# PRAGMATIC PORTFOLIO OPTIMIZATION: GAUGING BLACK-LITTERMAN MODEL IN EMERGING MARKETS


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March 2016

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## Abstract

With the advent of modern portfolio theory[1] in 1952 by Harry Markowitz, the investment management industry had witnessed an uprising. Yet the encountered shortfalls and rigidity of the methodologies lead to the development of Black- Litterman model by 1990s. The Black- Litterman model addressed those deficiencies and introduced the luxury of incorporating the unique views of Asset managers about the assets under management in their portfolios.

This projected research efforts implementing the difficult phases of the Black-Litterman model and depicts its practical and pertinent nature by comparing to other portfolio allocation methods which uses the historical and CAPM methods. The modeling of mean variance (reward and risk) and then the portfolio allocation has been done using these three distinct methods. Thereafter the benevolent leads of the BL method over others have been discussed.

To assess the BL model, eight stocks such as Samsung Electronics Co., Ltd (SAMSUNG-Korea), China Mobile Communications Corporation (CHINA MOB- China), Naspers Limited (NASPERS-South Africa), Emaar Properties (EMAR- United Arab Emirates), Koc Holding AS (KCHOL- Turkey), Akbank (AK BANK- Turkey), Braskem SA (BRKM5-Brazil) and Taiwan Cement Corporation (TAIWAN CE- Taiwan) which comes under Emerging markets have been considered. For the analysis, the monthly stock closing prices published by Bloomberg L.P. have been taken. In addition to this the monthly closings of the MSCI Emerging Markets Index and US Treasury rates have been obtained to use respectively as the market benchmark and market risk free rate.

Four outlooks/views about these stocks were evaluated and the vector of BL Expected Excess Return which is the weighted average of Equilibrium market return vector and the View vector have been established using the Black- Litterman model. The grandeur of the BL method that’s tailored portfolio weightages corresponding the Asset managers’ views was studied.

The model has been implemented using the scientific software MATLAB. Other than the Black-Littreman methodology, the concepts of Markowitz portfolio theory, efficient frontier, CAPM returns, Portfolio expected returns, Portfolio variances and the Sharp ratios have been used to describe the portfolio dynamics.

The portfolio weightages derived using BL Expected Excess Returns did accord with the four views. It has been clearly witnessed that the incorporation of View vector, had caused the Equilibrium market return vector to get adjusted with respect to the outlooks/views.

**Keywords:** Black- Litterman model, Asset/ Portfolio allocation, Portfolio Optimization, Corporate Finance, Investment management

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[1]Modern Portfolio Theory (MPT), a hypothesis put forth by Harry Markowitz in his paper "Portfolio Selection," (published in 1952 by the Journal of Finance) is an investment theory based on the idea that risk-averse investors can construct portfolios to optimize or maximize expected return based on a given level of market risk, emphasizing that risk is an inherent part of higher reward. It is one of the most important and influential economic theories dealing with finance and investment. [W1]

**Acknowledgement**

It’s my pleasure to be grateful and pay respects to my parents, sisters and husband who has been the backbone of all my endeavors giving moral support at all times.

I’m very much indebted to thank those who made this thesis a realistic, primarily supervisors Mr. A. R. Dissanayake and Mr. L. P. Ranasinghe who helped me providing research materials and guidance.

Further I extend my thanks to the Head of Equities, Asset Trust Management (Pvt) Ltd who had encouraged me to gauge the model to global markets by helping me to pick up the portfolio constituents and expressed outlooks for those stocks. Also I’m appreciative to our friend Thisal Weerasekara who had supported to set the model up in MATLAB.

Kumuthini Sivathas

**Table of Contents**

**List of Figures**

**List of Tables**

## List of Abbreviations

| | | |
|---|---|---|
| BL | - | Black- Litterman |
| CAPM | - | Capital Asset pricing Model |
| ETF | - | Exchange Traded Funds |
| MSCI | - | Morgan Stanley Capital International |

# CHAPTER 1: INTRODUCTION

## 1.1 Introduction

The Chapter discusses the background of the research and the chronological development that took place with investment analysis. The Objective and Scope of the research has been comprised under this chapter which defines the aim and the latitude of the research. In addition to this the development of modern portfolio theory and the shortfalls of it that led to the tactical asset allocation methods have been deliberated. Also the significance and boundaries of the research have been conversed here.

