

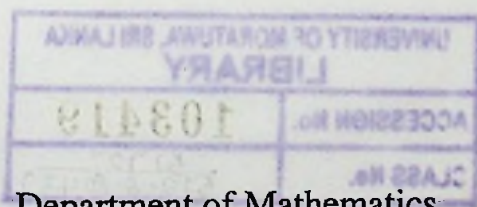
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# COMPARISON OF FACTOR EXTRACTION AND ROTATION METHODS IN EXPLORATORY FACTOR ANALYSIS

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(08/10308)

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of Science in Operational Research



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## Declaration of the Candidate

"I declare that this is my own work and this dissertation does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any University or other institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another except where the acknowledgment is made in the text"

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## Declaration of the Supervisor

“I have supervised and accepted this thesis/dissertation for the award of the degree”

Signature:

Date:

*To my Parents and Husband*

## ACKNOWLEDGEMENT

Writing a thesis has been a long journey which has taken a long time. I require various encouragements and support from many people, specially my supervisor. I would like to take the opportunity to thank my former manager and president of the company for C&A Parts, Florida, located in Suvalala, Department of Mechanical, University of Missouri, for his intellectual inspiration, positive attitude and participation. Without him this thesis would not have been completed.

Also, I would like to thank all of my colleagues in which I studied at the University of Missouri. Their encouragement and support are very much appreciated.

Finally, I would like to thank my parents and my husband who have supported me throughout this journey.

***To my Parents and Husband***

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**ABSTRACT**

Exploratory Factor Analysis (EFA) is a technique to explore the underlying factors of a large set of observed variables which cannot be measured directly. In general there are seven types of factor extraction methods. For a meaningful interpretation of occurred factor model, the extraction method usually followed by either Orthogonal rotation or Oblique rotation method. However it has not been recommended a particular method of EFA for a given set of data. Further most of the researchers are misusing Principal Component Analysis (PCA) with Exploratory Factor Analysis. Therefore, this study was carried out to investigate a possibility of recommending a particular method for a given set of data using a data set comprising seven variables on crimes. Data were analysed using the statistical software SPSS.

To illustrate the contrast of PCA and EFA, analysis was begun with Principle Component. For the comparison of different types of extraction methods under EFA, variables were extracted using Maximum Likelihood Factoring, Principle Axis Factoring and General Least Squares followed by all the Orthogonal rotation methods separately. The steps of the analysis in EFA were quite same with all the extraction methods, however the final result and the effect of the prior assumptions make difference. It is very important to confirm that KMO statistic to be greater than 0.6, prior to carry out EFA for the adequacy of sample size in order to derive valid statistical inferences. If the variables having multivariate normal distribution it is recommended to conduct Maximum Likelihood or General Least Squares. For the non normal distributions, Principle axis factoring is recommended. However it is recommended to compare the results from each method irrespective of the distribution of data set.

Among Orthogonal rotations Varimax rotation is recommended as it provides simple factor loadings to interpret. Quartimax generally does not provide simple factor loadings as in Varimax. It is not recommended to carry out all possible combinations of factor extraction methods and rotation methods to any set of data, as same results will not be produced by each combination. The recommendation given for the particular data set was confirmed using Jackknife validation method.

**Keywords:** Factor analysis, Generalized Least Squares, Maximum Likelihood Extraction, Orthogonal rotations, Principal Axis Factoring

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FA	Factor Analysis
PCA	Principal Component Analysis
CPA	Supplementary Factor Analysis
ML	Maximum Likelihood
PF	Principal Factor Method (Using the Principal Axis Method)
GLS	Generalized Least Squares
N	Number

# LIST OF ABBREVIATIONS

## CHAPTER 01 INTRODUCTION

Abbreviation	Description
FA	Factor Analysis
PCA	Principle Component Analysis
EFA	Exploratory Factor Analysis
ML	Maximum Likelihood
PF	Principle Factoring (Using for Principal Axis Factoring)
GLS	Generalized Least Squares
V	Variance

Principle Component Analysis (PCA) is sometimes confused with Factor Analysis as both methods share a variable reduction procedure. However, Principle Component Analysis cannot be considered as a Factor Analysis and it differs with the purpose of use. If the purpose is to reduce the information in many variables to a set of weighted linear combinations of those variables then PCA is the appropriate method to use. Factor Analysis is used when the purpose is to identify the latent variables which are contributing to the common variance in a set of measured variables. Thus the aim of factor analysis is to reveal any latent variables that leads to the covariance in the observed variables. Therefore during factor extraction the common variance of a variable is partitioned from its unique variance and error variance and only the common variance appears in the solution. In contrast Principle Component Analysis does not differentiate between common and unique variance as it treats the measurement errors that really exists. With this aim in some situations such as when the factors are uncorrelated and orthogonal are appropriate. It was found correlated values of variables accounted for by the components (Cortado, 2005).