demand.

The largest Cumulative Index of the above Table 3.5 is 6 corresponding to the column 2. The largest value of column 2 is 12 associated to the cell (1,2). The corresponding supply and demand of that cell is 90 and 120 respectively. Therefore 90 units are allocated to the cell (1,2) based on the supply and demand conditions. By assigning 90 units to the cell (1,2), the available supply units of the supply point 1 is completely assigned. Therefore, the process is continued by deleting the first row and subtracting the 90 units from the demand of the second column.

Table 3.6: CDRM with 1st allocation

	D1	**D2**	**D3**	**Supply**
~~**S1**~~	~~7~~	~~12~~ **(90)**	~~2~~	~~**0**~~
S2	2	6	7	**80**
S3	2	0	4	**100**
Demand	**70**	**120-90**	**80**	

Step 5: Continue Step 3 and Step 4 until ensuring that the allocation of all the supply or demand is completed

Loop 1:

Step 3:

The reduced Cumulative Difference Representation Matrix (CDRM) after deleting the first row is tabulated in the Table 3.7 below:

Table 3.7: Reduced CDRM after the 1st allocation

2	6	7
2	0	4

Table 3.8: Reduced CDRM with Cumulative Index after the 1st allocation

2	6	7	(1)
2	0	4	(2)
(0)	(6)	(3)	

Step 4:

The largest cumulative index of the above Table 3.8 is 6 corresponding to the column 2. The largest value of column 2 is 6 associated to the cell (1, 2). The corresponding supply and needed demand of that cell are 80 and 30 respectively. Therefore 30 units are allocated to the cell (1, 2) based on the supply and demand conditions. By assigning 30 units to the cell (1,2), the needed demand units of the demand point 2 is completely allocated. Therefore the process is continued by deleting the second column and subtracting the 30 units from the available supply of the second row.

Table 3.9: CDRM with 2nd allocation

	D1	**D2**	**D3**	**Supply**
S2	2	6 **(30)**	7	**80-30**
S3	2	0	4	**100**
Demand	**70**	**30**	**80**	

Loop 2:

Step 3:

The reduced Cumulative Difference Representation Matrix (CDRM) after deleting the second column is given in the Table 3.10 given below:

Table 3.10: Reduced CDRM with Cumulative Index after the 2nd allocation

2	7	(5)
2	4	(2)
(0)	(3)	

The reduced Cumulative Difference Representation Matrix (CDRM) as illustrated in Table 3.10 contains exactly two values in each row and column. Therefore, the Cumulative Index of the row-1 is 5 (7-2). Similarly, the Cumulative Index of the second row is 2 (4-2).

The Cumulative Index of the column-1 is 0 (2-2) and the Cumulative Index of the second column is 3 (7-4)

Step 4:

The largest cumulative index of the aforementioned Table 3.10 is 5 which corresponds to row 1. The largest value of row 1 is 7 associated to the cell (1,2). The corresponding supply and demand of that cell are 50 and 80 respectively. Therefore, 50 units are allocated to the cell

(1,2) based on the supply and demand conditions. By assigning 50 units to the cell (1,2), the available supply units of the supply point 2 is completely assigned. Therefore, the process is continued by deleting the first row and subtracting the 50 units from the demand point 2 of the second row.

Table 3.11: CDRM with 3rd allocation

	D1	D3	Supply
~~**S2**~~	~~2~~	~~7 **(50)**~~	~~**50**~~
S3	2	4	**100**
Demand	**70**	**80-50**	

Loop 3:

The reduced Cumulative Difference Representation Matrix (CDRM) after deleting the first row is given in the Table 3.12.

Table 3.12: Reduced CDRM after 3rd allocation

2 **70**	4 **30**	**100**
70	**30**	

Since the reduced CDRM after the 3rd allocation contains only one row assignment is done in that row in order to fulfil the demand and supply conditions.

Step 7: The allocation given is the Initial Basic Feasible Solution

The following Table 3.13 shows the complete assignment on the Cumulative Difference Representation Matrix (CDRM) after completion of the assignments.

Table 3.13: Complete transportation assignments on CDRM

7	12 **90**	2	**90**
2	6 **30**	7 **50**	**80**
2 **70**	0	4 **30**	**100**
70	**120**	**80**	

These assignments are transformed to the transportation cost matrix of the given problem as shown in the Table 3.14 below:

Table 3.14: Assignments of IBFS to the given transportation problem

	D1	**D2**	**D3**	**Supply**
S1	4	3 **90**	5	**90**
S2	6	5 **30**	4 **50**	**80**
S3	8 **70**	10	7 **30**	**100**
Demand	**70**	**120**	**80**	

The Initial Basic Feasible Solution to the given problem is summarized in the following Table 3.15:

Table 3.15: Summary of IBFS of the given transportation problem

Source	Destination	Number of units	Unit transportation cost	Cost
S1	D2	90	3	270
S2	D2	30	5	150
S2	D3	50	4	200
S3	D2	70	8	560
S3	D3	30	7	210
Total transportation cost				1390

CHAPTER 4

RESULTS AND DISCUSSION

This chapter is directed towards presenting the analysis of test results and the discussion on that. This chapter is presented with the intention of showing that the newly proposed Cumulative Difference method is capable of finding a better and more promising IBFS to balanced or unbalanced transportation problems by comparing with standard methods, namely North West Corner Rule, Least Cost method and Vogel's Approximation method and some of the selected alternative methods discussed in the literature. At the end of the chapter, results are shown based on the performance analysis conducted for large scale transportation problems.

4.1 Comparative Study of IBFS of the Cumulative Difference Method with the Standard Methods

In this section, the IBFS obtained by the Cumulative Difference method is compared with the IBFS obtained by the three standard methods mentioned above.

The proposed Cumulative Difference method is coded in Matlab environment along with the standard methods for finding the IBFS. Also MODI is coded in Matlab environment to arrive at the optimal solution from the IBFS. Obtained optimal solutions are verified using Excel Solver.

Twenty transportation problems with some degree of variation were selected and the obtained IBFS using the Cumulative Difference method was compared with respect to the standard methods in this study. The results are shown in the Table 4.1. The optimal solutions to the given problems are also indicated in the last column of the table.

Table 4.1: IBFS outcomes of the selected problems

Problem No	Dimension of the problem	Transportation cost corresponding to the IBFS				Optimal Solution
		North-West Corner Rule Method	Least Cost Method	Vogel's Approximation Method	Cumulative Difference Method	
01	6×6	4 105	2 455	2 310	2 170*º	2 170
02	3×4	273	231	204	204*	200
03	3×4	4 400	2 900	2 850	2 850*º	2 850
04	3×4	116	112	102	102*	100
05	3×4	1 500	1 450	1 500	1 390*º	1 390
06	3×3	914	674	750	674*º	674
07	4×4	540	435	470	430*	410
08	3×4	4 160	3 500	3 320	3 320*º	3 320
09	3×3	74	71	73	63*	46
10	3×3	1 180	1 080	1 020	1 020*º	1 020
11	5×4	629	477	390	390*	378
12	3×4	470	470	475	435*º	435
13	4×4	848	884	856	844*	821
14	3×4	2 820	2 090	2 170	2 040*º	2 040
15	4×3	102	83	80	76*º	76
16	3×3	5 925	4 550	5 125	4 525*º	4 525
17	5×5	1 977	1 046	1 118	1 038*	1 034
18	4×5	2 235	2 090	1 175	1 175*º	1 175
19	4×4	1 068	1 004	902	890*º	890
20	5×4	5 161	4 364	4 354	4 294*º	4 294

* IBFS obtained by the Cumulative Difference method is better than the standard method

º IBFS obtained by the Cumulative Difference method is itself optimum solution

The outcomes presented in the Table 4.1 are summarized in the Table 4.2 given below:

Table 4.2: Summary of the comparative study

	North-West Corner Rule Method		Least Cost Method		Vogel's Approximation Method		Cumulative Difference Method	
	Best IBSF	IBSF is optimal	Best IBSF	IBSF is optimal	Best IBSF	IBSF is optimal	Best IBSF	IBSF is optimal
No of occurrences	0	0	1	1	7	4	20	13

According to the Table 4.2, it is apparent that the proposed Cumulative Difference method gives the best IBFS in all the problems while for 7 problems the best IBFS is provided by Vogel's Approximation method. The Least Cost method gives only one IBFS and none in North-West Corner Rule method. Out of these 20 best IBFS obtained by the Cumulative Difference method 13 of them produced the optimum solution.

The following Table 4.3 shows the percentage of deviation of the transportation cost at IBFS obtained by the respective standard methods with respect to the Cumulative Difference method:

Table 4.3: Deviation between the transportation cost at IBFS and the optimum transportation cost

Problem number	Dimension of the problem	Deviation			
		North-West Corner Rule Method	Least Cost Method	Vogel's Approximation Method	Cumulative Difference Method
01	6×6	1 935	285	140	0
02	3×4	73	31	4	4
03	3×4	1 550	50	0	0
04	3×4	16	12	2	2
05	3×4	110	60	110	0
06	3×3	240	0	76	0
07	4×4	130	25	60	20
08	3×4	840	180	0	0
09	3×3	28	25	27	17
10	3×3	160	60	0	0
11	5×4	251	99	12	12
12	3×4	35	35	40	0
13	4×4	27	63	35	23
14	3×4	780	50	130	0
15	4×3	26	7	4	0
16	3×3	1 400	25	600	0
17	5×5	943	12	84	4
18	4×5	485	340	0	0
19	4×4	178	114	12	0
20	5×4	867	70	60	0

Table 4.3 given above exhibits that out of the tested transportation problems, 60% of them reached the optimal solution using the Cumulative Difference method. This indicates that 3 out of 5 times, method not only finds the best IBFS but also it reaches the optimal solution.

