CROSS-RELATION-BASED FREQUENCY DOMAIN BLIND CHANNEL ESTIMATION FOR MIMO COMMUNICATION SYSTEMS

Henarath Hettige Samantha Rathnasiri

(09/8432)

Thesis submitted in partial fulfillment of the requirements for the degree Master of Science in Telecommunications.

Department of Electronic & Telecommunication Engineering

University of Moratuwa
Sri Lanka

August 2015

DECLARATION

"I declare that this is my own work and this thesis does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any other University or Institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

Also, I hereby grant to University of Moratuwa the non-exclusive right to reproduce and distribute my thesis in whole or in part in print, Electronic or other medium. I retain the right to use this content in whole or part in future works (such as articles or books).

Signature: Date: 07/08/2015

The above candidate had carried out research for the Masters thesis under my supervision.

Signature of the supervisor: Date: 08/08/2015

UOM Verified Signature

ACKNOWLEDGEMENT

I would like to express my gratitude to my supervisor Dr. Chandika Wavegedara for the useful comments, remarks and engagement through out the learning process of this master thesis. I would like to thank you for introducing me to the topic and guiding me to the success. Furthermore, I would also like to acknowledge with much appreciation the guidance given by Professor Dileeka Dias and Dr. Ajith Pasqual during the progress reviews.

A special thank goes to my colleague Mr.Priyantha Appuhami who helped me in simulating the proposed model. I also like to express my gratitude to my wife for her understanding, and for providing me with the time and support I needed for my work.

ABSTRACT

Blind channel estimation is attractive for the application of in high-speed wireless communication systems due to its high spectral efficiency. Most of the conventional blind channel identification algorithms are based on the statistical properties of the transmitted signals. However, in practical communication systems, the statistical model of the transmitted signals may not be known or there may not be sufficient data to estimate the statistical properties. Alternatively, we can use Cross Relation (CR) principle for computationally-efficient blind channel estimation. CR principle-based frequency domain blind channel estimation schemes offer good performance when the data length is inevitably short. In this thesis, a frequency-domain CR-based blind channel estimation schemes are developed for both single-carrier and multicarrier multiple-input multiple-output (MIMO) systems. The proposed channel estimation scheme is able to identify the channel using a single received signal block. This channel estimation scheme is accompanied by a simple block pre-coding scheme. The channel is assumed time invariant within the signal block period which depends on the antenna configuration of the system. The numerical simulation shows that the proposed methods perform satisfactorily with only one or very few received signal blocks, with compared to existing correlation based methods which require more data blocks.

Keywords –MIMO, SVD , CR, Channel Estimation

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

Abbreviation	Description
MSE	Mean Squared Error
SNR	Signal to Noise Ratio
FFT	Fast Fourier Transform
CR	Cross Relation
SISO	Single Input Single Output
SIMO	Single Input Multiple Output
MISO	Multiple Input Single Output
MIMO	Multiple Input Multiple Output
OFDM	Orthogonal Frequency Division Multiplexing
SVD	Singular Value Decomposition
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate

LIST OF APPENDICES

1. INTRODUCTION

1.1 Introduction to Blind Channel Estimation

During the last decade there has been an enormous expansion in the area of wireless communication. As new services and devices are introduced, and more information is sent between an increasing numbers of users, more bandwidth is required and the spectrum has become more limited. Therefore, it was essential to utilize that limited bandwidth in a way that it gives out maximum availability to data transmission. In this process channel estimation plays an important role in providing maximum utilizable bandwidth for the data communication while reducing training information.

The channel estimation can be categorized in various attributes. Instantaneous channel estimation is used to identify the channel properties at the same time when the receiver detects the data belonging to the received symbol and on time channel estimation uses training symbols to identify the channel within the same symbol period. The training symbol aided channel estimation schemes were proposed in [7], [11] considering the fast fading channel condition.

The statistical approaches for channel estimation schemes were introduced when there are fewer or no training symbols were used for the channel estimation. Due to the statistical properties deployed of these schemes, previously received data blocks also will be considered for channel estimation process. These schemes were used in [1], [3], [5],[9],[20] to estimate the channel.

At the point of implementing channel estimation processes, it is further classified as training based channel estimation schemes [7], [11], [10], [11], [12], [18], [25], Semi blind channel estimation schemes [8] , [14], [26] and Blind channel Estimation schemes[1], [2], [3], [4], [5], [6], [9], [15], [16], [17]. [19], [21], [22], [23], [27]. [28], considering the nature of the training data use to estimate the channel.

The optimal training sequences and pilot tones for orthogonal frequency division multiplexing (OFDM) channel estimation were investigated in [10] [25]. The optimum training symbol for multiple input multiple output (MIMO) channel estimation was investigated in [7] [12]. The Peak to Average Power (PAP) reduction based semi blind channel estimation scheme was analyzed in [26] for MIMO-OFDM systems. A combination of MIMO and OFDM is enhancing the performance of next generation wireless systems. The training symbols were reduced in improved techniques used for channel estimation in recent studies. The Semi blind channel estimation scheme was studied in [8] for MIMO channels with an improved technique to reduce the number of training symbols unlike in training based channel estimation schemes.

Unlike in the other two schemes, the blind channel estimation scheme does not require any training data for its channel estimation process. This scheme has increased the availability of bandwidth for useful data communication, thus increasing the spectral efficiency in the system. Due to these advantages, blind channel estimation schemes are mostly used for high data rate applications. The Blind channel estimation schemes were proposed in [21], [22], [23] with the use of pre-coding technique at the transmitter. The pre-coding based methods use the statistical or deterministic properties to the input signals to estimate the channel. Hence, the type of the pre-coder is used considering the method use for the channel estimation with the propose channel estimation method.

In comparison to the statistical methods, the deterministic ones converge much faster. However, they involve high complexity, which becomes even higher as the constellation order increases. A deterministic blind channel estimation method takes advantage of receive diversity and detailed below gives the references therein. A deterministic channel estimation schemes was proposed for an oversampled single input single output (SISO) channel model by exploiting the cross relation (CR) [16] between each channel output pair. The CR principle is extended to single carrier single input multiple output (SIMO) systems in frequency domain [27] via the discrete Fourier transform (DFT). In [28], frequency domain CR principle is proposed for SIMO OFDM systems due to its property to compute the channel realizations with less number of received signal blocks. In our study, new schemes were proposed for both single carrier and multicarrier MIMO systems with the use of CR principle in frequency domain.

1.2 Problem Statement – MIMO Channel Estimation Problem

MIMO systems are widely used in recent high data rate applications due to its high spectral efficiency. The multiple antennas accompanied by a suitable spatial multiplexing scheme at the transmitter and receiver can significantly improve the capacity of a wireless communication system without increasing its operational bandwidth [7], [8]. Many advantages of MIMO systems over the SISO systems are identified.

The SISO systems were replaced with the introduction of MIMO systems in order to improve the bandwidth efficiency without increasing the frequency spectrum. The major challenge faced in MIMO systems is how to obtain the channel state information accurately for coherent detection of information symbols.

The most of the MIMO channel estimation schemes use training based algorithms to improve the performance, but the spectral efficiency of training based schemes are comparatively less when compared with the blind channel estimation schemes. The MIMO systems create multiple numbers of sub channels between transmitters and receivers, and hence it is required to send higher number of training data for the channel estimation compared to SISO channel estimation schemes. The main drawback of the training based MIMO channel estimation schemes is less spectrally efficient due to large number of training symbols requirement to estimate the channel. Therefore, the training based MIMO channel estimation schemes perform significantly very low compared with training based SISO channel estimation schemes.

The existing blind channel estimation schemes for MIMO systems are spectrally efficient, but they require large number of received samples for channel estimation. Therefore, the blind channel estimation schemes used for the short data length applications suffer from scarcity of data samples to estimate the channel coefficients.

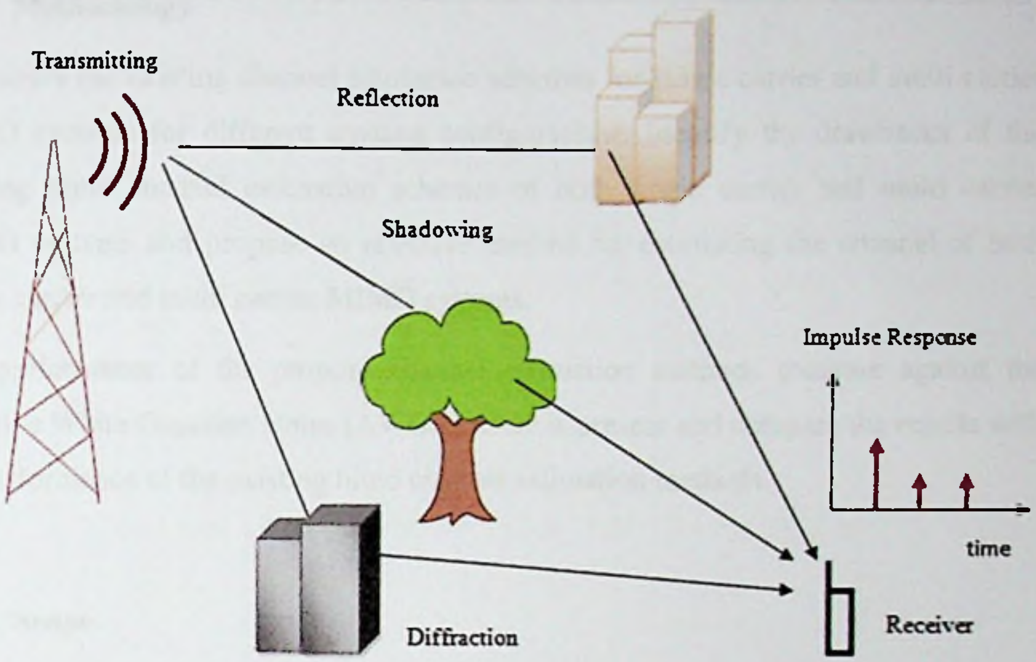

Figure 1 Wireless Signal propagation in typical environment

1.3 Objectives

The main objective of this research is to develop blind channel estimation schemes for both single-carrier and multicarrier MIMO systems.

Specific objectives are as follows,

The specific objectives are summarized as follows,

1. To study the existing blind channel estimation schemes for single carrier and multi carrier systems of different antenna configurations.
2. To develop an effective blind channel estimation scheme for both single carrier and multi carrier MIMO systems
3. To analyse the MSE performance & BER performance of these blind channel estimation schemes against the additive white Gaussian (AWGN) noise to the received signal.
4. To compare the performances of these proposed blind channel estimation schemes with the existing proposed channel estimation schemes.

1.4 Methodology

We study the existing channel estimation schemes for single carrier and multi carrier MIMO systems for different antenna configurations. Identify the drawbacks of the existing blind channel estimation schemes of both single carrier and multi carrier MIMO systems and propose an effective method for estimating the channel of both single carrier and multi carrier MIMO systems.

The performance of the propose channel estimation methods measure against the Additive White Gaussian Noise (AWGN) noise is present and compare the results with the performance of the existing blind channel estimation methods.

1.5 Scope

We have considered a blind identification of finite impulse response (FIR) MIMO schemes in this study. Additive White Gaussian Noise with zero mean and unit variance is applied to the channel outputs. The MIMO channel is modeled in this study as a superposition of SIMO channels to apply the cross relation principle. This is implemented by including a simple block pre-coder at the input of the transmitter to pre-code the input signal.

The MSE performance is simulated according to the CR principle used in [27] for SIMO channels and the results are compared with the existing results of the same principle used in [27]. Since the existing results shows for a SIMO system with four receiver antennas, but we have analysed the MSE performance even for the SIMO systems with more number receiver antennas Similarly, the MSE performance for the MIMO systems are analysed with the results shown , in the existing study. Finally, the MSE performance and BER performances of the proposed channel estimation schemes are evaluated for both single carrier and multicarrier MIMO systems.

1.6 Organization of this Thesis

The existing channel estimation schemes are studied in Chapter 2, SISO, SIMO and MIMO with single and multicarrier systems. The proposed channel estimation schemes in this research are presented in Chapter 3 for single-carrier and multicarrier systems. The simulation results of the existing channel estimation schemes and the performance comparison of the proposed channel estimation methods are presented in Chapter 4. The conclusion and further recommendations are presented in Chapter 5.

1.7 Notations

Upper case bold-face letters for matrices and bold face lower case letters for vectors, $(.)^T$ stand for transpose and * stands for convolution.

2. LITERATURE REVIEW

According to the Shannon-Hartley theorem, the channel capacity of clean data within a given bandwidth is proportional to the ratio between the average signal power and the noise of the channel. Therefore, the recent research studies mainly focus on achieving high signal to noise ratio (SNR) which improves the channel capacity. The wireless applications are categorized based on the number of transmit and receive antennas used in the systems such as SISO, SIMO and MIMO systems. These configurations are further categorized in to single carrier and multicarrier systems according to the number of frequency subchannels used for communication in between transmit and receive antenna pairs. With the complexity of the schemes used for channel estimation, bandwidth has effectively used for the useful data transmission.

Hence, developing improved methods of channel estimation is very important to maximize the channel capacity for sending and receiving useful data of different applications. Several literature related to this process is reviewed and the information gathered can be summarized as follows.

