

**PORTFOLIO OPTIMIZATION THROUGH QUADRATIC
PROGRAMMING WHEN THERE IS PERTURBATION IN
THE RETURN MATRIX**

N.T.Dharmathilaka

(148903X)

Degree of Master of Science

Department of Mathematics

University of Moratuwa
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Thesis/Dissertation submitted in partial fulfillment of the requirements for the degree
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DECLARATION OF THE CANDIDATE

I declare that this is my own work and this thesis/dissertation does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any University or other institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

.....

N.T.Dharmathilaka

.....

Date

DECLARATION OF THE SUPERVISOR

We have supervised and accepted this thesis for the submission of the degree.

.....

Mr. A. R. Dissanayake

Senior Lecturer,

Department of Mathematics,

University of Moratuwa.

.....

Date

.....

Mr. L. P. Ranasinghe

PhD,

Department of Mathematics,

University of Colombo.

.....

Date

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ABSTRACT

According to Finance the investor who invests in risky assets such as stocks, after forming a diversified portfolio or collection of securities; is interested in earning maximum return out of minimum risk, and it is more technically known as Portfolio Optimization(PO). The present problem is a Quadratic Programming problem is consisted of simultaneous variations of initial return vector of each company.

In this study the main objective was to study the behavior of covariance-variance matrix, correlation matrix and the optimum weights vector when there is small perturbation in the mean return vector. Then fitting a model between perturbation values versus optimum weights was also performed. The results show that there is a significant variation of optimum weights when there is small perturbation in the return matrix. And there is no change in the covariance or correlation matrices. This is done under the assumption that there is no short selling. Apart from that results show that when there is perturbation in the return matrix the expected return of the portfolio is also changing.

When the value of perturbation is increased individually for one company only, to drive away at least one company from the optimum weights (to zero the optimum weight of one company) it was observe that the perturbation value should be increased extensively for the SGX data sample. That means the weights are not very much sensitive to perturbations in the market.

If negatively mean companies are removed and perturbation is done for positively mean companies the effort to remove at least one company from the optimum weights is less.

Keywords: Portfolio Optimization, Quadratic Programming, Optimum weights, Expected return , Matlab

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LIST OF ABBREVIATIONS

CSV	Comma delimited
Matlab	Matrix Laboratory
MPT	Modern Portfolio Theory
MSE	Mean Squared Error
MV	Mean Variance
PO	Portfolio Optimization
QP	Quadratic Programming
SGX	Singapore Stock Exchange
3D	Three Dimensional