The figures 4.1, 4.2 and 4.3 graphically represent the percentage of deviation of the transportation cost at IBFS obtained by the respective standard methods with respect to the Cumulative Difference method:

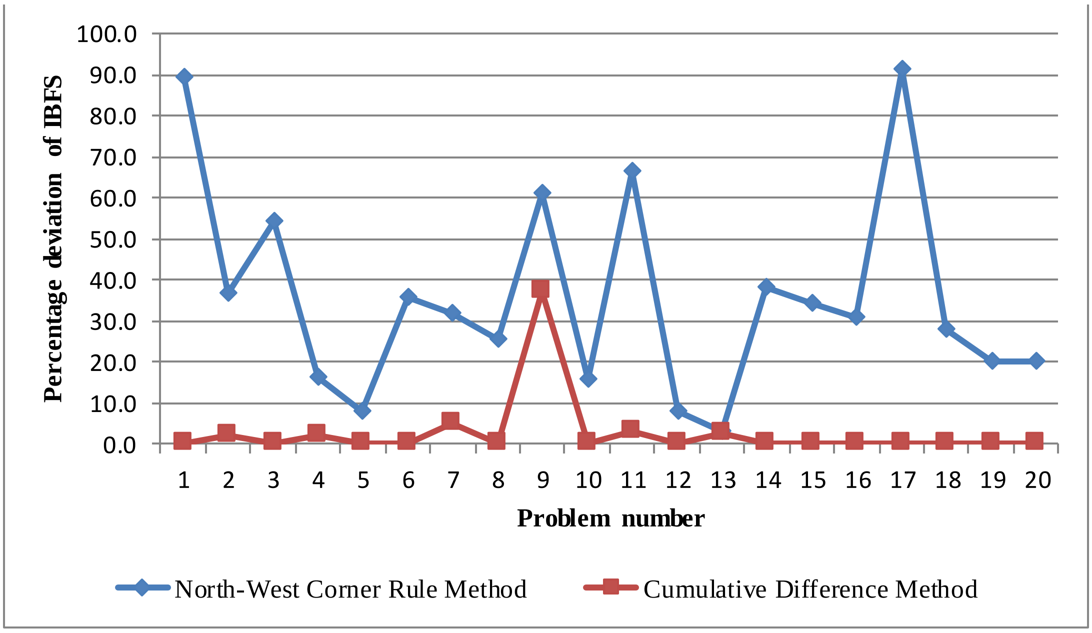

Figure 4.1: Percentage deviation of the IBFS of North-West Corner Rule and Cumulative Difference method

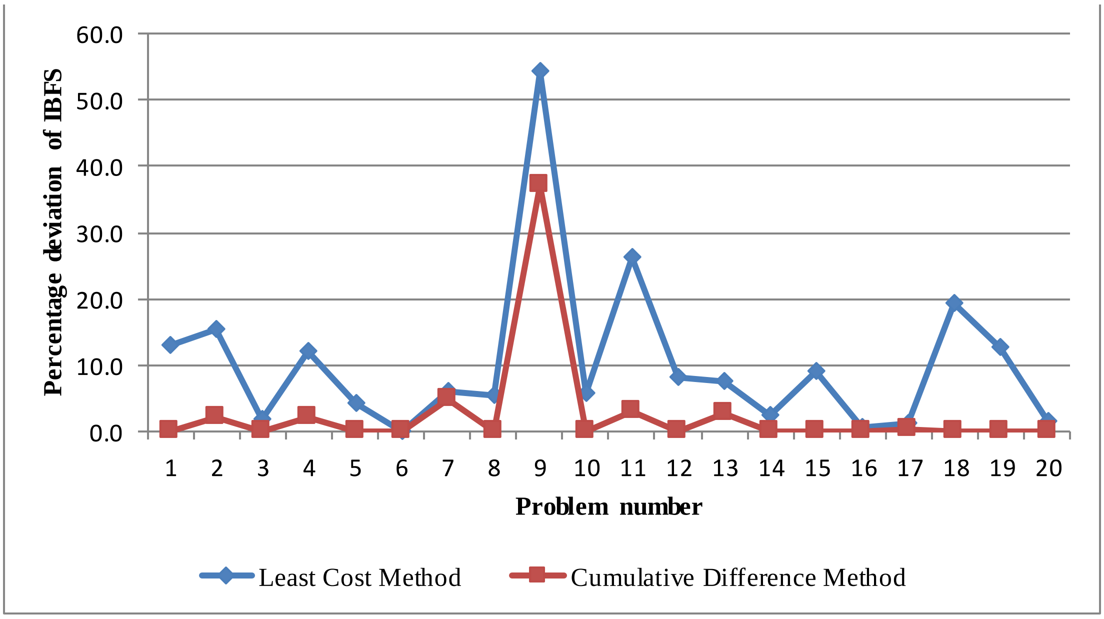

Figure 4.2: Percentage deviation of the IBFS of Least Cost method and Cumulative Difference method

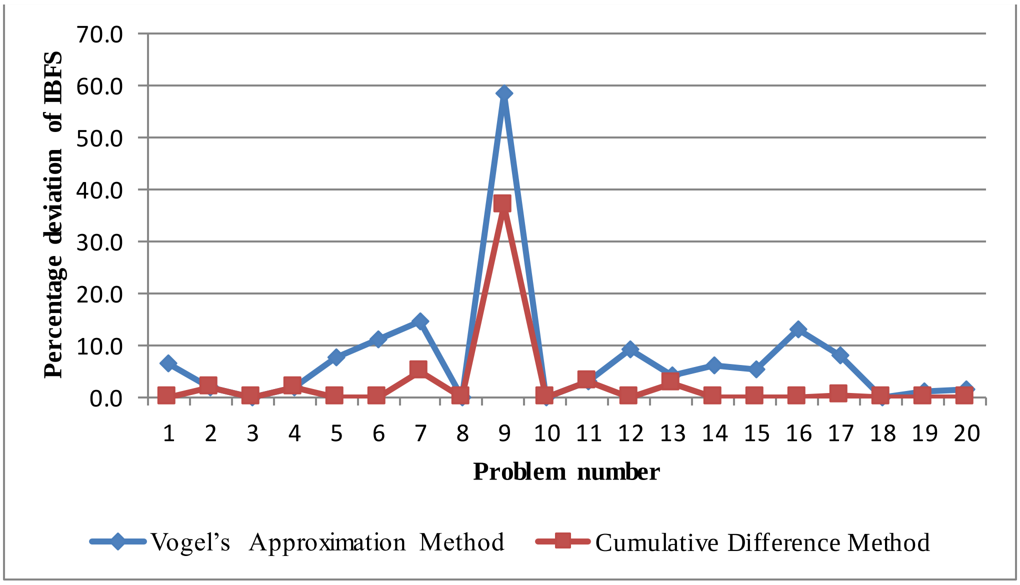

Figure 4.3: Percentage deviation of the IBFS of Vogel's Approximation method and Cumulative Difference method

A graphical illustration of the percentage of deviations of the transportation cost at IBFS from the optimum cost in each standard method and the Cumulative Difference method is given in Figure: 4.4.

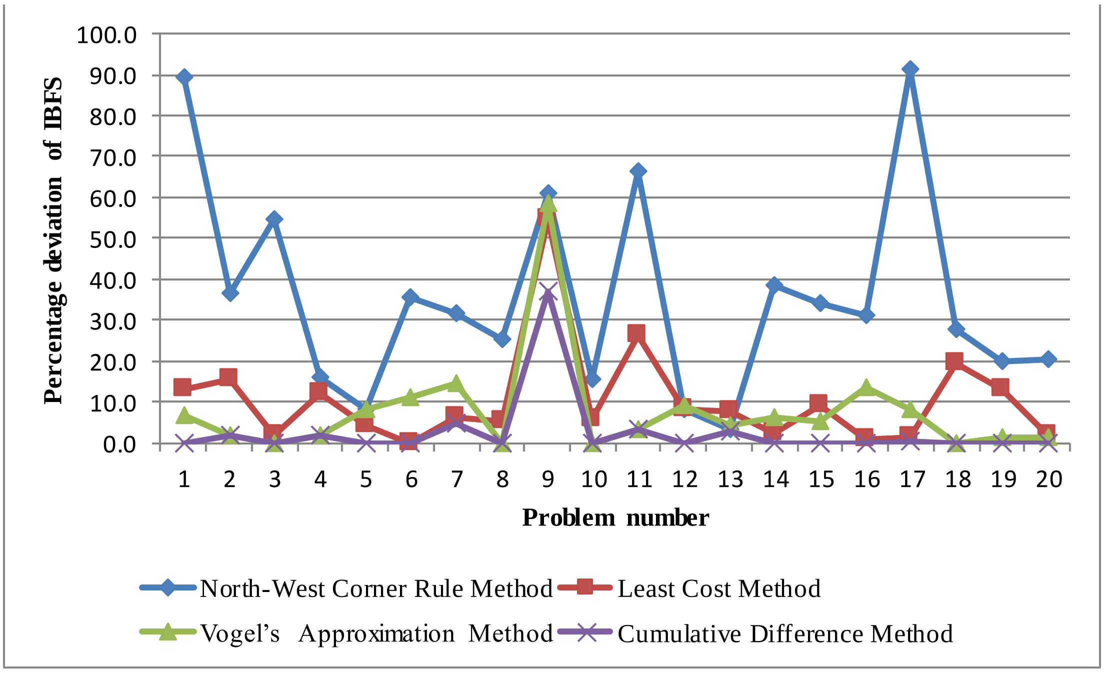

Figure 4.4: Percentage deviation of the IBFS Cumulative Difference method and standard methods

From the figures 4.1, 4.2, 4.3 and 4.4 it can be observed that the IBFS obtained using the Cumulative Difference method shows less deviation from optimal solution than all three standard methods.