2.1 The Wireless System Categorization for Channel Estimation Schemes

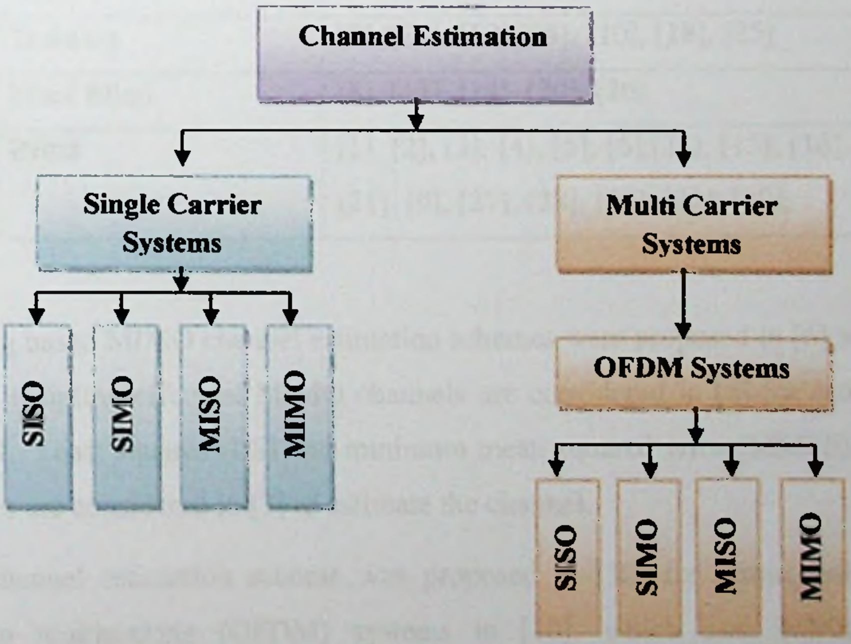

Figure 2 Categorization of wireless communication systems for channel estimation

In Figure 2 shows, wireless systems categorization according to the different antenna configuration of single-carrier and multi-carrier systems. While the SISO systems provide less computation for channel estimation methods at receiver side, MIMO systems provide comparatively higher computation power for the channel estimation process.

2.2 Channel Estimation Schemes

According to the literature studied, the channel estimation schemes can be basically categorized in to the following types,

- Training based channel estimation
- Semi blind channel estimation
- Blind channel estimation

For example, in table 1, the research papers for each type are shown.

Table 1 Categorisation of research papers based on the channel estimation type

	Channel Estimation Types	**References**
1.	Training	[7], [11], [12],[24], [10], [18], [25]
2.	Semi Blind	[8], [13], [14], [20], [26]
3.	Blind	[1], [2], [3], [4], [5], [6],[23], [15], [16], [17], [19], [21], [9], [27], [28], [29], [22], [30],

Training based MIMO channel estimation schemes were proposed in [6] and [7]. The Mutual coupling effect of MIMO channels are considered in [6] for estimating the channel. Least squares (LS) and minimum mean squared error (MMSE) estimation schemes are considered in [7] to estimate the channel.

Pilot channel estimation scheme was proposed in [10] for orthogonal frequency division multiplexing (OFDM) systems in [10], which uses MMSE and LS estimators for estimating the channel. Training-based channel estimation scheme was proposed in [18] for rapidly varying channels.

A training based channel estimation scheme was proposed in [12] for the spatially correlated channels by exploiting the spatial correlation between channels. This method is applied to rapidly varying OFDM channels and the interpolation method is used for the channel estimation.

All training based channel estimation methods use part of the channel bandwidth for sending pilot information and it reduces the spectral efficiency of the overall system. Therefore, these schemes are mostly used for low data rate application as large number of pilot information utilizes the effective bandwidth. Unlike, in semi blind or blind channel estimation schemes, training based channel estimation schemes can achieve higher performance for estimating the channel due to the some amount of training data sent along with data signals.

The references of pilot based channel estimation schemes can be further categorized as MIMO, OFDM and MIMO OFDM.

Table 2 Pilot-based channel estimation schemes

Channel Model	**Pilot**
MIMO	[7]
OFDM	[10], [18]
MIMO OFDM	[11], [24]

An Iterative channel estimation scheme was proposed in [8] for MIMO channels to use with small amount of training data. This algorithm is a combination of training-based channel estimation scheme and blind equalization scheme. Hence it is called as semi-blind channel estimation scheme.

Semi blind channel estimation scheme was proposed in [13] for MIMO flat fading channels and this scheme estimate the channel based on decomposition of channel matrix in to the product of a whitening matrix and a unitary rotation matrix. A subspace based semi blind channel estimation scheme was proposed in [20] for block-precoded OFDM systems. Singular value decomposition method is used in this paper to estimate the channel from the received signals.

A Naval semi blind channel estimation scheme was proposed with the combination of partial transmit sequence (PTS) method and selective mapping (SLM) approaches in [26] for MIMO OFDM systems. The performances of these schemes are relatively high when compare with the blind channel estimation schemes. Spectral efficiency in these systems are improved when compared to the training based channel estimation schemes by reducing the number of training symbols used for channel estimation process.

The references of semi blind channel estimation schemes can be further categorized as follows.

Table 3 Semi blind channel estimation schemes

Channel Model	Semi Blind
MIMO	[13], [14]
MIMO OFDM	[26]
OFDM	[20]

In blind channel estimation schemes, no pilot information is used to estimate the channel parameters. Hence it improves the spectral efficiency of the overall system. According to the scheme used to estimate the channel, these schemes can be categorized in to different methods, such as subspace based, precoder based and deterministic based methods etc.

A SISO channel was modeled as a SIMO channel by using the oversampling technique to apply cross relation principle for estimating the channel properties in [16]. In this study, a blind channel estimation scheme is proposed considering a multichannel finite impulse response (FIR) channel with an unknown deterministic input. With the proper channel order selection, the algorithm use to identify the channel blindly without any statistical information on the input process.

The SISO blind channel estimation scheme was proposed in [22] applying a non-redundant linear precoder on each pair of input signal blocks. It is composed of unitary sub-matrices, which spread each symbol onto multiple subcarriers, thus enabling robustness to subcarrier nulls. A precoder based schemes are most attractive for the blind channel estimation schemes, but it require having additional precoder circuitry at the transmitter side.

A data efficient blind channel estimation scheme was proposed in [15] for OFDM systems and the propose scheme utilizes frequency domain observation to estimate the channel impulse response, and it is intimately related to the cross relation method for blind channel estimation in single carrier systems. In the presence of noise, the scheme has very low complexity- only single eigenvalue decomposition is needed, no matter how many OFDM blocks are used.

A blind channel estimation scheme was proposed for SIMO OFDM schemes in [17] when there are no cyclic prefixes used. The proposed channel estimation scheme is capable of channel identification using a single received OFDM block. The correlation structure of the transmitted blocks was exploited at the receiver to recover the channel via cross correlation operation in [19] for OFDM systems. The blind channel estimation scheme was proposed for OFDM systems in [21] based on the assumptions that the transmitted symbols are independent and identically distributed. This is an improved channel estimation scheme of subspace based methods.

A subspace based blind channel estimation scheme was proposed in [1] for MIMO OFDM systems of time varying channel conditions. Especially subspace based schemes require large number of received samples to obtain a good time averaged correlation matrix and the method proposed in [1] reduced the time samples significantly by exploiting the frequency correlation among adjacent subcarriers. Subspace based blind channel estimation schemes were proposed in [2] and [3] for MIMO OFDM systems by using the presence of virtual carriers (VC). This method is proposed considering insufficient number of CP or when there is no CP. A blind channel estimation scheme is proposed in [4] for MIMO OFDM systems with the use of a non-redundant linear precoder. The channel is estimated blindly in this scheme by exploiting the correlation matrix of the received signals. An analytical method of obtaining the channel state information was proposed in [5] for the MIMO OFDM systems by obtaining the autocorrelation matrix of the received signals. A redundancy introduced by the CP was used for channel estimation in [9] for MIMO

OFDM systems. This is commonly used in most of the OFDM channel estimation schemes which can improve the bandwidth of the channel.

Definition of a deterministic channel can be introduced as follows. A deterministic system is in which no randomness is involved in the development of future states of the system. A deterministic model will thus always produce the same output from a given starting condition or initial state. A deterministic Blind Identification Fast Fourier Transform (BI-FFT) schemes were studied in [27], [28] for SIMO and SIMO-OFDM systems. The BI-FFT scheme is proposed for SIMO systems to estimate the channel blindly.

The references of blind channel estimation schemes can be further categorized as follows,

Table 4 Blind channel estimation schemes

Channel Model	Blind
SIMO	[27], [28]
SIMO OFDM	[29]
MIMO	[33], [34]
MIMO OFDM	[2], [3], [4], [5], [23],
OFDM	[15], [16], [17], [19], [21], [9], [27]

2.3 Channel Estimation schemes develop for communication systems with different antenna configurations

2.3.1 Channel estimation in SISO systems

The simplest form of radio link can be defined as SISO – Single Input Single Output. This is effectively a standard radio channel – this transmitter operates with one antenna as does the receiver. There is no diversity and no additional processing required. The advantage of a SISO system is its simplicity. The SISO system requires no processing in terms of the various forms of diversity that may be used. However the SISO channel is limited in its performance. Interference and fading will impact the system more than a MIMO system using some form of diversity.

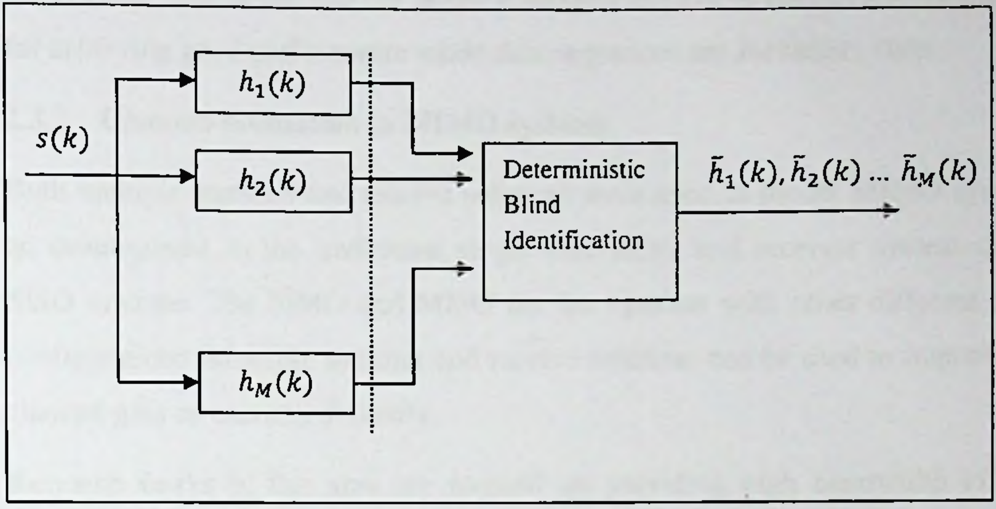

Figure 3 Multichannel identification (adopted from [16])

The SISO channel system is modeled as a SIMO system using oversampling technique in [16]. Here, the received signal is oversampled at a higher rate than the symbol rate in order to model the SIMO system. As a result of using this technique, one can obtain more samples during one symbol period time.

2.3.2 Channel estimation in SIMO systems

The SIMO – Single Input Multiple Output systems proposed where the transmitter has a single antenna and the receiver has multiple antennas. This is also known as receiver diversity. This is often used to enable a receiver system that receives signals from number of independent sources to combat the effects of fading.

The SIMO channel estimation schemes are studied under different channel estimation schemes, such as subspace based, deterministic based etc. A deterministic based channel estimation scheme was proposed for finite impulse response (FIR) SIMO systems with the use of cross relation principle in [16]. Here, the proposed CR scheme is given in time domain and it has exploited the cross relations between channel output pairs.

The frequency domain form of cross relation scheme (BI-FFT) was studied in [27] for considering SIMO systems. This scheme successfully handles even short length of received signal to identify the channel. A second-order statistics based methods suffer from performance degradation when the received data samples are short for the channel estimation, but the BI-FFT method in [27] is computationally efficient for achieving good performance when data sequences are inevitably short.

2.3.3 Channel estimation in MIMO systems

Both multiple transmit and receive antennas were used to model MIMO systems as an development to the traditional single transmitter and receiver system called as SISO systems. The SIMO and MISO are the systems with other different antenna configurations. Multiple transmit and receive antennas can be used to improve either channel gain or transmit diversity.

Research works in this area are focused on providing high bandwidth efficiency through MIMO systems and hence they have become more attractive for high data rate wireless applications.

2.3.4 Channel estimation in OFDM systems

The multimedia applications are implemented on OFDM considering more efficient and robustness of the system. The OFDM sometimes referred to as multi-carrier or discrete multi-tone modulation, utilizes multiple subcarriers to transport information from one particular user to another. The benefits of OFDM are high spectral efficiency, resiliency to radio frequency (RF) interference, and lower multi-path distortion. The orthogonal nature of OFDM allows sub channels to overlap, having a positive effect on spectral efficiency. Each one of the subcarriers transport information is just far enough apart from each other to theoretically avoid interference.

A statistical based time domain channel estimation scheme was proposed in [10] for FIR OFDM channel systems. The pre-coder based channel estimation schemes were proposed in [19], [21] for multicarrier systems.

2.4.5 Channel estimation in SIMO-OFDM systems

The SIMO OFDM system is the multicarrier system which is developed with one transmitter antenna multiple antennas at the receiver. The multiple received antennas provide diversity gain to the system and such systems are most effective for frequency- selective fading channels. A deterministic channel estimation scheme was proposed in [29] in frequency domain for SIMO OFDM systems and the proposed scheme is an extension to the method proposed in [27] for the SIMO systems. Therefore, the method proposed in [29] is more reliable even for short data length applications of SIMO OFDM systems. The CR-based deterministic time domain channel estimation scheme was proposed in [15]. The Non-CP based channel estimation schemes were proposed in [17] and [30] for SIMO OFDM systems.

