SEMI-ELLIPTICAL EXPONENTIALLY WEIGHTED MOVING AVERAGE SCHEME FOR JOINTLY MONITORING MEAN AND VARIANCE OF GAUSSIAN PROCESSES

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DECLARATION

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Abstract

Shewhart, cumulative sum and exponentially weighted moving average control charts were introduced for monitoring process mean. These charts were subsequently used for monitoring process variance. Later, it was realized that process monitoring is a bivariate problem and several joint monitoring scheme for process mean and variance were introduced by many authors. The challenge in the advanced joint monitoring scheme is that it should be sensitive for both small and larger changes either in process mean, variance or both. In this thesis, a new advanced joint monitoring scheme for process mean and variance called semi-elliptical exponentially weighted moving average scheme is proposed for Gaussian processes with its design procedure for the industry. The performance of this new scheme is compared with the joint monitoring schemes suggested by other authors using a new comparison index proposed in this thesis. Application of this new scheme is tested with real and simulated data sets.

Most frequently, this new scheme detected various magnitudes of shifts in mean and variance quicker than any other schemes. In overall, the new scheme developed in this study performs better than the existing schemes with some limitations when the shift in mean, variance or both is large. A big advantage of this new scheme is, the design parameters are independent of sample size. As this scheme use the standardized mean and variance, this scheme can be used to monitor several parameters at a time in a single display. Unlike most of the joint monitoring scheme, this new scheme takes the drop in variance as the desirable state when the mean is on target. Therefore this scheme can be recommended for advanced joint monitoring of process mean and variance. The new methodology is very useful for many industrial applications. Furthermore improvements are suggested on this scheme to monitor multi quality parameters simultaneously.

Keywords: Average run length, Control limits, Exponentially weighted moving average, Joint monitoring, Process mean, Process variance, Shifts

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Appendix A: SAS Programmes Developed for this Study

List of Abbreviations

ARL	average run lengths
CUSUM chart	cumulative sum chart
EWMA Chart	Exponential Weighted Moving Average chart
LCL	Lower Control limit
SEEWMA Scheme	Semi elliptical EWMA Scheme
UCL	Upper control limit
SS_r	Shewhart scheme with rectangular control region
SS_e	Shewhart scheme with elliptical control region
CS_r	CUSUM scheme with rectangular control region
ES_r	EWMA scheme with rectangular control region
ES_e	EWMA scheme with elliptical control region
ES_m	Max Charting scheme
ES_{me}	Max EWMA scheme
ES_{sc}	EWMA-semicircle scheme