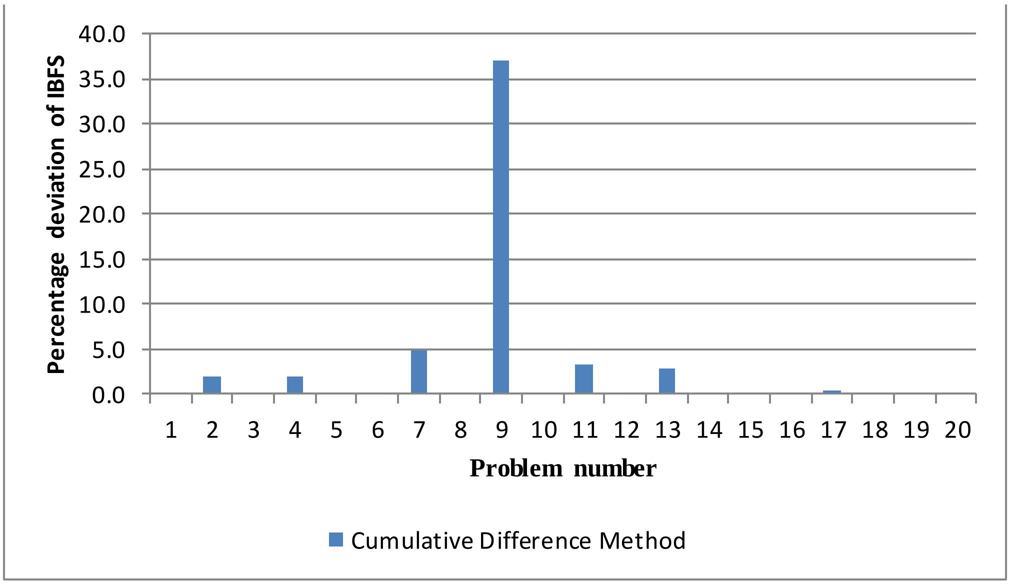

Figure 4.5: Histogram of percentage deviation of transportation cost at the IBFS

The above histogram shows the percentage deviations of the transportation cost at the IBFS of the Cumulative Difference method from the optimal solutions for the selected problems. Out of these 20problems, 13 problems do not have any deviation, 6 problems show a deviation of less than 5% and only one problem shows a deviation of more than 5%. Hence from the Figure 4.5 it is evident Cumulative Difference method is capable of providing an improved IBFS over the three standard methods.

One of the most important performance indicators in the analysis of an algorithm is the convergence rate or how fast the algorithm reaches the optimal solution. In an iterative algorithm, convergent rate can be measured by counting the number of iterations to reach the optimal solution. Table 4.4 shows this result for the Cumulative Difference method along with the standard methods.

Table 4.4: Number of Iterations needed to reach the optimality from the IBFS obtained from the Cumulative Difference method and the standard methods

Problem number	Number of Iterations			
	North-West Corner Rule Method	Least Cost Method	Vogel's Approximation Method	Cumulative Difference Method
1	6	4	3	0
2	4	2	1	1
3	2	0	0	0
4	4	3	2	2
5	2	1	2	0
6	3	0	1	0
7	5	2	3	1
8	2	0	0	0
9	2	2	2	2
10	3	2	0	0
11	6	5	1	1
12	2	2	3	0
13	3	3	2	2
14	4	1	2	0
15	2	2	0	0
16	3	1	2	0
17	4	1	2	1
18	4	1	0	0
19	6	4	1	0
20	5	3	2	0

From the Table 4.4 give above it can be observed that the IBFS obtained by the Cumulative Difference method always take lesser number of iterations to reach the optimal solution than the IBFS obtained by other standard methods.

The graphical representation of the Table 4.4 is given in the Figure 4.6:

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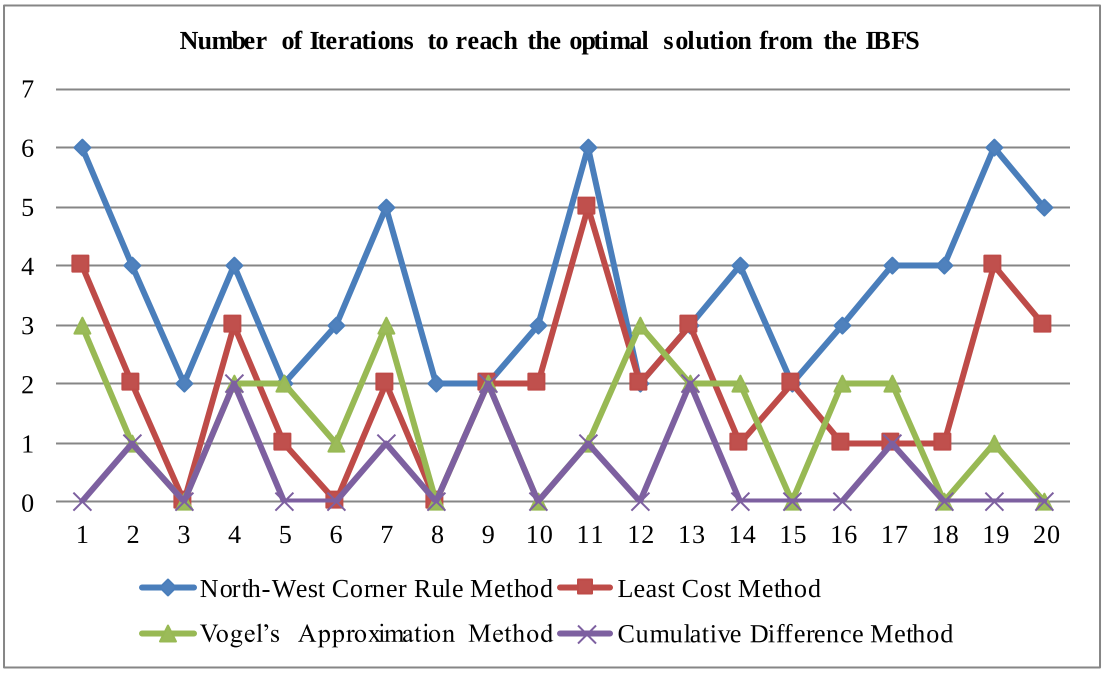

Figure 4.6: Number of iterations needed to reach the optimality from the IBFS

By referring to the Table 4.4 and Figure 4.6 it can be concluded that 60% of the time the IBFS obtained from the Cumulative Difference method itself is the optimal solution to the transportation problem and rest of the time IBFS is closer to the optimal solution than to the standard methods.

The Table 4.5 given bellow displays the time consumed to reach the optimal solution from the IBFS obtained using the Cumulative Difference method and the standard methods for the twenty problems considered in this section.

Table 4.5: Time consumed to reach the optimal solution from the IBFS

Problem number	Time needed to reach the optimal solution from the IBFS (in milliseconds)			
	North-West Corner Rule Method	Least Cost Method	Vogel's Approximation Method	Cumulative Difference Method
1	0.598	0.399	0.532	0.042
2	0.882	0.441	0.230	0.041
3	0.852	0.045	0.038	0.040
4	0.530	0.386	0.036	0.040
5	0.779	0.390	0.785	0.046
6	0.842	0.038	0.228	0.039
7	0.501	0.385	0.420	0.037
8	0.847	0.047	0.039	0.040
9	0.678	0.360	0.370	0.340
10	0.838	0.049	0.044	0.044
11	0.557	0.390	0.049	0.046
12	0.860	0.800	0.920	0.034
13	0.858	0.855	0.050	0.049
14	0.678	0.110	0.240	0.033
15	0.740	0.730	0.041	0.032
16	0.844	0.210	0.430	0.030
17	0.720	0.061	0.302	0.049
18	0.833	0.180	0.033	0.031
19	0.843	0.423	0.148	0.032
20	0.846	0.394	0242	0.034

Table 4.5 shows that more often the proposed Cumulative Difference method converges to the optimal solution from the IBFS in lesser time.

Table 4.6 displays the total time consumed to reach the optimal solution for the selected problems. In which the time consumed to reach the IBFS as well as the time consumed to reach the optimal solution using MODI method from the IBFS are included.

Table 4.6: Time consumed to reach the optimal solution

Problem number	Time consumed to reach the optimal solution (in milliseconds)			
	North-West Corner Rule Method	Least Cost Method	Vogel's Approximation Method	Cumulative Difference Method
1	0.997	0.588	0.877	0.521
2	1.325	0.642	0.575	0.525
3	1.285	0.250	0.374	0.511
4	1.010	0.551	0.309	0.415
5	1.185	0.595	1.165	0.700
6	1.287	0.234	0.571	0.537
7	0.913	0.583	0.756	0.517
8	1.279	0.241	0.393	0.536
9	1.027	0.518	0.646	0.727
10	1.259	0.242	0.397	0.524
11	0.990	0.596	0.411	0.546
12	1.290	1.000	1.260	0.508
13	1.282	1.049	0.396	0.523
14	1.029	0.271	0.448	0.429
15	1.083	0.885	0.411	0.435
16	2.010	0.404	0.778	0.499
17	1.173	0.261	0.656	0.545
18	1.282	0.370	0.386	0.413
19	1.277	0.613	0.487	0.453
20	1.279	0.566	0.578	0.423

Table 4.6 shows that more often the selected problems converge to optimal solution in less time when using IBFS of the Cumulative Difference method as the starting solution.