2.3.6 Channel estimation in MIMO-OFDM systems

The MIMO OFDM became the next extension to the multicarrier systems by introducing multiple antennas at the both transmitter and receiver sides. This configuration is aimed at improving the overall capacity of the system using multiple antenna arrangements.

2.4 The CR-Based Frequency Domain Channel Estimation Scheme

The CR-based frequency domain channel estimation scheme was proposed in most of the blind channel estimation schemes as an effective method for estimating channel properties. This principle is implemented in SIMO systems for single carrier as well as for multicarrier systems.

Further studies on this area had been conducted to apply the same principle for the systems with different antenna configurations such as,

1. Single transmitter and single receiver antennas (SISO).
2. Single Transmitter and multiple receiver systems (SIMO)
3. Multiple Transmitter and multiple receiver systems (MIMO)

Table 5 shows the summary of literature in which the BI-FFT scheme is used for channel estimation.

Table 5 Summary of the literature of CR scheme

	SISO	SIMO	SIMO OFDM		MIMO	
			CP OFDM	Non -CP OFDM	MIMO	MIMO OFDM
CR Algorithm	[16]	[16]	[15] [29]			
FFT-based CR Algorithm		[27]	[28]	[30]		

After reviewing these studies, it was decided to use the BI-FFT algorithm for single carrier MIMO system as new research avenue in this study.

SISO

The SISO channel is considered for applying CR-based frequency domain channel estimation scheme.

SIMO

The SIMO channels are considered for applying CR-based frequency domain channel estimation scheme.

SIMO OFDM

The SIMO channels with multi-carrier (OFDM) systems are considered for applying CR-based frequency domain channel estimation scheme. The SIMO OFDM systems are studied considering systems with cyclic prefix and the systems without cyclic prefix in literature.

3. PROPOSED BLIND CHANNEL ESTIMATION SCHEMES

3.1 A CR-based Frequency Domain Channel Estimation Scheme for MIMO

In this section, a Cross Relation (CR)-based frequency domain channel estimation scheme is proposed for MIMO systems. The principle of CR algorithm was originally proposed for both single carrier and multi carrier SIMO systems. Therefore, the MIMO system is modeled in a way such that system parameters (channel, input signal and output signal models) are compatible to apply CR algorithm as in SIMO systems. The propose channel estimation algorithm in this study perform well for the short data length applications.

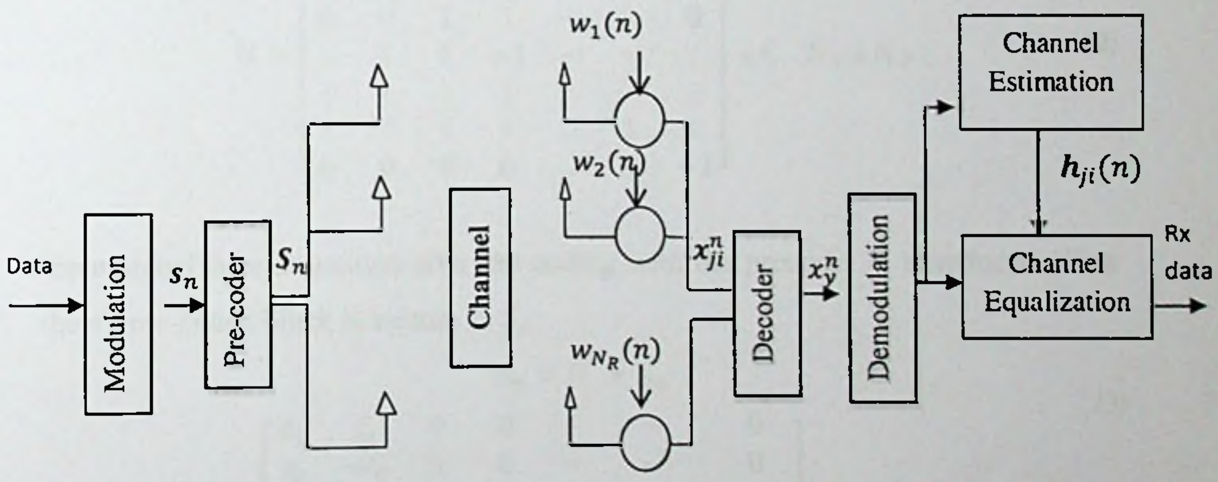

Figure 4 MIMO system model

A simple block pre-coder is introduced before the transmitter in order to model the output signal and hence, apply the Singular Value Decomposition (SVD) technique to find the optimized solution of this algorithm.

Let us consider the discrete FIR-channel model, received signal model at the nth block period,

$$x_m(n) = s(n) * h_m(n) + w_m(n) \qquad m = 1,2,\ldots\ldots, L \tag{1}$$

where, $x_m(n)$ is the mth channel output at time n, $s(n)$ is the common input sequence, $h_m\,(n)$ is the channel impulses responses of the mth sub channel, $w_m(\mathrm{n})$ is the additive noise of the mth channel.

We consider that the number of available output samples of each channel is $\mathrm{N_s}$ and $\mathrm{N_s}$ is small. Therefore, the minimum required length of the input sequence $s(n)$ to generate Ns output samples is $N_s - M$, where $N_s \geq M+1$.

Input sequence, $s(n) = [s(0)\, s(1) \ldots\ldots s(N_s - M - 1)]^T$ is pre-coded before transmission.

The structure of the precoder matrix introduced before the transmitter is given as (2),

$$\boldsymbol{U} = \begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & \cdots & 0 \\ 1 & -1 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & 1 & \cdots & \cdots & 0 \\ \vdots & \vdots & 1 & -1 & \cdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ldots & 1 & 1 \\ 0 & 0 & 0 & 0 & \ldots & 1 & -1 \end{bmatrix} \in \mathrm{C}\ \ \mathrm{N}_R \,\mathrm{x}\, \mathrm{N}_T\,. \qquad (2)$$

Input signal is re-structured after pre-coding with the precoder $\boldsymbol{U}$ introduced. Thus the nth pre-coded block is written as $\boldsymbol{S_n}$.

$$\boldsymbol{S_n} = \boldsymbol{U}\ .\!*\ \boldsymbol{s_n} \qquad (3)$$

$$\boldsymbol{S_n} = \begin{bmatrix} s_1 & s_1 & 0 & 0 & \cdots & \cdots & 0 \\ s_2 & -s_2 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & s_3 & s_3 & \cdots & \cdots & 0 \\ \vdots & \vdots & s_4 & -s_4 & \cdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ldots & s_{N_T-1} & s_{N_T-1} \\ 0 & 0 & 0 & 0 & \ldots & s_{N_T} & -s_{N_T} \end{bmatrix} \in \mathrm{C}\ \mathrm{N}_R \,\mathrm{x}\, \mathrm{N}_T\ . \qquad (4)$$

where, $.*$ is element by element operator.

The first row of the post precoder matrix $\boldsymbol{S_n}$, $[s_1\, s_1\, \mathbf{0}\, \mathbf{0} \ldots 0] \in \boldsymbol{C}^{\,N_T}$ is associated with transmit antenna 1, the second row $[s_2\, -s_2\, \mathbf{0}\, \mathbf{0} \ldots 0] \in \boldsymbol{C}^{\,N_T}$ is associated with transmit antenna 2 and the $\mathrm{N_T}$th row $[\mathbf{0}\, \mathbf{0} \ldots s_{N_T}\, -s_{N_T}] \in \boldsymbol{C}^{\,N_T}$ is associated with transmit antenna $\mathrm{N_T}$ from the expression (3). Since the each antenna repeatedly transmits the same signal blocks multiple times, code rate of the whole system is reduced.

We consider a MIMO system operating over a time invariant channel. Let $h_{ji}(n)$, $(i = 1, \ldots\ldots, N_{N_T}, \ j = 1, \ldots\ldots, N_R, \ n = 0,1, \ldots M)$ denote the time domain channel impulse response between the i^{th} transmit antenna and j^{th} receive antenna pair, where M is the channel order which can be written as;

$$h_{ji} = [h_{ji}(0)\ h_{ji}(1) \ldots\ldots h_{ji}(M)] \tag{5}$$

$$h_{ji}(n) = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1\,N_T} \\ h_{21} & h_{22} & \cdots & h_{2\,N_T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_R\,1} & \cdots & \cdots & h_{N_R\,N_T} \end{bmatrix} \in \mathrm{C}\ \mathrm{N}_R \times \mathrm{N}_T \tag{6}$$

During the transmission, input signal $s(n)$ is passing through a wireless channel, is contaminated by AWGN noise $w_m(n)$.

$$w_m = [w_m(1)\ w_m(2) \ldots w_m(Ns-1)]^T$$

The nth received signal block corresponding to the j^{th} receiver antenna is modeled as follows,

$$x_j^n = h_{ji}(n) * S_n$$

The $x_1^n, x_2^n, \ldots . x_{N_R}^n$ are the received signal vectors corresponding to the receiver antenna $1,2, \ldots\ldots, N_R$ respectively.

We consider a 2x2 MIMO system to derive the relationships corresponding to the received signals x_j^n,

$$x_j^n = h_{ji}(n) * S_n + w_m^n \tag{7}$$

The nth received signal block x^n is shown as follows,

$$x^n = \begin{bmatrix} x_1^n & x_1^{n+1} \\ x_2^n & x_2^{n+1} \end{bmatrix} = \begin{bmatrix} h_{11}^n * s_1 + h_{12}^n * s_2 & h_{11}^n * s_1 - h_{12}^n * s_2 \\ h_{21}^n * s_1 + h_{22}^n * s_2 & h_{21}^n * s_1 - h_{22}^n * s_2 \end{bmatrix} \tag{8}$$

The following relationships can be obtained from (10) assuming that the noise free received signal,

$$h_{11}^{n} * s_1 = \frac{[x_1^{n} + x_1^{n+1}]}{2}$$

$$h_{12}^{n} * s_2 = \frac{[x_1^{n} - x_1^{n+1}]}{2}$$

$$h_{21}^{n} * s_1 = \frac{[x_2^{n} + x_2^{n+1}]}{2}$$

$$h_{22}^{n} * s_2 = \frac{[x_2^{n} - x_2^{n+1}]}{2}$$

Let us construct the relationships in matrix form as (12): (9)

$$\begin{array}{ccccc} \frac{[x_1^{n} + x_1^{n+1}]}{2} & = & h_{11}^{t} & * & s_1 \\ \frac{[x_2^{n} + x_2^{n+1}]}{2} & = & h_{21}^{t} & * & s_1 \\ \boldsymbol{x_y} & = & \boldsymbol{h_y} & * & \boldsymbol{s} \end{array} \qquad (10)$$

Let us rearrange the equations as (11) & (12)

$$\boldsymbol{x_k} = \boldsymbol{h_k} * \boldsymbol{s}$$
$$k = 1,2, \dots, N_R, l \neq k \qquad (11)$$

$$\boldsymbol{x_l} = \boldsymbol{h_l} * \boldsymbol{s}$$
$$l = 1,2, \dots, N_R, l \neq k \qquad (12)$$

Similarly, the MIMO system can be modeled as a SIMO system as in (10) in order to apply the frequency domain CR algorithm for channel estimation. In the absence of noise, with respect to the received symbols for any block m for any antenna pair (i, j), the following signaling property is given in [27],

$$\boldsymbol{x_k} * \boldsymbol{h_l} = \boldsymbol{x_l} * \boldsymbol{h_k} \qquad (13)$$

After DFT operation of the time domain CR fromula, the following relationship can be obtained,

$$X_k(a)H_l(a) = X_l(a)H_k(a)$$

$$a = 0,1,\dots,N-1 \tag{14}$$

Where, $X_m(a)$ and $H_m(a)$ represent the frequency domain samples of $x_m(n)$ and $h_m(n)$ respectively, with $m = k, l$. As the N- point DFT sequence $H_m(a)$ is defined as,

$$H_m(a) = \sum_{n=0}^{N-1} h_m(n)e^{-(\frac{j2\pi an}{N})} \tag{15}$$

Where zero padding is applied to the $h_m(n)$ for $M < n \leq N-1$ and (13) can be rewritten as,

$$[-F_l \quad F_k]\begin{bmatrix} h_k \\ h_l \end{bmatrix} = 0 \tag{16}$$

Where,
$\boldsymbol{h_m} = [h_m(0)h_m(1)h_m(2) \dots\, h_m(M)]^T$ and
$\boldsymbol{F_m} = diag\{X_m(0)X_m(1)X_m(2) \dots\, X_m(N-1)\}\, \boldsymbol{V}$, $with\, m = k, l$
$v = e^{-j2\pi/N}$ and

$$\boldsymbol{V} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & v & v^2 & \dots & v^M \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & v^{N-1} & \dots & \dots & v^{M(N-1)} \end{bmatrix} \tag{17}$$

We have proposed a FIR channel estimation scheme for MIMO channels in this study.

The CR-based frequency domain channel estimation scheme proposed in (14) can be applied for MIMO channels and the equation can be rearranged as in (18),

$$[-F_l \;\; F_k]\begin{bmatrix} h_k \\ h_l \end{bmatrix} = 0 \tag{18}$$

with the l^{th} entry being $-F_l$ and the k^{th} entry being F_k. Based on F_m and (18), we have

$$F_m \, h = 0 \tag{19}$$

where the vectors **F** and h are as shown in (20),

$$F = [-F_l \;\; F_k]$$

$$h = [h_k^T \, h_l^T]^T \tag{20}$$

In the presence of noise, the channel estimation $\hat{h}$ is obtained by,

$$\hat{h} = arg \min_{\|h\|=1} \|Fh\|_2 \tag{21}$$

The computation procedure is illustrated as follows, and the number of output samples Ns is assumed as fixed for a given channel model. Therefore, N is selected according to the channel condition of the system.