Summarizing the results obtained from all the information gathered, it can be concluded that the IBFS obtained by the Cumulative Difference method converges to the optimal solution is more efficient than the standard methods in terms of number of iterations as well as time consumed.

This implies that the proposed Cumulative Difference method is more capable of providing an improved IBFS than the standard methods.

4.2 Comparative study of IBFS of the Cumulative Difference method with the alternative methods in the literature

In order to study the efficiency of the proposed Cumulative Difference method the IBFS obtained by the Cumulative Difference method is compared with the IBFS obtained by the selected related works in the literature.

Twenty eight transportation problems presented in the selected literature were selected and the obtained IBFS using the Cumulative Difference method was compared with respect to the alternative method proposed in the respective literature along with the standard methods in this study. The results are shown in the Table 4.7. The optimal solutions to the given problems are also indicated in the last column of the table.

Table 4.7: IBFS outcomes of the selected problems from literature

Problem No	Transportation cost corresponding to the IBFS					
	Method discussed in the literature	North-West Corner Rule Method	Least Cost Method	Vogel's Approximation Method	Cumulative Difference Method	Optimal Solution
01	1 005	1 220	1 075	1 005	1 000	1 000
02	80	102	83	76	76	76
03	11 500	19 700	13 100	12 250	11 800	11 500
04	200	273	231	204	204	200
05	1 690	1 815	1 885	1 745	1 650	1 650
06	2 170	4 285	2 455	2 310	2 170	2 170
07	15 910	30 570	26 310	16 130	15 910	15 910
08	112	139	114	112	112	112
09	796	1 095	922	796	796	796
10	1 580	1 950	1 870	1 580	1 580	1 580
11	100	116	112	102	102	100
12	743	1 015	814	779	743	743
13	2 040	2 820	2 090	2 170	2 040	2 040
14	674	914	674	750	674	674
15	988	1010	988	988	988	988
16	381	621	423	391	381	381
17	20 550	25 530	21 450	21 030	20 550	20 550
18	2 850	4 400	2 850	2 850	2 850	2 850
19	3 320	4160	3 320	3 320	3 320	3 320
20	430	540	435	470	430	410
21	1 390	1 500	1 450	1 500	1 390	1 390
22	1 200	1 490	1 200	1 200	1 200	1 200
23	412	484	516	476	412	412
24	267	878	555	267	267	267
25	695	1 095	705	695	695	695
26	248	320	248	240	240	240
27	68	93	79	68	68	68
28	139	150	145	150	139	139

The following Table 4.8 summarizes the results of the comparative study of the proposed Cumulative Difference method with the related works in literature. The table shows that the number of optimum solutions obtained at the IBFS using the methods proposed in literature as well as the proposed Cumulative Difference method for the problems discussed in each literature.

Table 4.8: Summary of the comparison of IBFS between the Cumulative Difference method and the methods proposed in the literature

S. No	Methods discussed in the literature	No of Problems Discussed	Transportation cost at IBFS is Optimum	
			Methods in Literature	Cumulative Difference Method
01	Best Candidate Method	1	0	1
02	Cost Minimization Approach	2	1	1
03	TOCM-SUM Approach	3	2	2
04	Maximum Difference Method	4	4	4
05	Monalisha's Approximation Method	2	2	1
06	Modified Vogel's Approximation Method	5	5	5
07	Allocation Table Method	4	3	4
08	Approach Proposed by Reena and Bhathawala	3	3	3
09	MDMA Method	1	1	1
10	Average Penalty Method	3	3	3
	Total Problems	**28**	**24**	**25**

By referring to the Table 4.8, the transportation cost at IBFS of the proposed Cumulative Difference method is the optimal transportation solution for twenty five transportation problems out of 28 problems presented in the selected literature.

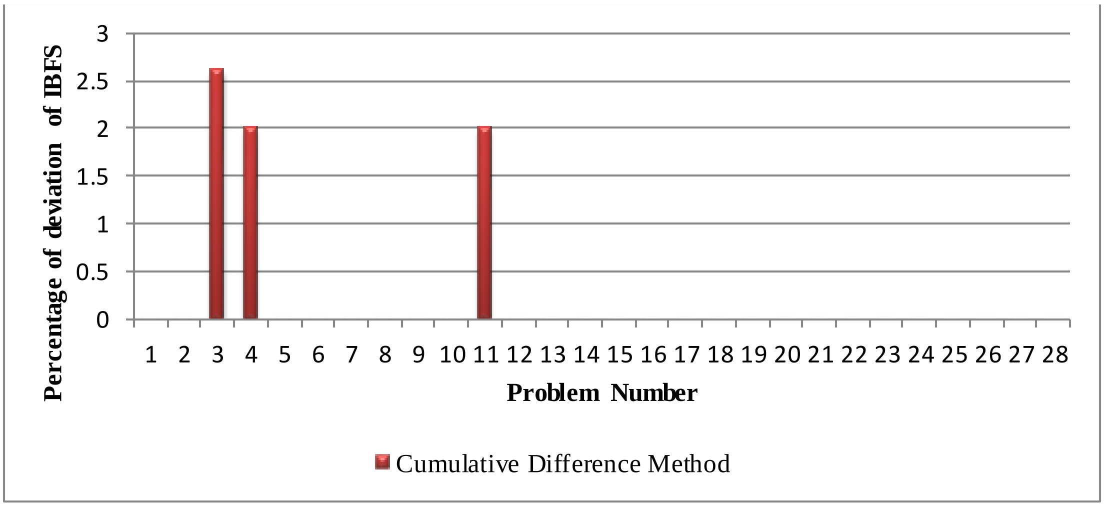

Figure 4.7: Histogram of percentage of deviation of the transportation cost at IBFS of the Cumulative Difference method from optimum transportation cost

The histogram in Figure 4.7 shows the percentages of deviation of the transportation cost at IBFS of the Cumulative Difference method from the optimum solution for the 28 problems discussed in the ten selected literature. From the histogram it can be noticed that out of these 28 problems, 25 problems do not have any deviation since itself they are optimal solutions. It is an indication that the IBFS are optimum solutions. The rest of the four problems show a deviation of less than three percentage.

The histogram shown in Figure: 4.8 reveals the percentage of deviation of transportation cost at IBFS obtained using the three standard methods as well as the proposed Cumulative Difference method from the optimal solution for 28 transportation problems presented in the selected literature. From the histogram, it can be observed that the IBFS obtained by the standard methods show a higher deviation from the IBFS to optimal transportation cost compared to the Cumulative Difference method.

Hence, the IBFS obtained using the Cumulative Difference method is better than that of standard methods.

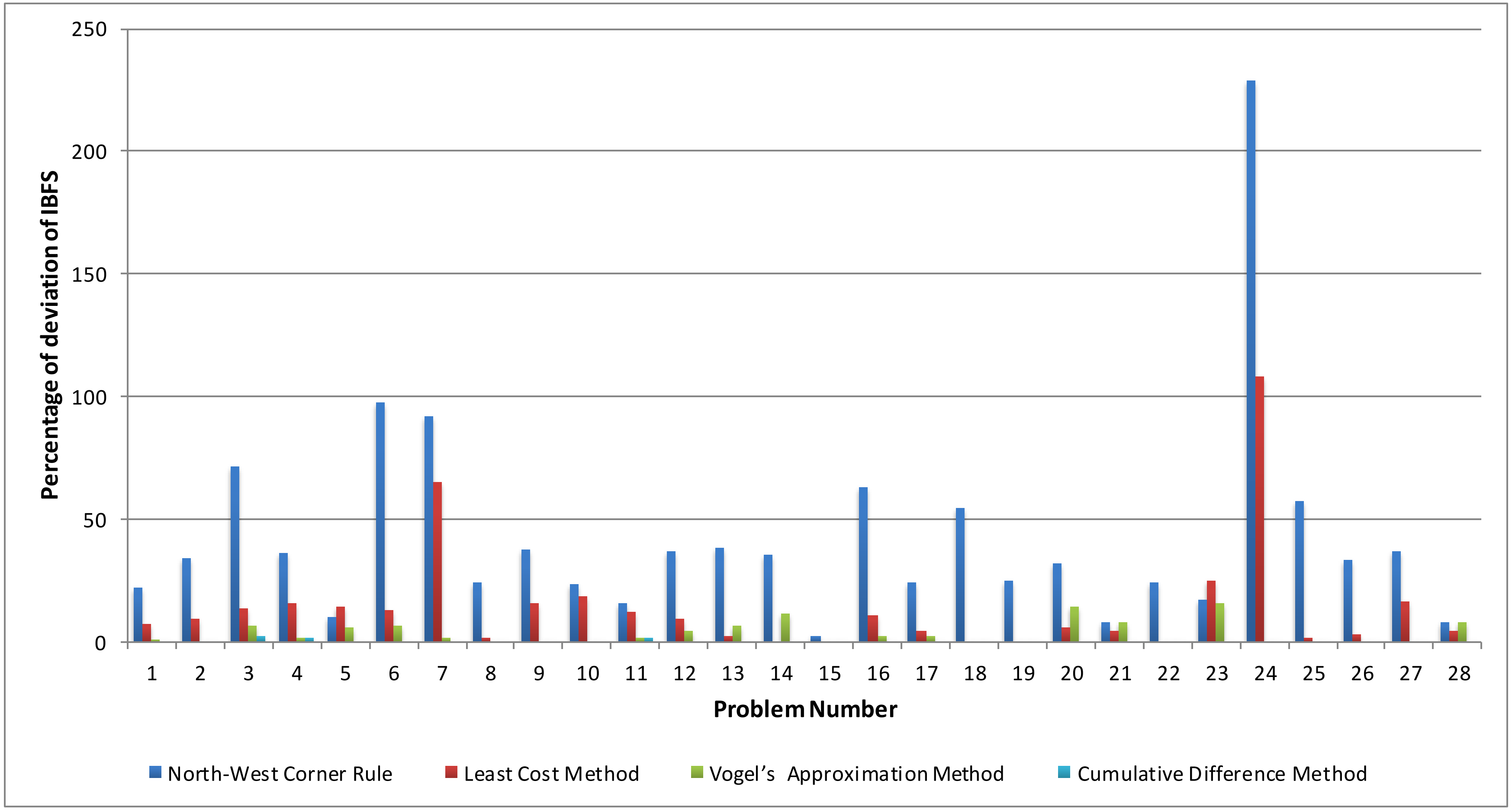

Figure 4.8: Percentage deviation from the optimal transportation cost to IBFS

4.3 A Comparative Study between Cumulative Difference Method and Standard Methods for Large Scale Problems

For this study, 22 transportation problems were randomly selected with the dimension in the range of 10×10 to 26×26and obtained IBFS with respect to the standard methods and the Cumulative Difference method which are shown in Table 4.9 along with the optimal solutions.

Table 4.8: IBFS of the selected large scale problems

Problem No	Dimension of the problem	Transportation cost corresponding to the IBFS				Optimal Solution
		North-West Corner Rule	Least Cost Method	Vogel's Approximation Method	Cumulative Difference Method	
01	10×10	20 880	14 115	12 750	12 660* O	12 660
02	10×11	41 736	26 519	27 734	25 942*	24 685
03	10×12	6 7961	43 037	37 499	37 499*	37 390
04	10×14	493 385	205 146	198 587	196 163*	183 133
05	16×15	334 116	134 191	128 731	113 657*O	113 657
06	15×15	245 618	122 928	118 977	117 783*	112 053
07	15×15	362 149	150 349	132 184	131 179*	123 319
08	17×15	275 777	101 719	106 462	99 273*	88 745
09	18×15	2 005 297	899 287	857 944	832 848*	729 183
10	19×15	1938152	1 186 027	1 128 156	1 108 143*	1 064 772
11	15×16	22 988	11 748	10 786	10 222*	9 883
12	15×17	18 801	10 199	9 961	9 228*	8 671
13	15×18	42 309	24 206	25 249	22 736*	22 354
14	15×19	29 646	16 110	14 338	13 722*	13 324
15	15×22	164 819	85 384	82 219	78 551*	73 631
16	20×20	140 902	62 620	54 487	52 758*	50 859
17	21×21	169 101	67 840	53 487	51 431* O	51 431
18	22×22	183 665	72 792	74174	64 167*	60 657
19	23×23	158 812	61 940	54 146	5 0821*	48 731
20	24×24	165 210	64 668	55 788	53 010*	50 357
21	25×25	77 967	28 523	30 653	27 813*	25 498
22	26×26	65 026	33 479	33 961	30 216*	28 568

* IBFS obtained by the Cumulative Difference method is better than the standard method

O IBFS obtained by the Cumulative Difference method is itself the optimum solution

By referring to the Table 4.9, it is apparent that the Cumulative Difference method gives a better IBFS in all 22 problems as compared to the standard methods and also three of them are optimal solutions as well.

The Table 4.10 shows the deviations of the transportation cost at IBSF from the optimal solution.

Table 4.10: Deviation of the IBSF from the optimal solution

Problem No	Dimension of the problem	Deviations			
		North-West Corner Rule	Least Cost Method	Vogel's Approximation Method	Cumulative Difference method
01	10×10	8 220	1 455	90	0
02	10×11	17 051	1 834	3 049	1 257
03	10×12	30 571	5 647	109	109
04	10×14	310 252	22 013	15 454	13 030
05	16×15	220 459	20 534	15 074	0
06	15×15	133 565	10 875	6 924	5 730
07	15×15	238 830	27 030	8 865	7 860
08	17×15	187 032	12 974	17 717	10 528
09	18×15	1 276 114	170 104	128 761	103 665
10	19×15	873 380	121 255	63 384	43 371
11	15×16	13 105	1 865	903	339
12	15×17	10 130	1 528	1 290	557
13	15×18	19 955	1 852	2 895	382
14	15×19	16 322	2 786	1 014	398
15	15×22	91 188	11 753	8 588	4 920
16	20×20	90 043	11 761	3628	1 899
17	21×21	117 670	16 409	2 056	0
18	22×22	123 008	12 135	13 517	3 510
19	23×23	110 081	13 209	5 415	2 090
20	24×24	114 853	14 311	5 431	2 653
21	25×25	52 469	3 025	5 155	2 315
22	26×26	36 458	4 911	5 393	1 648

As it can be seen from the Table 4.8 the IBFS of the Cumulative Difference method shows a lesser deviation from the optimal solution compared to the IBFS of the standard methods.

Figures 4.9, 4.10 and 4.11 graphically represent the percentage of deviation of the transportation cost at IBFS obtained by the standard methods and the Cumulative Difference method.

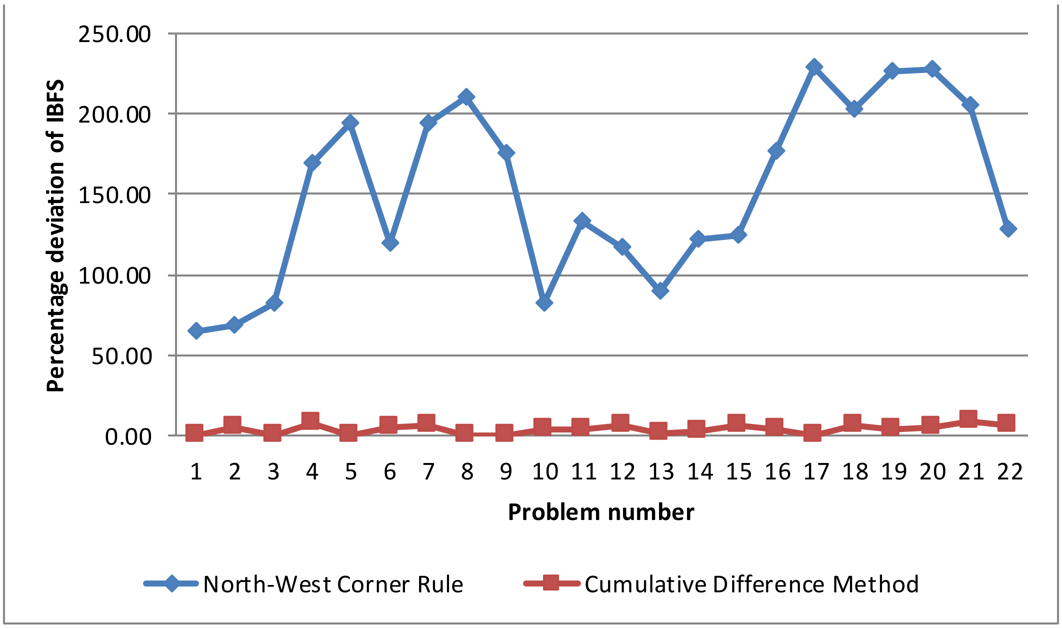

Figure 4.9: Percentage deviation of the IBFS of North-West Corner Rule vs. Cumulative Difference method

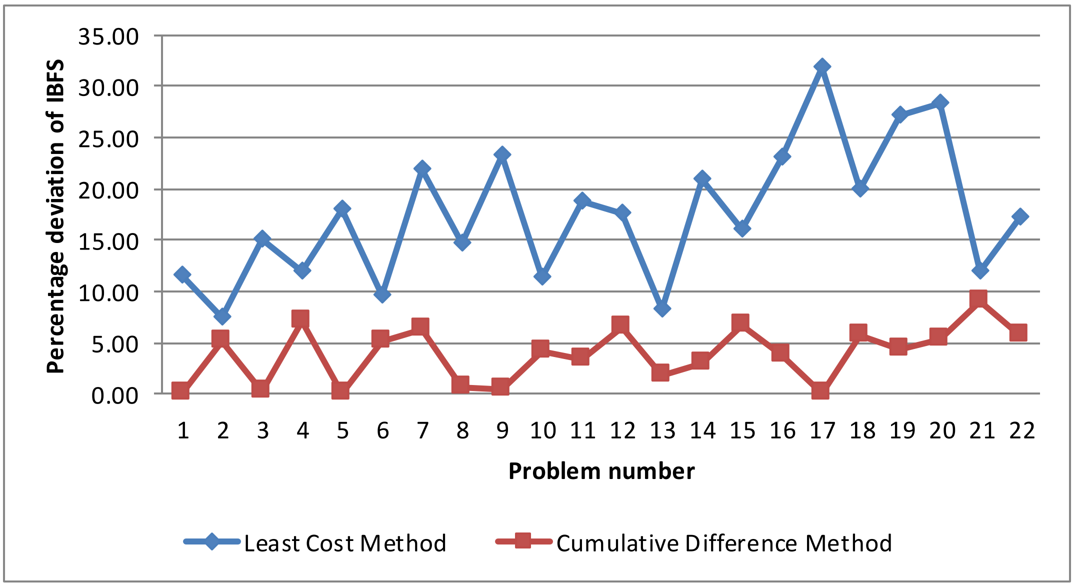

Figure 4.10: Percentage deviation of the IBFS of Least Cost method vs. Cumulative Difference method