Step 1 – Construct the F matrix using F_m matrices.

Step 2–Find $\hat{h}$ that minimizes $\|Fh\|_2$. $\hat{h}$ Comprises the estimated channel impulse response $\hat{h}_m(n)$, m = 1,….,L, n = 0,1,..M.

Singular Value Decomposition (SVD) of the **F** matrix results,

$$F = U\,S\,V^T \tag{22}$$

Where,

U – Unitary matrix (Left Singular vector of matrix **F**), **S** – Diagonal matrix with Singular values, **V** – Unitary matrix (Right singular vector of matrix **F**). The optimization solution of the channel $\hat{h}$ is obtained by taking the right singular vector corresponding to the smallest singular vector of Matrix **F**.

3.2 The CR- Based Channel Estimation Scheme for MIMO OFDM systems

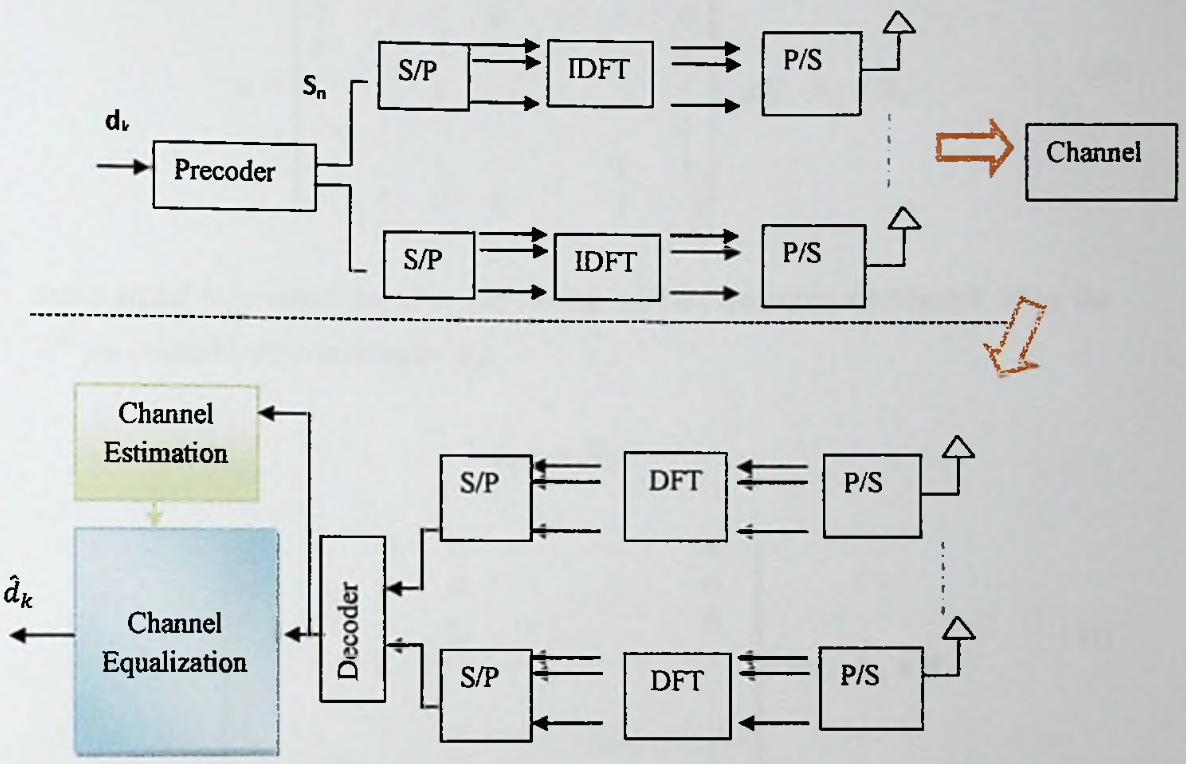

Figure 5 MIMO non-CP OFDM channel model

A blind channel estimation scheme was developed for MIMO OFDM system without cyclic prefix (CP). This method is proposed with the use of a pre-coder matrix introduced before the transmitter.

The propose MIMO non-CP OFDM channel consists of N_T number of transmitter antennas and N_R number of receiver antennas. Here, the relationships of channel impulse responses and the received signal blocks are shown below,

The i^{th} OFDM frame of N symbols sent to the transmitter is represented as (29),

$$s^i = [s_1^i s_2^i \; s_3^i \ldots\ldots s_N^i \,] \tag{29}$$

The structure of the precoder matrix u is introduced before the transmitter is given as in (30),

$$u = \begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & \cdots & 0 \\ 1 & -1 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & 1 & \cdots & \cdots & 0 \\ \vdots & \vdots & 1 & -1 & \cdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \cdots & 1 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix} \in C \;\; N_R \times N_T \tag{30}$$

Input signal is re-structured after pre-coding with the precoder introduced. Thus the nth pre-coded block is written as S_n.

$$S_n = U \circledR s^i ,$$

$$S_n = \begin{bmatrix} s_1 & s_1 & 0 & 0 & \cdots & \cdots & 0 \\ s_2 & -s_2 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & s_3 & s_3 & \cdots & \cdots & 0 \\ \vdots & \vdots & s_4 & -s_4 & \cdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \cdots & s_{N_T-1} & s_{N_T-1} \\ 0 & 0 & 0 & 0 & \cdots & s_{N_T} & -s_{N_T} \end{bmatrix} \in C \;\; N_R \times N_T \;\; . \tag{31}$$

The first row of the post precoder matrix S_n, $[s_1 \; s_1 \;\; \cdots \;\; 0] \in C^{N_T}$ is associated with transmit antenna 1, the second row $[s_2 - s_2 \;\; \cdots \; 0] \in C^{N_T}$ is associated with transmit antenna 2 and the N$_T$ th row $[\,0 \;\; \cdots s_{N_T} \;\; -s_{N_T}] \in C^{N_T}$ is associated with transmit antenna N$_T$ from the expression (31). Since, the each antenna repeatedly transmit the same signal multiple times code rate of the whole system is reduced.

The MIMO-OFDM system considered is shown in figure 5, operating over a time invariant channel. Let $h_{ji}(n), (i = 1, \dots\dots, N_{N_T}, \; j = 1, \dots\dots\dots, N_R, \; n = 0,1, \dots M)$ denote the time domain channel impulse response between the i^{th} transmit antenna to j^{th} receive antenna,

$$h_{ji} = [h_{ji}(0) \; h_{ji}(1) \dots\dots\dots h_{ji}(M)] \tag{32}$$

$$h_{ji}(n) = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1\,N_T} \\ h_{21} & h_{22} & \cdots & h_{2\,N_T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_R\,1} & \cdots & \cdots & h_{N_R\,N_T} \end{bmatrix} \in C \quad N_R \times N_T \tag{33}$$

During the transmission, input signal $s(n)$ is passing through a wireless channel, is contaminated by AWGN noise $w_m\,(n)$.

$$w_m\,(n) = [w_m\,(1)\ w_m\,(2) \ldots\ldots w_m\,(Ns-1)]^T \tag{34}$$

The nth received signal corresponding to the j^{th} receiver antenna after DFT operation can be written as (35),

$$x_{ji}^n = h_{ji}(n)\, S_n \tag{35}$$

The $x_{1i}^n, x_{2i}^n, \ldots. x_{N_Ri}^n$ are the received signal vectors corresponding to the receiver antenna $1, 2, \ldots\ldots, N_R$ respectively.

We consider a 2x2 MIMO OFDM system to derive the relationships corresponding to the received signals,

$$x_{ji}^n = h_{ji}(n)\ S_n + w_m^n \tag{36}$$

$$x_{ji}^n = \begin{bmatrix} x_{11}^n & x_{12}^n \\ x_{21}^n & x_{22}^n \end{bmatrix} = \begin{bmatrix} h_{11}^n\ s_1 + h_{12}^n\ s_2 & h_{11}^n\ s_1 - h_{12}^n\ s_2 \\ h_{21}^n\ s_1 + h_{22}^n\ s_2 & h_{21}^n\ s_1 - h_{22}^n\ s_2 \end{bmatrix} \tag{37}$$

The following relationships can be obtained from (37)

$$h_{11}^n\ s_1 = \frac{[x_{11}^n + x_{12}^n]}{2}, \qquad h_{12}^n\ s_2 = \frac{[x_{11}^n - x_{12}^n]}{2} \tag{38}$$

$$h_{21}^n\ s_1 = \frac{[x_{21}^n + x_{22}^n]}{2}, \qquad h_{22}^n\ s_2 = \frac{[x_{21}^n - x_{22}^n]}{2} \tag{38}$$

We can construct the relationships in matrix form as (41):

(39)

$$\frac{[x_{11}^{n} + x_{12}^{n}]}{2} = h_{11}^{n} \qquad s_1$$

$$\frac{[x_{21}^{n} + x_{22}^{n}]}{2} = h_{21}^{n} \qquad s_1 \tag{40}$$

$$x_y = h_y \qquad s \tag{41}$$

Let us rearrange the equations as follows in (42)

$$x_k = h_k \qquad s \qquad , k = 1,2,\ldots,N_{N_R}, l \neq k \tag{42}$$

$$x_l = h_l \qquad s \qquad , l = 1,2,\ldots,N_{N_R}, l \neq k \tag{43}$$

Similarly, the MIMO OFDM system is modeled as a SIMO OFDM system as (41) in order to apply the CR scheme in frequency domain to estimate the channel.

In the absence of noise, with respect to the received signal blocks for any block m for any antenna pair (k, l), the following signaling property is given by [30]

$$\mathrm{x}_k(n) * \mathrm{h}_l(n) = \mathrm{x}_l(n) * \mathrm{h}_k(n) \qquad 1 \leq k, l \leq L, k \neq l \tag{44}$$

A CP free MIMO OFDM system can be considered with N subcarriers and L receive antennas, where L ≥ 2. A stream of digitally modulated symbols is split into parallel sub streams on N adjacent subcarriers. Each block of N complex-valued source symbols

$$d_m = [d(mN) \quad d(mN+1) \quad \ldots \quad d(mN+N-1)] \tag{45}$$

Where, $d(k) \neq 0$, inverse discrete Fourier transformed to form the complex baseband time-domain signal block $s_m = [s(mN) \quad s(mN+1) \quad \ldots\ldots\ldots\ldots s(mN+N-1)]^T$ for transmission , m is the block index on the OFDM signal. Note that no CP is introduced to the OFDM signal. It is assumed that there is perfect timing and carrier synchronization of transmitter and receiver.

Although there are N symbols in each received OFDM block $x_m^{(i)}$ the resulting signal on either side of (45) is of valid length N-M, and coincides with the N- point circular convolution of $x_m^{(k)}[x_m^{(l)}]$ with $h^l(n)$ $[h^k(n)]$ after discarding the first M points of the circular convolution result. The circular convolution involved are equivalent to the point wise multiplication of the DFTs of $x_m^{(k)}[x_m^{(l)}]$ with $h^l(n)$ $[h^k(n)]$, given by $y_m{}^k(a)H^l(a)$ *and* $y_m{}^l(a)H^k(a)$, where $y_m{}^b(a)$ is given as,

$$y_m{}^b = [y_m{}^b(0)y_m{}^b(1) \ldots \quad y_m{}^b(N-1)]^T \tag{46}$$

$\mathbf{H}^l(a)$ represents the N-point DFT of $\mathbf{h}^l(n)$ and a = 0, 1, ...N-1.

Hence we obtain,

$$\mathbf{CW}\begin{pmatrix} y_m{}^k(a)H^l(a) \\ y_m{}^k(a)H^l(a) \\ \\ y_m{}^k(N-1)H^l(N-1) \end{pmatrix} = \mathbf{CW}\begin{pmatrix} y_m{}^l(a)H^k(a) \\ y_m{}^l(a)H^k(a) \\ \ldots \\ y_m{}^l(N-1)H^k(N-1) \end{pmatrix} \tag{47}$$

Where, W denotes the N x N IDFT matrix whose elements are $\{W\}_{p,q} = 1/N \; W^{-pq}$ with $W = e^{-i2\pi/N}$, for p,q= 0,1,.....N-1, and $\boldsymbol{C} = [0_{(N-M)xM} \; I_{N-M}] \in \boldsymbol{R}^{(N-M)xN}$. As the N point DFT $\mathbf{H}^l(a) = \sum_{n=0}^{N-1} h^l(n)\mathrm{W}^{an} = \sum_{n=0}^{M} h^l(n)\mathrm{W}^{an}$, we rewrite the (48) as,

$$\mathrm{CW}\left[-\mathbf{F}_m^{(l)}\mathbf{F}_m^{(k)}\right]\begin{bmatrix}\boldsymbol{h}_k \\ \boldsymbol{h}_l\end{bmatrix} = 0 \tag{48}$$

$$\text{Where } \mathbf{h}^l = [h^l(0)h^l(1) \; \ldots\ldots h^l(M)]^T \tag{49}$$

$$\mathbf{F}_m^{(l)} = diag\{y_m^l(0)y_m^l(1) \; \ldots\ldots y_m^l(N-1)\} \; \boldsymbol{V} \tag{50}$$

Where the V is the first M+1 columns of the N x N DFT matrix . Taking into account all combinations of antenna pair (i, j), defined in [30] (51)

$$F_m = [(F_m^{1,2})^T\ (F_m^{1,3})^T \ \dots\dots (F_m^{1,L})^T\ (F_m^{1,2})^T \dots (F_m^{2,3})^T\ \dots (F_m^{2,L})^T \dots$$
$$\dots (F_m^{L-1,L})^T]^T \ \in\ C^{\frac{NL(L-1)}{2}\, x\, L(M+1)}$$

With the i^th entry is being$-F_m^j$ and the j^th entry being F_m^i. Based on F_m in (51) we have,

$$\bar{F}_m h = 0 \tag{52}$$

Where

$$\mathrm{h} = [(h^{(1)})^T (h^{(2)})^T \ \dots\dots (h^{(L)})^T]^T \ \in\ C^{L\,(M+1)} \text{ and} \tag{53}$$

$$\tilde{F}_m = diag\{CW, CW, \dots. CW\} F_m. \tag{54}$$

The dimension of $\tilde{F}_m$ $is \frac{L(L-1)(N-M)}{2} x\ \ L(M+1)$.

Hence, the channel coefficient vector h can be determined by solving (52) based on just one block of observation data y_m^l.

The least squares optimization solution for the minimization problem is given with the right singular vector associated with the minimum singular value of the matrix $\tilde{F}_m$.

Singular Value Decomposition (SVD) of the $\tilde{F}_m$ matrix results,

$$\tilde{F}_m = U\, SV^T, \tag{55}$$

where,

U – Unitary matrix (Left Singular vector of matrix **F**)

S – Diagonal matrix with Singular values

V – Unitary matrix (Right singular vector of matrix **F**)

Optimization solution for the channel impulse response $\hat{h}$ is obtained from the right singular vector corresponding to the smallest singular value of Matrix $\bar{F}_m$.

Similarly, the simulation procedure for channel estimation is same as in MIMO system.

4. SIMULATION RESULTS AND DISCUSSION

Simulations were conducted to verify the performance of the frequency domain CR [27] scheme for different lengths of the input signal block and for different number of transmitter antennas and receiver antennas. The performance of the propsoed blind channel estimation scheme for MIMO and MIMO OFDM systems were compared. A typical MIMO channel with random channel coefficients and channel assumed to be time invariant within a signal blocks period.

The source signal is white binary (-1, 1) sequence with white spectrum and unit power. A different realization of noise (independent on each channel) with power σ_n^2 is added to the channel outputs during the simulation to obtain performance of the system. The simulation is performed each of 100 times for a given SNR to measure the performance corresponding to a given SNR. The L is the number of subchannels between each transmitter and receiver antennas in the system. Without loss of generality, we assume that all the subchannels have constant multipath components and the maximum order of the subchannels is assumed a priory for estimating the channel during the simulation.

Additive White Gaussian Noise (AWGN) is applied to the received signal in the SNR range of 5 dB to 40 dB during the simulation process in this simulation. The noise σ_n^2 is corresponding to the desired SNR is added to the output signal of each subchannels.

$$SNR = 10 * log10\left(\frac{signal_{power}}{Noise_{power}}\right) \tag{56}$$

For this SIMO channel model, SNR is defined as ,

$$SNR = 10 \log_{10} \frac{\|h\|^2}{L\sigma_n^2}$$

The noise power σ_n^2 can be extracted as,

$$\sigma_n^2 = \frac{\|h\|^2}{10^{\frac{SNR}{10}}} L \tag{57}$$

The performance of the proposed channel estimation is assessed in terms of mean square error (MSE) as given by

$$MSE(dB) = 10\log_{10}\left(\frac{1}{t}\sum_{i=1}^{t}\|\hat{h}_i - h\|^2\right) \tag{58}$$

In which $\|.\|^2$ stands for the Frobenious norm.

Where the number of Monte Carlo runs (t), true unit norm coefficient vector (**h**) and estimated unit norm coefficient vector ($\hat{h}_i$).

The performance of the proposed MIMO OFDM system is measured in terms of normalized mean square error value (NMSE).

$$NMSE(dB) = \frac{1}{LM}\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{1}{t}\sum_{i=1}^{t}\|\hat{h}_i - h\|^2 / \|h\|^2\right) \tag{59}$$

Where M is the number of total channel realizations and L is the number subchannels in each channel realization.

The performance of the proposed MIMO OFDM system is measured in terms of normalized root mean square error value (NRMSE).

$$NRMSE(dB) = \sqrt{\frac{1}{LM}\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{1}{t}\sum_{i=1}^{t}\|\hat{h}_i - h\|^2 / \|h\|^2\right)} \tag{60}$$

4.1 Verification of the system model using the existing results

4.1.1 Verification of the existing results for single carrier SIMO

In this section, the MSE performance of the existing CR based frequency domain channel estimation scheme proposed for SIMO systems [27] is further investigated. The channel estimation scheme proposed in [27] for the SIMO systems. The performance of this SIMO channel estimation scheme is further investigated when there are different numbers of antennas used at the receiver and for the multiple number of received signal blocks are used for the channel estimation.

Figure 6 shows the MSE performance of the frequency domain CR based channel estimation scheme proposed for SIMO systems with different number of receiver antennas exist.

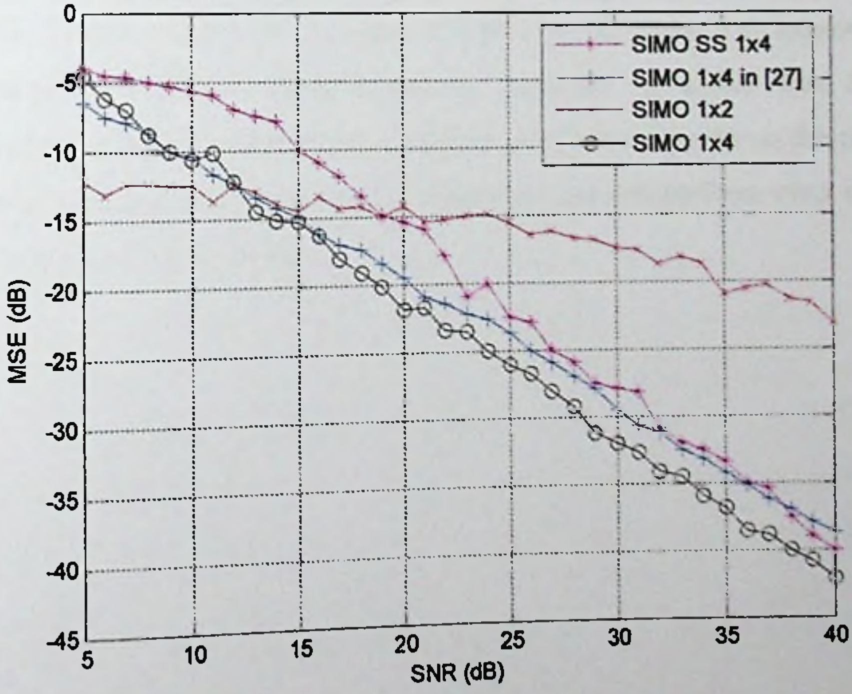

Figure 6 MSE performance of SS and CR based schemes

The MSE performance of SIMO system is simulated with BPSK modulation for 5 dB to 40 dB SNR range. It is observed from the result that the simulated MSE performance for 1x4 SIMO is similar to the result shown in [27]. The SIMO system with different number of receiver antennas is considered to investigate the performance of the existing frequency domain CR channel estimation scheme. The results obtained for this simulation show that the MSE performance of SIMO 1x 2 systems comparatively better at low SNR regions, but the MSE performance comparatively low in high SNR regions. It is observed from the results that CR based 1x4 SIMO systems comparatively better than MSE performance result of 1x4 SIMO subspace based channel estimation scheme proposed in [35]. The MSE performance for a proposed channel estimation scheme based on CR principle for SIMO channels was investigated. It can be observed that the MSE performance obtained through the Matlab simulation for the 1x4 SIMO channels follow the same trend as it is in [27]. It was also observed that the MSE performance for SIMO systems was improved when there are more number of receiver antennas in the system. Further, the number of received signal in a signal block is varied to simulate the MSE performance of the 1x4 SIMO systems, and it was observed the MSE performance was improved when the number of received signal blocks is increased. It shows that the MSE performance of the CR based channel estimation scheme depends on the number of samples of received signals but the CR algorithm can still perform even with short length of received signal blocks.

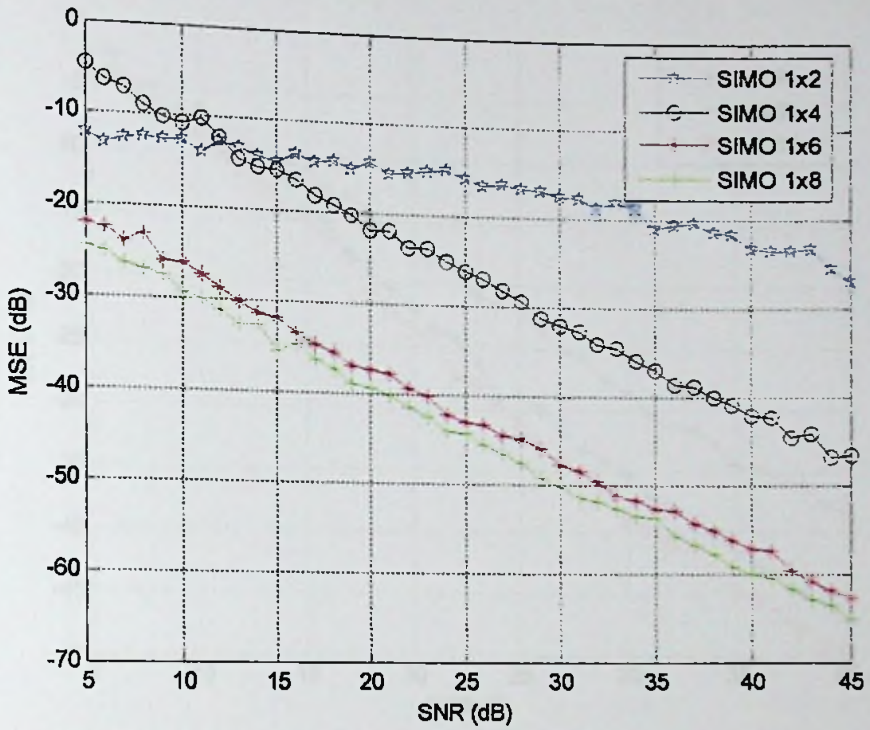

Figure 7 MSE performances for SIMO systems with different number of receiver antennas

Figure 7 shows the MSE performance of SIMO systems with BPSK modulation for 5 dB to 40 dB SNR range. It is observed from the result that the better MSE performance for the frequency domain CR algorithm can be achieved with the increased number of receiver antennas. The optimization algorithm consists of several computations which process all the received signal vectors. Hence, the number of samples for the channel estimation gets increased. Therefore, the results obtained for this simulation shows comparatively better performance when the number of receiver antennas is increased.

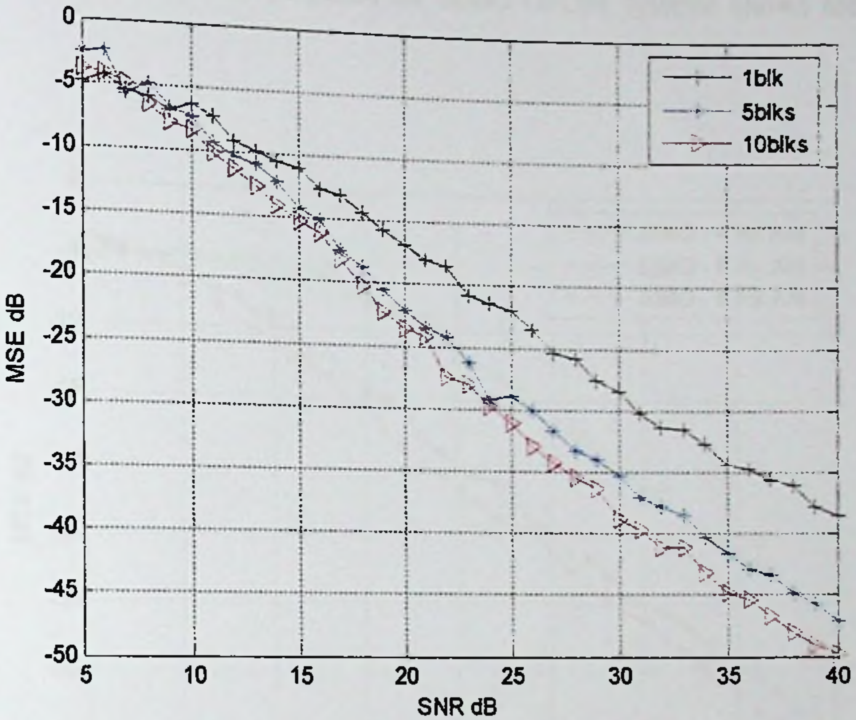

Figure 8 MSE performance for SIMO systems with multiple received signal blocks

Figure 8 shows the MSE performance of SIMO systems with BPSK modulation for different number of received signal blocks. It is observed that the simulated results show better performance for the SIMO systems when higher number of received signal block are used for channel estimation.

The results of the simulation depends the number received signal blocks to estimate the channel. Therefore, MSE performance of the frequency domain channel estimation scheme increases with increase of number of received signals blocks to identify the channel.

4.1.2 Verification of the existing results for multicarrier SIMO

The MSE performance is analysed for the scheme proposed in [28] for the multicarrier MIMO systems which is analysed with the simulated results for the same channel estimation scheme. The channel estimation scheme proposed in [28] for the frequency domain for OFDM systems without cyclic- prefix (CP) is considered to analysed the results. The simulated result shows that the channel estimation scheme proposed in [28] show similar results. Therefore, the CR based channel estimation scheme proposed for SIMO OFDM systems shows the verified results.