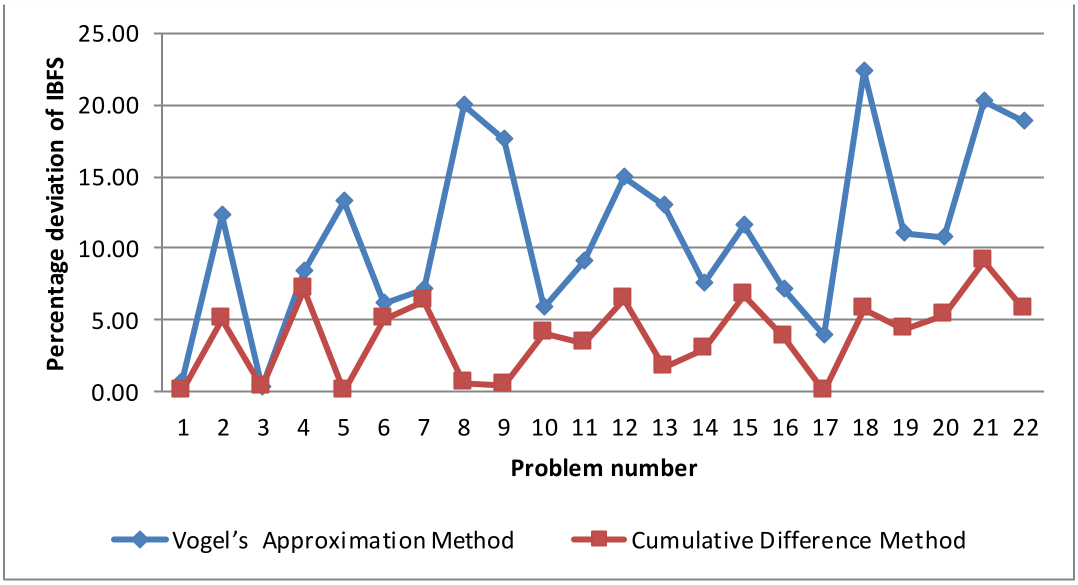

Figure 4.11: Percentage deviation of the IBFS of Vogel's Approximation method vs. Cumulative Difference method

From the Figures 4.8, Figure 4.9 and Figure 4.10 it can be observed that the percentage of deviation of the transportation cost at IBFS of the North-West Corner Rule lies in between 50 percentage and 250 percentage. The percentage of deviation of the Least Cost method lies in between 5 percentage and 35 percentage. The percentage of deviation of Vogel's Approximation method goes up to 25. Meanwhile, the percentage of deviation of the transportation cost at IBFS of the proposed Cumulative Difference method shows a deviation only up to 10 percentage. It implies the Cumulative Difference method is capable of find a better IBFS for large scale transportation problems.

As an overall summary the IBFS obtained by the proposed Cumulative Difference method shows less deviation from the optimal solution as compared to the standard methods namely North-West Corner Rule, Least Cost method and Vogel's Approximation method. The IBFS obtained by Cumulative Difference method converges the optimal solution with highest convergent rate in terms of number of iteration as well as computational time. It exemplifies that the newly proposed Cumulative Difference method is sound enough to provide an improved primal solution to the transportation problems.

CHAPTER 5

CONCLUSION

In this research study, we focused on designing a better algorithm in finding an initial feasible solution to solve the transportation problem, which is a main concern in industries which manufacture goods. In the history of transportation problem, many researchers attempted to design different algorithms in finding an IBSF. Among the numerous methods proposed by researchers, most celebrated ones are namely, North-West Corner Rule, Least Cost method and Vogel's Approximation method which have weaknesses.

In this study, the three standard methods are analysed in order to find the weaknesses and thereby, to provide a better IBFS for a wider range of transportation problems by introducing a new heuristic approach. The North-West Corner Rule is one of the easiest techniques to apply. The technique is designed in such a way that it considers only supply and demand constraints and does not incorporate transportation cost minimization into consideration. As a consequence, the obtained IBFS deviates highly from the optimal solution. In the Least Cost Method, the IBFS is found by considering the available minimum costs for assignments at each instant. But this method does not pay any consideration for all consecutive assignments to find the best possible minimum costs. Therefore, this method does not guarantee that all the assignments are made into the best possible minimum cost. The Vogel's approximation method does not guarantee that the next immediate assignment is made into a minimum cost. This method seems to be a better method compared to the other two methods. However, this method also does not ensure the best IBFS to many transportation problems. It is revealed that none of these methods has the ability to ensure that all the assignments which are made to minimize the cost.

In transportation problems, the transportation assignments have to be made repeatedly until the complete supply and demand with respect to rows and columns are met. While making a particular assignment to a cell in a row or column, it is ideal to consider all the following assignments which fall in the possible minimum cost. It is impractical to keep track of all the following assignments while making a particular assignment. In fact, it is more time consuming and especially for large scale problems, it is almost impossible.

Considering all these issues, in this research project, an algorithm named as an Cumulative Difference method is proposed. In which a Cumulative Difference Representation is used as an alternative to the direct transportation cost. In the new approach, each iteration assignment is made after considering all the costs in the cost matrix and their respective excess cumulative cost. By making the assignments based on this newly proposed Cumulative Difference method, a more improved IBFS can be obtained compared to other existing methods.

It was found that the newly proposed method is slightly time consuming to reach the IBFS compared to other existing methods. But, this is compensated by the less time consumed to reach the optimal solution. In further studies, the new algorithm can be revised carefully by analysing the data structure to manage the execution time in a more robust way. Also in further, some techniques can be introduced to find a better IBFS with less computations or the optimal solution with less number of iterations.

Based on this study, it can be concluded that the newly proposed method to find IBFS is capable enough to provide an improved IBFS over the other methods discussed in this report resulting in finding the optimal solution with less computations and computational time.

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APPENDIX

Appendix-01 MATLAB 2015 Code

Code for MODI Method

```
function [ x,tc ] = modi( x,a)

    b=a;
    r=size(a,1);
    c=size(a,2);

    lc=0;
    k=0;
     for i=1:r
         for j=1:c
             if(x(i,j)~=0)
                 k=k+1;
             end
         end
     end


    nbc=zeros(r,c);
    loop=zeros(r,c);

    u=zeros(r,1);
    v=zeros(1,c);
iter=0;
    mi=1;
    mj=1;

     while(k==r+c-1)
    %      FOR BASIC CELL
    %      Counting the no.of allocations in row & column */
        for i=1:r
         for j=1:c
          if x(i,j)~=0
              u(i)=u(i)+1;
          end
         end
        end

        for j=1:c
            for i=1:r
                if x(i,j)~=0
                    v(j)=v(j)+1;
                end
            end
        end
```

```
%  Selecting the row or column having max no.of allocations

    max=0;
    flag=0;
    for i=1:r
        if max<u(i)
            max=u(i);
            mi=i;
            flag=1;
        end
    end

    for j=1:c
        if max<v(j)
            max=v(j);
            mj=j;
            flag=2;
        end
    end
u=zeros(r,1);
v=zeros(1,c);

%       Assigning value for u and v
    if(flag==1)
        for j=1:c
            if x(mi,j) ~=0
                v(j)=b(mi,j);
            end
        end

        for k=1:r
            for i=1:r
                for j=1:c
                        if (x(i,j)~=0 && v(j)~=0)
                            u(i)=b(i,j)-v(j);
                        end
                end
            end

            for j=1:c
                for i=1:r
                        if(x(i,j)~=0 && u(i)~=0)
                            v(j)=b(i,j)-u(i);
                        end
                end
            end
        end
    end

if(flag==2)
        for i=1:r
            if (x(i,mj)~=0)
                u(i)=b(i,mj);
            end
        end
```

```
        for k=1:r
            for j=1:c
                for i=1:r
                    if(x(i,j)~=0 && u(i)~=0)
                        v(j)=b(i,j)-u(i);
                    end
                end
            end

            for i=1:r
                for j=1:c
                    if(x(i,j)~=0 && v(j)~=0)
                        u(i)=b(i,j)-v(j);
                    end
                end
            end
        end
end

%     FOR NON BASIC CELL
    max=0;
    for i=1:r
        for j=1:c
            if(x(i,j)==0)
                nbc(i,j)=b(i,j)-(u(i)+v(j));
                if(max>nbc(i,j))
                    max=nbc(i,j);
                    mi=i;
                    mj=j;
                end
            end
        end
    end

    if(max>=0)
      break;
    end


%        Loop Formation

    for i=1:r
        for j=1:c
            if(x(i,j)~=0)
                loop(i,j)=1;
            else
                loop(i,j)=0;
            end
            sign(i,j)=' ';

        end

    end

    for k=1:r
        for i=1:r
```

```
              for j=1:c
                  if(loop(i,j)==1)
                      lc=lc+1;
                  end
              end

              if(lc==1 && i~=mi)
                  for j=1:c
                      loop(i,j)=0;
                  end
              end
              lc=0;
          end

          lc=0;
          for j=1:c
              for i=1:r
                  if(loop(i,j)==1)
                      lc=lc+1;
                  end
              end

              if(lc==1 && j~=mj)
                  for i=1:r
                      loop(i,j)=0;
                  end
              end
              lc=0;
          end
      end



  %     Assigning the Sign
      sign(mi,mj)='+';
      i=mi;
      for k=1:15%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
          for j=1:c
              if(loop(i,j)==1 && sign(i,j)==' ')
                  sign(i,j)='-';
                  break;
              end
          end

          for i=1:r
              if(loop(i,j)==1 && sign(i,j)==' ')
                  sign(i,j)='+';
                  break;
              end
          end
      end
sign;
      %  Finding @ Value
      min=9999;

      for i=1:r
```

```
            for j=1:c
                if(sign(i,j)=='-' && min>x(i,j))
                        min=x(i,j);
                end
            end
        end

        for i=1:r
             for j=1:c
                 if(sign(i,j)=='+')
                     x(i,j)=x(i,j)+min;
                 elseif(sign(i,j)=='-')
                     x(i,j)=x(i,j)-min;
                 end
             end
        end

        % Checking m+n-1 Condition
        k=0;
         for i=1:r
             for j=1:c
                 if(x(i,j)~=0)
                     k=k+1;
                 end
             end
         end
                 iter=iter+1;
                 k;
    end   % End of While

    disp('The Optimum Solution Using Modi Method');
    disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~');
    x
    disp('Total Cost:');
    disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~');
    tc= sum(sum(x.*a))
    disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~');
    disp('Total Number iteration:');
    iter


end
```

Code for Cumulative Difference Method

```
function [ Solution, OverallCost ] = Newmtd(CostsMtx,
resources_col,demands_row)

C_startcost =CostsMtx;

Cumalative_Mat=Ccost(CostsMtx);


C_startcost =CostsMtx;

C_start = Cumalative_Mat;
C = C_start;
m = size(C,1);
n = size(C,2);
a = resources_col;
b = demands_row;
X = zeros(m,n);

stop = 0;

while stop == 0

for i = 1:m
for j = 1:n

if a(i,1) == 0
   C(i,j) = min(C(:,j));
end
if b(1,j) == 0
    C(i,j) = min(C(i,:));
end
end
end
C_sort_col = sort(C,1,'descend');
C_sort_row = sort(C,2,'descend');

Diff_customer = abs(C_sort_col(1,:) - C_sort_col(2,:));
Diff_supplier = abs(C_sort_row(:,1) - C_sort_row(:,2));

for i = 1:m
if a(i,1) == 0
Diff_supplier(i,1) = 0;
end
end
for j = 1:n
if b(1,j) == 0
Diff_customer(1,j) = 0;
end
end
Max_Diff_customer = max(Diff_customer);
Max_Diff_supplier = max(Diff_supplier);
```

```
Customer_nr =
find(Diff_customer==max(Max_Diff_customer,Max_Diff_supplier));
Supplier_nr =
find(Diff_supplier==max(Max_Diff_customer,Max_Diff_supplier));

if isempty(Customer_nr) == 0
Supplier_nr_ = find(C(:,Customer_nr(1)) ==
max(C(:,Customer_nr(1))));
X(Supplier_nr_(1),Customer_nr(1)) =
min(a(Supplier_nr_(1),1),b(1,Customer_nr(1)));

a(Supplier_nr_(1),1) = a(Supplier_nr_(1),1) -
X(Supplier_nr_(1),Customer_nr(1));
b(1,Customer_nr(1)) = b(1,Customer_nr(1)) -
X(Supplier_nr_(1),Customer_nr(1));
Supplier_nr = [];
end
if isempty(Supplier_nr) == 0
Customer_nr_ = find(C(Supplier_nr(1),:) ==
max(C(Supplier_nr(1),:)));
X(Supplier_nr(1),Customer_nr_(1)) =
min(a(Supplier_nr(1),1),b(1,Customer_nr_(1)));

a(Supplier_nr(1),1) = a(Supplier_nr(1),1) -
X(Supplier_nr(1),Customer_nr_(1));
b(1,Customer_nr_(1)) = b(1,Customer_nr_(1)) -
X(Supplier_nr(1),Customer_nr_(1));
end
a1 = a > 0;
b1 = b > 0;
if sum(a1) == 1
stop = 1;
for j = 1:n
if b(j) > 0;
X(a1 == 1,j) = b(j);
end
end
end
if sum(b1) == 1
stop = 1;
for i = 1:m
if a(i) > 0;
X(i,b1 == 1) = a(i);
end
end
end
end
Solution = X;
OverallCost = sum(sum(C_startcost .* X));

    disp('The Initial Feasible Solution Using New Method');
    disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~');
    X
    disp('Initial Cost:');
    OverallCost
    disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~');
 end
```

Code Vogel's Approximation Method

```
function [ Solution, OverallCost ] = VAM(CostsMtx, resources_col,
demands_row)

[ CostsMtx,ss,sd] = inputFun(CostsMtx, resources_col, demands_row);

C_start = CostsMtx;
C = C_start;
m = size(C,1);
n = size(C,2);
a = resources_col;
b = demands_row;
X = zeros(m,n);

stop = 0;

while stop == 0

for i = 1:m
for j = 1:n
if a(i,1) == 0
C(i,j) = max(C(:,j));
end
if b(1,j) == 0
C(i,j) = max(C(i,:));
end
end
end
C_sort_col = sort(C,1);
C_sort_row = sort(C,2);

Diff_customer = abs(C_sort_col(1,:) - C_sort_col(2,:));
Diff_supplier = abs(C_sort_row(:,1) - C_sort_row(:,2));

for i = 1:m
if a(i,1) == 0
Diff_supplier(i,1) = 0;
end
end

for j = 1:n
if b(1,j) == 0
Diff_customer(1,j) = 0;
end
end

Max_Diff_customer = max(Diff_customer);
Max_Diff_supplier = max(Diff_supplier);
```

```
Customer_nr =
find(Diff_customer==max(Max_Diff_customer,Max_Diff_supplier));
Supplier_nr =
find(Diff_supplier==max(Max_Diff_customer,Max_Diff_supplier));

if isempty(Customer_nr) == 0
Supplier_nr_ = find(C(:,Customer_nr(1)) ==
min(C(:,Customer_nr(1))));
X(Supplier_nr_(1),Customer_nr(1)) =
min(a(Supplier_nr_(1),1),b(1,Customer_nr(1)));

a(Supplier_nr_(1),1) = a(Supplier_nr_(1),1) -
X(Supplier_nr_(1),Customer_nr(1));
b(1,Customer_nr(1)) = b(1,Customer_nr(1)) -
X(Supplier_nr_(1),Customer_nr(1));
Supplier_nr = [];
end
if isempty(Supplier_nr) == 0
Customer_nr_ = find(C(Supplier_nr(1),:) ==
min(C(Supplier_nr(1),:)));
X(Supplier_nr(1),Customer_nr_(1)) =
min(a(Supplier_nr(1),1),b(1,Customer_nr_(1)));

a(Supplier_nr(1),1) = a(Supplier_nr(1),1) -
X(Supplier_nr(1),Customer_nr_(1));
b(1,Customer_nr_(1)) = b(1,Customer_nr_(1)) -
X(Supplier_nr(1),Customer_nr_(1));
end

%Stop condition:

if (max(a)==0||max(b)==0)
     stop = 1;
end
a1 = a > 0;
b1 = b > 0;
if sum(a1) == 1
stop = 1;
for j = 1:n
if b(j) > 0;
X(a1 == 1,j) = b(j);
end
end
end
if sum(b1) == 1
stop = 1;
for i = 1:m
if a(i) > 0;
X(i,b1 == 1) = a(i);
end
end
end
end

 if (isempty(a1) || isempty(b1))
      stop = 1;
```

```
 end

Solution = X;
OverallCost = sum(sum(C_start .* X));
a;
b;

    disp('The Initial Feasible Solution Using Vogels Method');
    disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~');
    X
    disp('Initial Cost:');
    OverallCost
    disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~');
end
```

Code for North-West Corner Rule Method

```
function [x,tc]=nwc(a,s,d)
  [ a,ss,sd] = inputFun(a,s,d);
  r=size(a,1);
  c=size(a,2);


    x=zeros(size(a));
    k=0;i=1;j=1;
    while(k<(r+c)-1)
        if(s(i)>d(j))
            k=k+1;
            x(i,j)=d(j);
            s(i)=s(i)-d(j);
            ss=ss-d(j);
            sd=sd-d(j);
            d(j)=0;
            j=j+1;
        elseif(s(i)<d(j))
            k=k+1;
            x(i,j)=s(i);
            d(j)=d(j)-s(i);
            ss=ss-s(i);
            sd=sd-s(i);
            s(i)=0;
            i=i+1;
        else
            k=k+1;
            x(i,j)=s(i);
            ss=ss-s(i);
            sd=sd-s(i);
            s(i)=0;
            d(j)=0;
            i=i+1;
            j=j+1;
        end
```

```
        if((ss==0) && (sd==0))
            break;
        end
    end
    tc= sum(sum(x.*a));

    disp('The Initial Feasible Solution Using North-West Corner
Method');
    disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~');
    x
    disp('Initial Cost:');
    tc
    disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~');
end
```