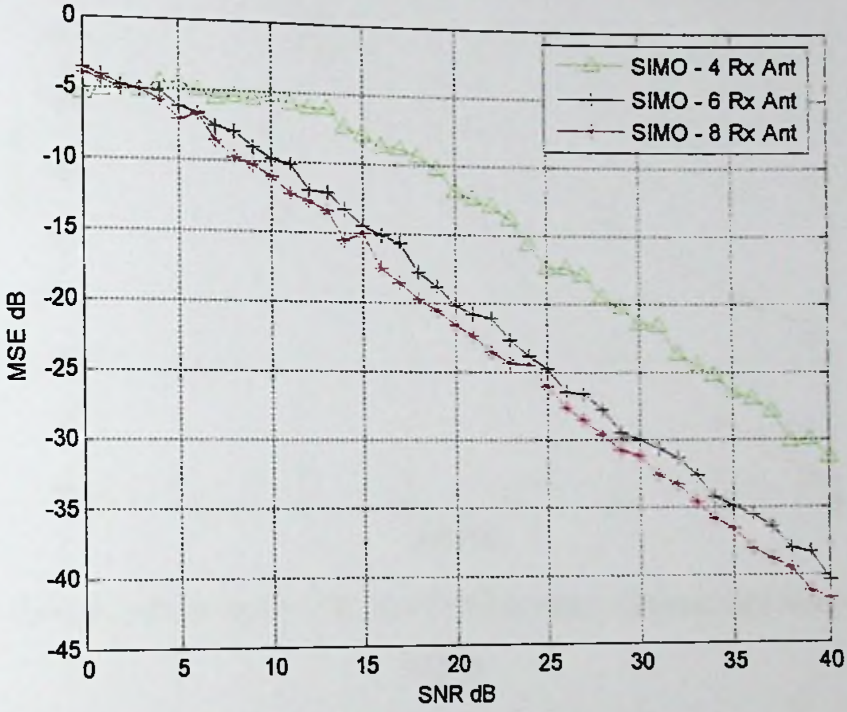

Figure 9 MSE performance for SIMO-OFDM systems with different number of received antennas

In Figure 10 shows the MSE performance results for 1x4 SIMO OFDM systems. The results were obtained for the different number of received OFDM blocks used for channel estimation. The multiple number of received OFDM signal block improve the MSE performance due to the higher number of received signal blocks for the channel estimation.

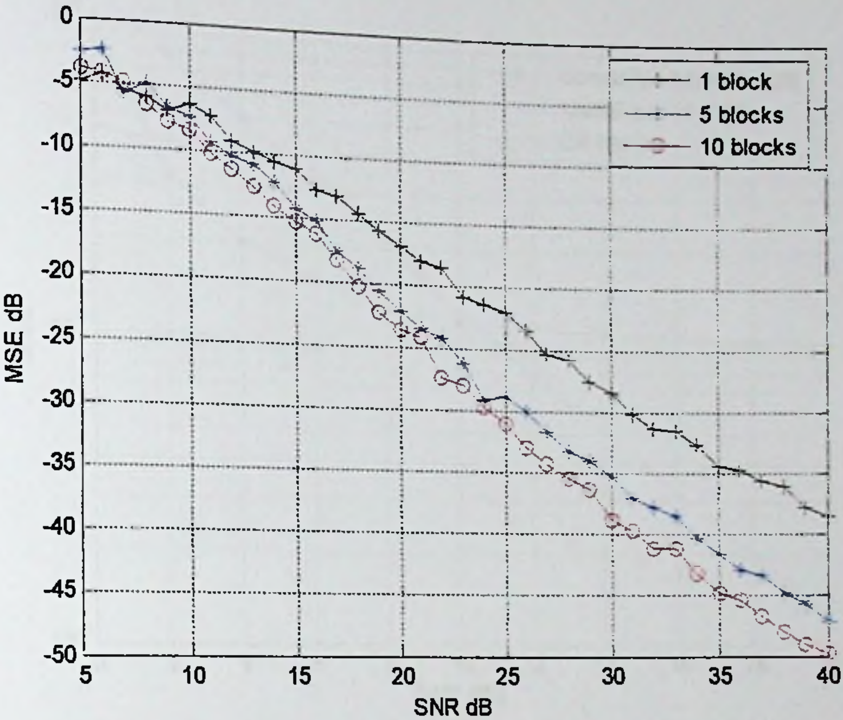

Figure 10 MSE performance for SIMO-OFDM systems with multiple received signal blocks

Figure 10 shows the MSE performance of the channel estimation scheme for varying number of received signal blocks. The number of received signal blocks are considered as 1, 5 and 10 received signal blocks. It is shown in Figure 9 that more data blocks lead to better estimation accuracy.

4.2 Performance investigation of the proposed MIMO CR-based Blind channel estimation scheme

The validity of the proposed blind channel estimation algorithm for a 2x2 MIMO is investigated via Matlab simulations. In Figure 11 shows the MSE performance of MIMO systems of [33] for 2x2 MIMO systems which are cross correlation based channel estimation scheme. It is observe that comparatively higher MSE performance of our proposed CR based channel estimation scheme compared with the scheme proposed in [33] at higher SNR region.

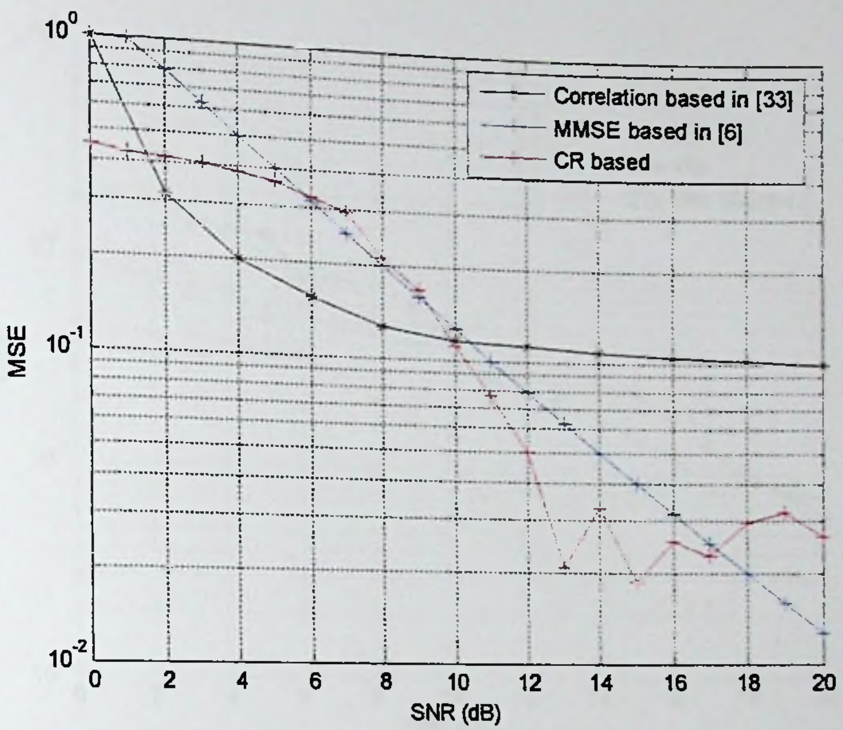

Figure 11 MSE performance for MIMO systems with $N_T = 2$ & $N_R = 2$

Figure 11 shows the MSE performance of MIMO systems with BPSK modulation for the range from 0dB to 20dB. It can be observed that the MSE performance of the proposed channel estimation scheme for 2x2 MIMO channel shows comparatively better performance with the channel estimation scheme proposed in [6] for 2x2 MIMO systems.

The results of the simulation study show better MSE performance in the specified region of SNR. Hence, the proposed channel estimation scheme shows comparatively better performance compared with correlation based schemes.

Figure 12 shows the MSE performance for different number of receiver antennas deployed at the receiver. The result in Figure 12 show that the proposed model can be used effectively with the minimum of 2x2 MIMO channel systems. It also proves that the MSE performance of the proposed model can be increased with the use of higher number received signal blocks for channel estimation. The results shown in simulation shows that the higher number of transmitter receiver antenna pairs will cause to reduce the MSE performance of our proposed channel estimation scheme.

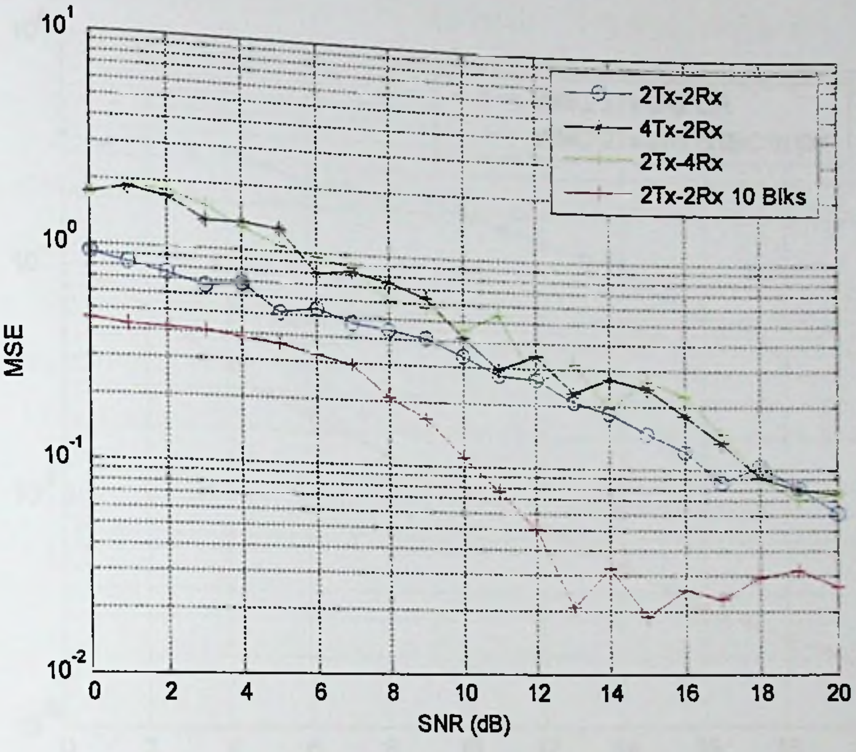

Figure 12 MSE performance MIMO systems for different number antennas

In Figure 13 and Figure 14 shows the BER performance of frequency domain channel estimation scheme and the channel estimation scheme of Space Time Block Coded (STBC). The results are obtained for 2x2 MIMO system with BPSK modulation. It is clear that the results obtained for frequency domain CR based channel estimation scheme shows better performance at low SNR regions. The results are compared with the theoretical BER simulations for a 2x2 MIMO systems.

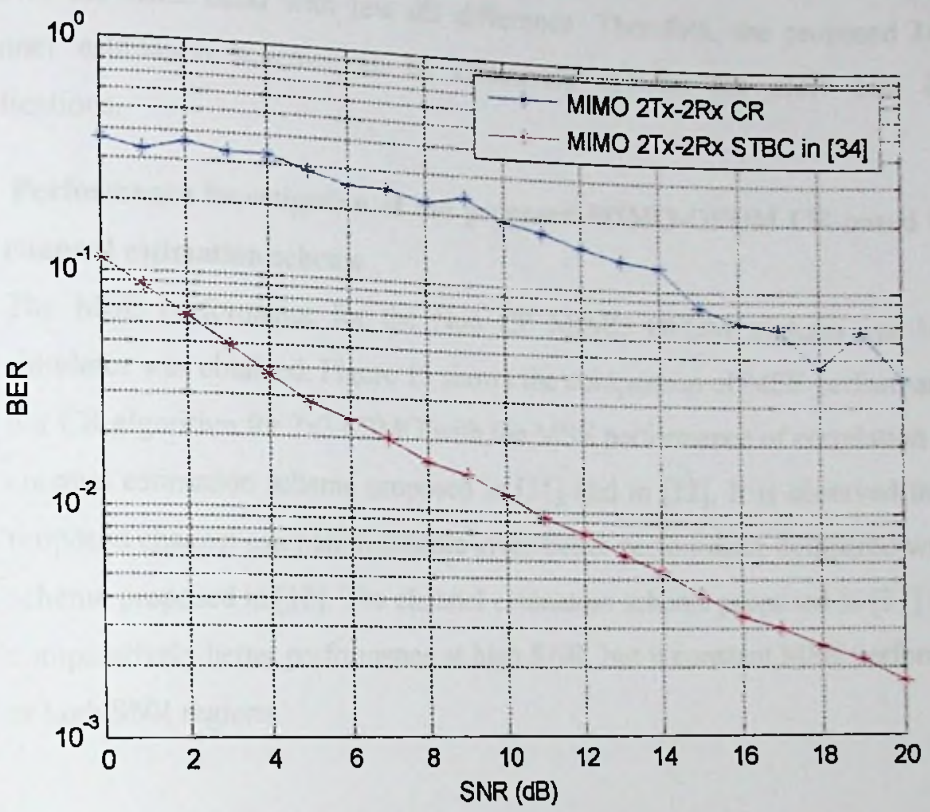

Figure 13 BER performance comparison for MIMO systems with existing results

In Figure 13 shows the BER performance for CR based and STBC [34] based schemes. The results shows comparatively low BER performance for the frequency domain CR based channel estimation scheme at higher ranges of SNR. The MSE performance of the proposed CR based 2x2 MIMO channel estimation scheme with correlation based channel estimation scheme proposed in [33] and in [6]. The proposed CR based channel estimation scheme in this research for 2x2 MIMO systems shows comparatively better performance with the scheme proposed in [33]. It can be observe comparatively better performance compared with the MMSE based channel estimation scheme proposed in [6]. Therefore, the proposed CR based MIMO channel estimation scheme outperform the schemes proposed in [33] and [6]. It can be observed that the MSE performance of the proposed MIMO system is improved when the number of subchannels is decreased in a MIMO system. The BER performance is analysed for the proposed CR based channel estimation scheme in this research as another performance measure. The BER performance is less compared with the STBC based channel estimation scheme proposed in [34], but it follows the same trend with few dB difference. Therefore, the proposed MIMO channel estimation scheme can be effectively use for the short data length applications.

4.3 Performance investigation of the proposed MIMO-OFDM CR-based Blind channel estimation scheme

The MSE performance for the Non CP MIMO OFDM systems via Matlab simulator was obtained. Figure 15 shows the comparison of MSE performance of our CR algorithm for 2x2 MIMO with the MSE performance of correlation based channel estimation scheme proposed in [31] and in [12]. It is observed that the proposed channel estimation scheme gives better performance compared with the scheme proposed in [12]. The channel estimation scheme proposed in [31] shows comparatively better performance at high SNR, but it constant MSE performance at high SNR regions.

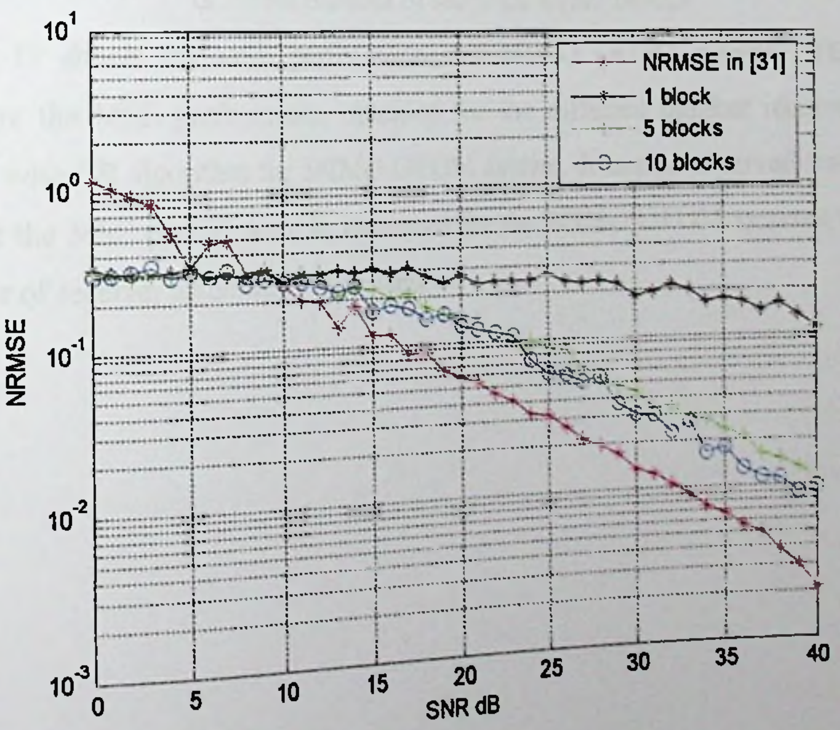

Figure 14 NRMSE Performance comparison for MIMO OFDM system with CR & Correlation based schemes

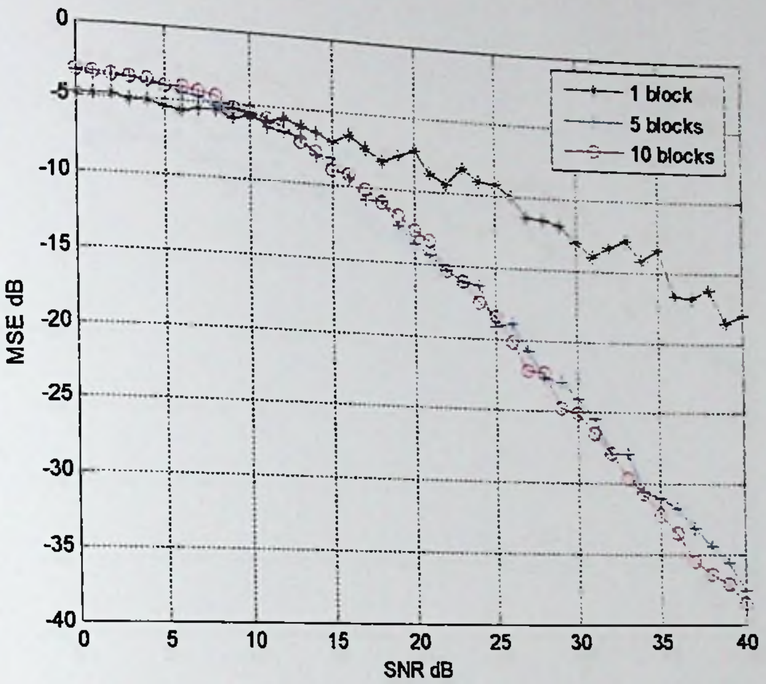

Figure 15 MSE Performance comparison of CR based MIMO OFDM system with different number of received signal blocks

Figure 17 shows the MSE performance for MIMO OFDM systems. The results compare the MSE performance obtained for the different number received signal blocks with CR algorithm for MIMO OFDM system. It can be observed from Figure 17 that the MSE performance is increased for the MIMO OFDM systems when the number of receiver antennas is increased.

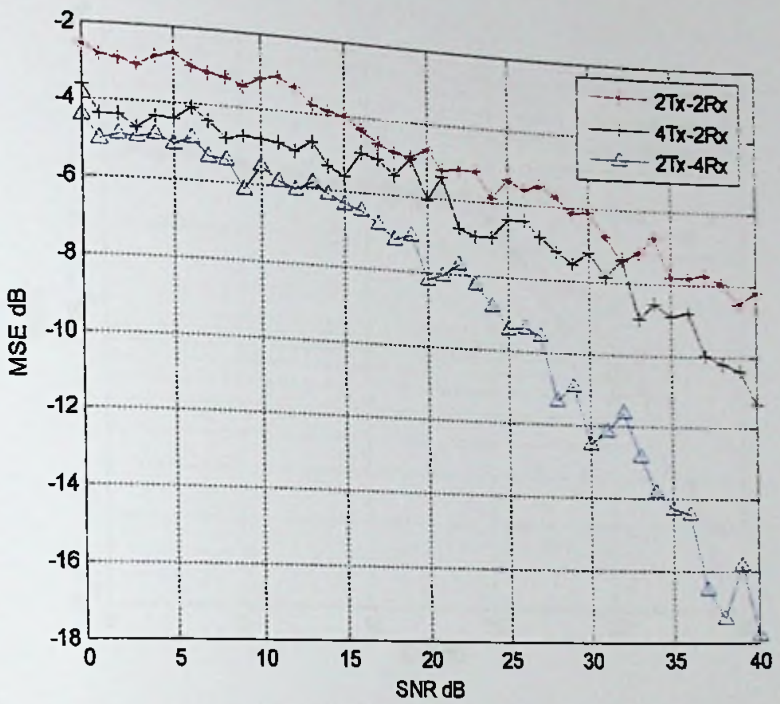

Figure 16 MSE Performance comparison of CR based MIMO OFDM systems with different number of received signal blocks

Figure 18 shows the BER performance of MIMO OFDM systems for different number of received signal blocks are used for channel estimation. BER performance was obtained for the SNR range of 0 dB to 40 dB, and the BER performance was obtained for the SNR regions.

It is observed that the BER performance of the proposed channel estimation scheme increases when the number of received signal blocks increases. It shows that the channel estimation accuracy directly affect BER performance of the system.

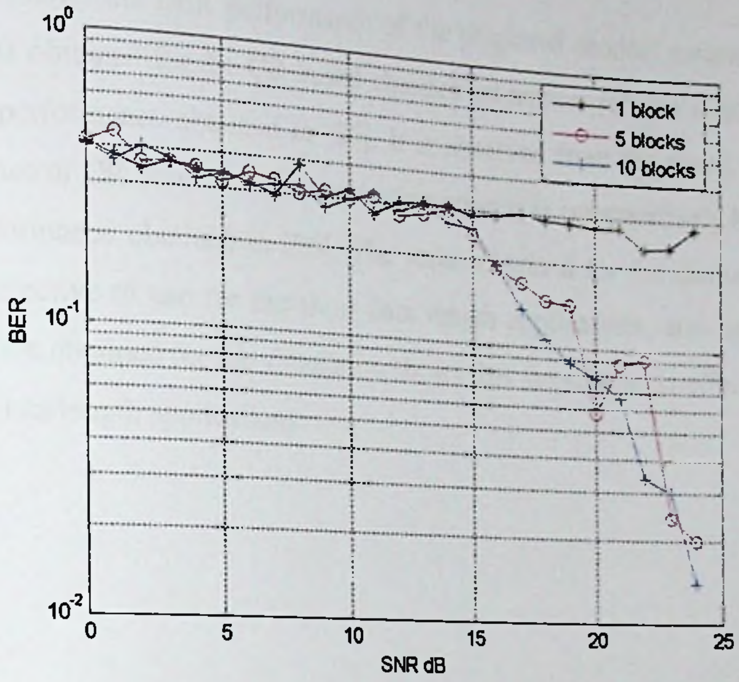

Figure 17 BER Performance comparison of CR based MIMO OFDM system with different number of received signal blocks

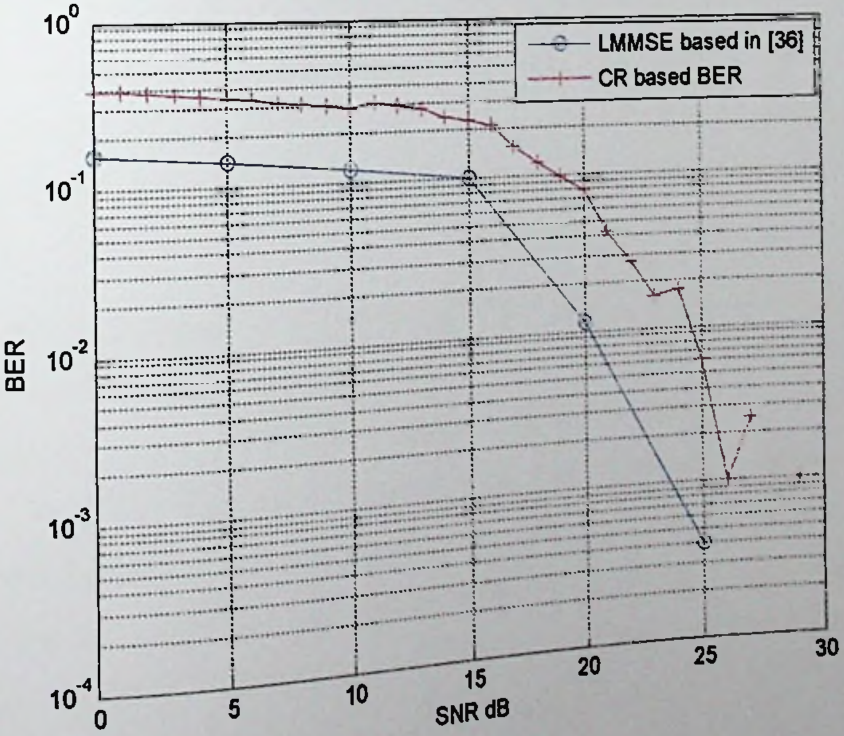

Figure 18 BER Performance comparison of CR based with results in [36] for MIMO OFDM system

Figure 19 shows the BER performance of the proposed channel estimation scheme. The results obtained for the CR based channel estimation scheme is compared with the BER performance obtained in [36]. It is observed from the result that the BER performance of CR based channel estimation scheme is comparatively better than the BER performance obtained in [36]. The result obtained for the channel estimation scheme effective to use for the short data length applications, and hence the BER performance obtained for the simulation shows that the scheme is effectively use for the short data length applications.

5. CONCLUSIONS AND FURTHER RESEARCH

5.1 Conclusion

A FFT based blind channel estimation scheme for FIR MIMO and MIMO Non-CP OFDM systems with CR principle was developed in this research. The proposed channel estimation scheme seeks multiple copies of the same input signal blocks to be transmitted to estimate the channel. The computation of this scheme involves few FFTs operations, mathematical calculations and seeking the right singular vector corresponding to the smallest singular value.

The MSE performance of CR based SIMO Non CP OFDM system proposed in [28] is investigated. The similar performance could be obtained with the simulation results of this research and the similar results are shown in [28]. Since the results are obtained in [28] for 1x4 SIMO channels, the MSE performance was investigated with the same scheme for different number of receiver antenna systems. It proves that the error of the channel estimation scheme in [28] can be minimized by increasing the number receiver antennas in a system. Similarly, the MSE performance was observed by varying the number of received signal blocks to collect more samples of received signal blocks, and it shows improved MSE performance for channel estimation with the increase number of received signal blocks for channel estimation.

The MSE performance of the proposed CR based MIMO Non CP OFDM system in this research is compared with the scheme proposed in [36] and in [4]. The MSE performance of this proposed channel estimation scheme reveals similar performance as in [36]. The results are obtained for our proposed scheme with the use of 10 number received signal blocks. Therefore, the MSE performance can also be further improved by increasing the number received signal blocks for the channel estimation. The BER performance for our proposed channel estimation is obtained, and it maintains few dB differences with less performance.

The proposed CR based MIMO Non-CP OFDM channel estimation scheme can be effectively use for short data length applications. Therefore, the proposed CR based channel estimation schemes for both single carrier and multi carrier MIMO systems can be effectively use to estimate the channel specially for short data length applications.

5.2 Further Research

1. Proposed blind channel estimation scheme requires the knowledge of the channel order, but it can be considered as a further research area.
2. Another research area was identified with the CR based frequency domain scheme for MIMO & MIMO OFDM systems with improved code rate. Since, the proposed channel estimation scheme in this study performs effectively with the use of multiple copies of input signals blocks, spectral efficiency of the system reduces. Therefore, the CR-based MIMO & MIMO OFDM channel estimation scheme is an another area to study in order to improve the spectral efficiency.