Code for Least Cost Method

```
function [x,tc]=lcm(a,s,d)
  [ a,ss,sd] = inputFun(a,s,d);
  r=size(a,1);
  c=size(a,2);
  x=zeros(size(a));
  b=a;


  for i=1:r
      for j=1:c
          b(i,j)=a(i,j);
      end
  end

  k=0;mi=1;mj=1;
  while(k<(r+c)-1)
      min=9999;
      for i=1:r
          for j=1:c
              if(min>b(i,j) && b(i,j)~=-1)
                  min=b(i,j);
                  mi=i;
                  mj=j;
              end
          end
      end
    if(s(mi)>d(mj))
        k=k+1;
        x(mi,mj)=d(mj);
        s(mi)=s(mi)-d(mj);
        ss=ss-d(mj);
        sd=sd-d(mj);
        d(mj)=0;
        for i=1:r
            b(i,mj)=-1;
```

```
        end
    end

    if(s(mi)<d(mj))
        k=k+1;
        x(mi,mj)=s(mi);
        d(mj)=d(mj)-s(mi);
        ss=ss-s(mi);
        sd=sd-s(mi);
        s(mi)=0;
        for j=1:c
            b(mi,j)=-1;
        end
    end

    if(s(mi)==d(mj))
        k=k+1;
        x(mi,mj)=s(mi);
        ss=ss-s(mi);
        sd=sd-s(mi);
        s(mi)=0;
        d(mj)=0;
        for i=1:r
            b(i,mj)=-1;
        end

        for j=1:c
            b(mi,j)=-1;
        end
    end

    if((ss==0)&&(sd==0))
      break;
    end
  end
  tc= sum(sum(x.*a));

    disp('The Initial Feasible Solution Using Least Cost Method');
    disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~');
    x
    disp('Initial Cost:');
    tc
    disp('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~');
end
```

Appendix-02 Problem set

20 Selected problems

```
01
a=[12 4 13 18 9 2; 9 16 10 7 15 11; 4 9 10 8 9 7;  9 3 12 6 4 5;
    7 11 5 18 2 7; 16 8 4 5 1 7 ]
s=[120;80;50;90;100;60]
d=[75;85;140;40;95;65]'

02
a=[3 6 8 4;  6 1 2 5;7 8 3 9 ]
s=[20;28;17]
d=[15;19;13;18]'

03
a=[3 1 7 4; 2 6 5 9;8 3 3 2 ]
s=[300;400;500]
d=[250;350;400;200]'

04
a=[2 3 11 7; 1 0 6 1; 5 8 15 9]
s=[6;1;10]
d=[7;5;3;2]'

05
a=[4 3 5;6 5 4;8 10 7]
s=[90; 80; 100]
d=[70; 120; 80]'

06
a=[4 5 8 4; 6 2 8 1;8 7 9 10]
s=[52;57;54] d=[60;45;8;50]'

07
a=[7 5 9 11;  4 3 8 6;3 8 10 5;  2 6 7 3]
s=[30;25;20;15]
d=[30;30;20;10]'

08
a=[50 60 100 50;80 40 70 50; 90 70 30 50]
s=[20;38;16]
d=[10;18;22;24]'

09
a=[2 7 4 ; 3 3 1 ;   5 4 7]
s=[5;8;7;14]
d=[7;9;18]'

10
a=[8 6 10 9;9 12 13 7 ; 14 9 16 5]
s=[35;50;40]
d=[45;20;30;30]'
```

```
11
a=[5 2 4 1 0; 5 2 1 4 0 ;6 4 8 2 0; 4 6 5 4 0;2 8 4 5 0]
s=[30;20;12;46;46]
d=[30;50;30;27;17]'

12
a=[0 2 20 11;12 7 9 20 ;4 14 16 18]
s=[15;25;10]
d=[5;15;15;15]'

13
a=[6 3 8 7; 8 5 2 4 ; 4 9 8 4; 7 8 5 6]
s=[110;60;54;3;27]
d=[20;70;78;86]'

14
a=[4 19 22 11;1 9 14 14 ; 6 6 16 14]
s=[100;30;70]
d=[40;20;60;80]'

15
a=[2 7 4; 3 3 1 ; 5 4 7;1 6 2]
s=[5;8;7;14]
d=[7;9;18]'


16
a=[6 8 10;7 11 11;4 5 12]
s=[150;175;275]
d=[200;100;300]'

17
a=[14 19 10 7 16;6 7 7 14 12;5 14 17 9 6;13 12 9 15 8;17 5 13 16 19]
s=[30;20;12;46;46]
d=[30;50;30;27;17]'

18
a=[17 14 11 29 11;17 7 15 29 14;13 25 7 19 26;28 15 8 6 5]
s=[55;45;30;50]
d=[40;20;50;30;40]'


19
a=[31 27 20  23;30 20 19 39;20 36 20 26;22 20 26 19]
s=[6;8;16;15]
d=[9;10;12;14]'


20
a=[42 33 25 29; 41 38 45 35;26 38 28 32;33 31 27 44; 36 34 32  44]
s=[30;20;12;30;46]
d=[31;50;30;27]'
```

28 Selected problems from literature

```
1
a=[10 30 25 15;20 15 20 10;10 30 20 20;30 40 35 45]
s=[14;10;15;12]
d=[10;15;2;15]'

%2
a=[2 7 4;3 3 1;5 4 7;1 6 2]
s=[5;8;7;14]
d=[7;9;18]'

3
a=[10 2 16 14 10;6 18 12 13 16;8 4 14 12 10;14 22 20 8 18]
s=[300;500;825;375]
d=[350;400;250;150;400]'

4
a=[3 6 8 4;6 1 2 5;7 8 3 9]
s=[20;28;17]
d=[15;19;13;18]'

5
a=[6 10 14;12 19 21;15 14 17]
s=[50;50;50]
d=[30;40;55]'

6
a=[12 4 13 18 9 2;9 16 10 7 15 11;4 9 10 8 9 7;9 3 12 6 4 5;7 11 5
      18 2 7;16 8 4 5 1 10]
s=[120;80;50;90;100;60]
d=[75;85;140;40;95;65]'

7
a=[25 55 40 60;35 30 50 40;36 45 26 66;35 30 41 150]
s=[60;80;160;150]
d=[90;100;120;140]'

8
a=[9 12 9 6 9 10 ;7 3 7 7 5 5;6 5 9 11 3 11;6 8 11 2 2 10]
s=[5;6;2;9]
d=[4;4;6;2;4;2]'

9
a=[21 16 25 13;17 18 14 23; 32 27 18 41]
s=[11;13;19]
d=[6;10;12;15]'

10
a=[6 4 4 7 5;5 6 7 4 8;3 4 6 3 4]
s=[100;125;175]
d=[60;80;85;105;70]'
```

```
11
a=[2 3 11 7;1 0 6 1;5 8 15 9]
s=[6;1;10]
d=[7;5;3;2]'

12
a=[19 30 50 10;70 30 40 60;40 8 70 20]
s=[7;9;18]
d=[5;8;7;14]'

13
a=[4 19 22 11;1 9 14 14;6 6 16 14]
s=[100;30;70]
d=[40;20;60;80]'

14
a=[4 5 8 4;6 2 8 1;8 7 9 10]
s=[52;57;54]
d=[60;45;8;50]'

15
a=[6 3 8 7;8 5 2 4; 4 9 8 4; 7 8 5 6]
s=[110;60;54;30]
d=[20;70;78;86]'

16
a=[5 2 4 1;5 2 1 4; 6 4 8 2; 4 6 5 4;2 8 4 5]
s=[30;20;12;30;46]
d=[31;50;30;27]'

16
a=[3 15 17;45 30 30;13 25 42]
s=[580;240;330]
d=[310;540;300]'

17
a=[3 1 7 4;2 6 5 9;8 3 3 2]
s=[300;400;500]
d=[250;350;400;200]'

18
a=[50 60 100 50;80 40 70 50;90 70 30 50 ]
s=[20;38;16]
d=[10;18;22;24]'

19
a=[7 5 9 11;4 3 8 6;3 8 10 5;2 6 7 3]
s=[30;25;20;15]
d=[30;30;20;10]'

20
a=[4 3 5;6 5 4;8 10 7]
s=[90;80;100]
d=[70;120;80]'
```

```
21
a=[11 9 6;12 14 12;10 8 10]
s=[40;50;40]
d=[55;45;30]'

22
a=[13 18 30 8;55 20 25 40;30 6 50 10]
s=[8;10;11]
d=[4;7;6;12]'

23
a=[3 4 6 8 9;2 10 1 5 8;7 11 20 40 3;2 1 9 14 16]
s=[20;30;15;13]
d=[40;6;8;18;6]'

24
a=[12 4 9 5 9;8 1 6 6 7;12 4 7 7;10 15 6 9 1]
s=[55;45;30;50]
d=[40;20;50;30;40]'

25
a=[12 4 9 5 9;8 1 6 6 7;1 12 4 7 7; 10 15 6 9 1]
s=[55;45;30;50]
d=[40;20;50;30;40]'

6
a=[9 8 5 7;4 6 8 7;5 8 9 5]
s=[12;14;16]
d=[8;18;13;3]'

27
a=[2 2 2 1;10 8 5 4;7 6 6 8]
s=[3;7;5]
d=[4;3;4;4]'

28
a=[4 3 5;6 5 4;8 10 7]
s=[9;8;10]
d=[7;12;8]'
```