REFERENCES

1. Chao-Cheng Tu, and Benoît Champagne, *"IEEE,* "Subspace-Based Blind Channel Estimation for MIMO-OFDM Systems With Reduced Time Averaging" IEEE Trans. on Vehicular Technology, vol. 59, pp. 1539-1544, Mar. 2010.

2. Changyong Shin and Edward J. Powers "Blind Channel Estimation for MIMO-OFDM Systems Using Virtual Carriers" IEEE Global Telecommunication conf, vol. 4, pp. 2465-2469, 2004.

3. C. Li and S. Roy, "Subspace-based blind channel estimation for OFDM by exploiting virtual carriers," *IEEE Trans. Wireless Commun.*,vol.2, pp. 141–150, Jan. 2003.

4. *Xi Chen†, A. Rahim Leymanand Jun Fang* "Blind Channel Estimation for Linearly Precoded MIMO-OFDM" in Proc. IEEE International conf. on Acoustics, speech and signal processing, vol. 4, 2006.

5. Jin -Goog Kim, Jun-Han Oh and Jong-Tae Lim " Subspace-based Channel Estimation for MIMO-OFDM Systems with Few Received Blocks" Signal processing letters,IEEE, vol. 19, pp. 435-438, January 2012.

6. Xia Liu, ShiyangLu, Marek E .Bialkowski and Hon Tat Hui "MMSE Channel Estimation for MIMO System with Receiver Equipped with a Circular Array Antenna"Proc. Asia-Pacific conf. on Microwave, 2007.

7. MehrzadBiguesh, and Alex B. Gershman, " *IEEE* "Training-Based MIMO Channel Estimation: A Study of Estimator Tradeoffs and Optimal Training Signals" *IEEE Trans. Signal Processing,* vol.54, pp. 884–893, Mar. 2006.

8. TianbinWo and Peter Adam Hoeher ""Semi-Blind Channel Estimation for MIMO Systems," University of Kiel, Oct. 2004."

9. Bertrand Muquet, Marc de Courville, and Pierre Duhamel, "Subspace-Based Blind and Semi-Blind Channel Estimation for OFDM Systems" *IEEE Trans. Signal Processing,* vol.50, pp. 1699–1712, July. 2002.

10. Jan-Jaap van de Beek , OveEdfors, Magnus Sandell, Sarah Kate Wilson , Per Ola B˜orjesson "On Channel Estimation in OFDM Systems" IEEE Vehicular Technology conf, vol. 2, pp. 815-819, 1995.

11. Prof. Susmita Das, Kala Praveen Bagadi "MIMO-OFDM Channel Estimation Using Pilot Carries", International Journal of computer applications, vol. 2, May 2010.

12. Hua Zhang, Ye (Geoffrey) Li, Anthony Reid, and John Terry "Optimum Training Symbol Design for MIMO OFDM in Correlated Fading Channels" IEEE Trans. on Wireless com, vol. 5, pp. 2343-2347, Sep. 2006.

13. Aditya K. Jagannatham, and Bhaskar D. Rao, "Whitening-Rotation-Based Semi-Blind MIMO Channel Estimation" IEEE Trans., Signal Processing, vol. 54, pp. 861-869, Mar. 2006.

14. M. Abuthinien, S. Chen, and L. Hanzo "Semi-blind Joint Maximum Likelihood Channel Estimation and Data Detection for MIMO Systems" IEEE Letters., Signal Processing, vol. 15, pp. 202-205, Oct. 2008.

15. Hao Wang, Ying Lin, and Biao Chen, "Data-Efficient Blind OFDM Channel Estimation Using Receiver Diversity" " IEEE Trans., Signal Processing, vol. 51, pp. 2613-2622, Oct. 2003.

16. G. Xu, H. Liu, L. Tong, and T. Kailath, "A least-squares approach to blind channel identification," *IEEE Trans. Signal Processing*, vol. 43, pp. 2982–2993, Dec. 1995.

17. Song Wang and Jonathan H. Manton " Blind Channel Estimation for Non-CP OFDM Systems Using Multiple Receive Antennas" IEEE Letters., Signal Processing, vol. 16, pp. 299-302, April. 2009.

18. Won-Gyu Song and Jong-Tae Lim "Pilot-Symbol Aided Channel Estimation for OFDM With Fast Fading Channels" IEEE Trans., Broadcasting, vol. 49, pp. 398-402, Dec. 2003.

19. AthinaPetropulu, Ruifeng Zhang, and Rui Lin" Blind OFDM Channel Estimation Through Simple Linear Precoding" IEEE Trans., Wireless Communication, vol. 3, pp. 647-655, Mar. 2004.

20. Shengli Zhou, BertrandMuquet, and Georgios B. Giannakis, "Subspace-Based (Semi-) Blind Channel Estimation for Block Precoded Space-Time OFDM". IEEE Trans., Signal Processing, vol. 50, pp. 1215-1227, May. 2002.

21. FeifeiGao, and ArumugamNallanathan, . "Blind Channel Estimation for OFDM Systems via a Generalized Precoding". IEEE Trans., Vehicular Technology, vol. 56, pp. 1155-1164, May. 2007.

22. Rui Lin AthinaP.Petropulu "Blind channel Estimation for OFDM systems based on Non-Redundant Linear Precoding" IEEE Workshop, Statistical Signal Processing, pp. 351-354, 2003.

23. FeifeiGao and A. Nallanathan, "Blind Channel Estimation for MIMO OFDM Systems via NonredundantLinearPrecoding". IEEE Trans., Signal Processing, vol. 55, pp. 784-789, Feb. 2007

24. Ming-Fu Sun, Ta-Yang Juan, Kan-Si Lin, and Terng-Yin Hsu, "Adaptive Frequency-Domain Channel Estimator in 4 4 MIMO-OFDM Modems". IEEE Trans., VLSI systems, vol. 17, pp. 1616-1625, Nov. 2009

25. Hua Zhang, Ye (Geoffrey) Li, Anthony Reid, and John Terry "Optimum Training Symbol Design for MIMO OFDM in Correlated Fading Channels". IEEE Trans., Wireless Communication, vol. 5, pp. 2343-2347, Sep. 2006

26. Myung-Sun Baek, Mi-Jeong Kim, Young-Hwan You, and Hyoung-Kyu Song "Semi-Blind Channel Estimation and PAR Reduction for MIMO-OFDM System With Multiple Antennas". IEEE Trans., Broadcasting, vol. 50, pp. 414-424, Dec. 2004.

27. Song Wang, Jonathan Manton, David B. H. Tay, Cishen Zhang, and John C. Devlin, "An FFT-Based Method for Blind Identification of FIR SIMO Channels". IEEE Signal Processing Letter, vol. 14, pp. 437-440, July. 2007.

28. S. Wang and J. H. Manton, "A cross-relation-based frequency-domain method for blind SIMO-OFDM channel estimation," IEEE Signal Process. Lett., vol. 16, pp. 299–302, Oct. 2009.

29. M.Ahmadi and A.S. Mehr, "Blind channel identification and data detection for SIMO OFDM systems,"in Proc. Can. Conf. Electrical and Computer Engineering, Apr. 2007, pp. 56-58.

30. S. Wang and J. H. Manton, "Blind channel estimation for non-CP OFDM systems using multiple receive antennas," IEEE SignalProcess. Lett. vol. 16, pp. 299–302, 2009.

31. X. Liu and M. E. Bialkowski "SVD-based blind channel estimation for a MIMO OFDM system employing a simple block precoding scheme", Proc. Int. Conf. Comput. Tool EUROCON, pp.926 -929 2007

32. Paresh Naik1, Nisha S L2 "Comparative Performance of MIMO Channel Estimation Techniques" vol. 02, pp, 1925-1930, June 2015.

33. Xia LIU, Marek E. Baikowski, Feng Wang, "A Novel Blind Channel Estimation for a 2x2 MIMO System, Int. J. Communications," Network and System Sciences, Vol 5, pp. 344-350, 2009

34. OFDM in Correlated Fading Channels" IEEE Trans. on Wireless com, vol. 5, pp. 2343-2347, Sep. 2006.

35. E. Moulines, P. Duhamel, J. F. Cardoso, and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," *IEEE Trans. Signal Process.*, vol. 43, no. 2, pp. 516–525, Feb. 1995.

36. S. Sun, I. Wiemer, C. K. Ho, and T. T. Tjhung, "Training Sequence assisted channel estimation for MIMO OFDM", *IEEE WCNC 2003*, Vol. 1, pp. 38 - 43.

APPENDIX A – SIMULATION MODEL DETAILED FLOW CHART

1. Matlab codes for the simulation of SIMO channel with CR algorithm

Initialize variables

SNR = Signal to noise ratio

MSE_vec = Mean squared error vector

Initialize Channel Parameters

h_{11} = Channel 1 impulse response 1st transmitter antenna to 1st receive antenna

h_{12} = Channel 2 impulse response 1st transmitter antenna to 2nd receive antenna

h_{21} = Channel 3 impulse response 2nd transmitter antenna to 1st receive antenna

h_{22} = Channel 4 impulse response 2nd transmitter antenna to 2nd receive antenna

Binary phase shift keying

```
ip1    = rand(1,5)>0.5;
ip2    = rand(1,5)>0.5;
s1     = (2*ip1-1)'; % BPSK signal generation
s2     = (2*ip2-1)'; % BPSK signal generation
s11 = s1/sqrt(2)
s12 = s2/sqrt(2)
```

Calculate received signal

$x_{11} = H1 * s$: Antenna 1 transmitted signal received by the antenna 1

$x_{12} = H2 * s$: Antenna 1 transmitted signal received by the antenna 1

$x_{21} = H3 * s$: Antenna 2 transmitted signal received by the antenna 2

$x_{22} = H4 * s$: Antenna 2 transmitted signal received by the antenna 2

Noise addition to received signals

n = 1/sqrt(2)*[randn(1,N) + j*randn(1,N)]; % white gaussian noise, 0dB variance

n1 = 1/sqrt(2)*(randn(size(x13))+j*randn(size(x13))); AWGN noise addition to the received signals

k1 = x13+ (n1*((norm(h11))^2)/(2*(10^(SNR(ii)/20)))); Noise power varying in the range of different SNR values

FFT conversion

H = fft(h,N) - Conversion of channel impulse response matrix to frequency domain

X = fft(x,N) - Conversion of received signal matrix to frequency domain

Model the optimization parameter "Fi"

Fi = (diag(X))*V; where, V is the FFT matrix which is developed with according to the size of the channel impulse response

Model the optimization parameter "Fij"

Fij=[-Fj Fi Fd Fd], Fij is the result of the cross relation formula for 4 channel system. Where, Fd is a matrix used to model the Fij matrix when the number received signals are more than two.

Model the optimization parameter "F"

F = [F12' F13' F14' F23' F24' F34']' , F matrix consists of all the combinations of received signals. All the possible combinations are considered to model this matrix.

Optimization Methodology

[a b c]=svd(F) , Singular value decomposition of matrix F is calculated to get the left singular vector (a) , singular values (b) and right singular vector (c)..

2. **MSE calculation**

f =10*log10(trace(conj(transpose(hhat-h))*(hhat-h))); MSE calculation for the estimated channel matrix (hhat) and the real channel matrix (h).

Received signals corresponding to the multiple copies of transmitted signals

```
k1 =conv(h11,s1)
k2 =conv(h12,s2)
k3 =conv(h11,s1)
k4= conv(h12,-s2)
k5 =conv(h21,s1)
k6 =conv(h22,s2)
k7 =conv(h21,s1)
k8 =conv(h22,-s2)

y11 = k1+k2
y12 = k3+k4
y21 = k5+k6
y22 = k7+k8

x11 = ((y11+y12)/2)
x12 = ((y11-y12)/2)
x13 = ((y21+y22)/2)
x14 = ((y21-y22)/2)
```

Add Additive White Gaussian Noise

```
n1 = 1/sqrt(2)*(randn(size(k1))+j*randn(size(k1)));
x111 = x11+(n1*((((norm(h11)))*(std(x11)))/(sqrt(4)*std(n1)*(10^(SNR(ii)/20))))); % add noise to signal
x112 = x12+(n1*((((norm(h12)))*(std(x12)))/(sqrt(4)*std(n1)*(10^(SNR(ii)/20))))); % add noise to signal
x113 = x13+(n1*((((norm(h21)))*(std(x13)))/(sqrt(4)*std(n1)*(10^(SNR(ii)/20))))); % add noise to signal
x114 = x14+(n1*((((norm(h22)))*(std(x14)))/(sqrt(4)*std(n1)*(10^(SNR(ii)/20))))); % add noise to signal
```

Adding DFT matrix parameters to model CW matrix

```
N  = 16
n = [0:1:N-1];                % row vector for n
k = [0:1:N-1];                % row vecor for k
WN = exp(-j*2*pi/N);              % Wn factor
nk = n'*k;                   % creates a N by N matrix of nk values
WNnk = (1/N)*(WN .^ (-nk));                % IDFT matrix

C = [0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0;0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 ;
   0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0;0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0;
   0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0;
   0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0;0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0;
   0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0;0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0;
   0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0;0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0;
   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1];

cw = C*WNnk;
```

Optimization Methodology – SVD operations are performed to find the eigen vector corresponding to lowest eigen value

```
[a1 b1 c1]= svd(F1) ;
y1 = c1(:,[10]) ;
k1 = y1(1,1)/l1(1,1);
hhat1 = y1./k1;

[a2 b2 c2]= svd(F2) ;
y2 = c2(:,[10]) ;
k2 = y2(1,1)/l2(1,1);
hhat2 = y2./k2;
```