## 1.2 Background of the Research

Investment management is concerned, the portfolio asset allocation and return optimization is vital. On early period, the security or asset selection models focused only on returns. The time worn investment strategy was to construct a portfolio only by recognizing the securities that offered the better returns with least risk by doing a fundamental analysis of the firm, its financial statements and its dividend policy. Here the securities were selected merely on the idea of better risk-return characteristics. Later on by mid of twentieth century, Harry Markowitz introduced his portfolio selection technique called as Modern Portfolio Theory or Markowitz Portfolio Selection Model which presented the ground-breaking concept of portfolio risk. Finance professor Markowitz began the revolution by suggesting that the value of a security to an investor might best be evaluated by its mean, its standard deviation, and its correlation to other securities in a portfolio.

He gave risk coequal level of importance as return and showed how the variance of the portfolio could be reduced through the impact of diversification of assets which carry varied level of risks. After the advent of his theory, it was concentrated on selecting portfolios based on their overall risk reward characteristics instead of purely compiling portfolios from securities that independently had attractive risk-reward characteristics. But Markowitz Portfolio Selection Model had a major insufficiency, that the model did not allow the investors to incorporate their opinions about future performance of certain assets or markets.

Markowitz model results in portfolio weights just in a way to increase the return per a unit risk. His model did not permit to comprise information about the firm such as its earnings, dividend policy, capital structure, market, competitors to the model and looked at only calculating a few simple statistics using the market performances of stocks (mainly stock prices). Hence these weights did not reflect the logic, especially when the investors had opinions about how the market or certain assets would perform in the future. Also the Markowitz model requires the assets' expected return vector and risk vector as inputs to the mean variance optimization model whereas in practice the investors would not have the return expectations and risk measures for all the portfolio constituents. Since the whole focus of the model is to result in weights in a way maximizing the return per unit risk, the results are heavily input sensitive such as on risk and return vectors. Hence any errors in the estimated inputs would be enlarged in the results to cause slanted portfolio weights.

Addressing these issues, The Black-Litterman model was then developed to provide neutral weights for the investors that can be adjusted according to their opinions about the market.

### 1.3 Objective of the Research

The Overall objective of the research is to study the application of Black - Litterman model which has been regarded as an instrument for portfolio optimization. The distinct treatment of Black - Litterman model would be illustrated by analyzing a portfolio of eight stocks from emerging markets[2] and it would be risk return optimized using Black - Litterman model for given outlooks of the stocks.

### 1.4 Scope

The research evaluates the Black- Litterman model that has been very practical and pertinent approach for portfolio allocation as it allows the Investors/ Asset managers to add in their own views to the optimization model.

[2] An Emerging market economy (EME) is defined as an economy with low to middle per capita income. [W2]

This paper attempts implementing the difficult phases of the Black Litterman model understandable even to a layman investor.

By using the Black Litterman model, the vector of BL Expected Excess Return which is the weighted average of Equilibrium market return vector and the View vector would be established. Meanwhile the traditional modeling of mean variance using historical method and CAPM method would be done for the comparison purposes with BL findings.

Then based on these three different modeling of mean variances, the portfolio asset allocation would be done. Here the portfolio allocation would be done with respect to the formula derived from the Utility maximization function.

The fundamental techniques towards the portfolio optimization such as Markowitz portfolio theory, efficient frontier, CAPM returns, Portfolio expected returns, Portfolio variances and the Sharp ratios would be applied whenever required.

The benevolent advantages of the BL method would be illustrated by comparing to the portfolio allocations done using Historical measures and Capital Asset Pricing Methods. These would be implemented using the scientific software MATLAB. Basic numerical methods associated with the portfolio statistics would be directly called from MATLAB inbuilt functions.

## 1.5 Black-Litterman Model: An Effective Portfolio Optimization

The Black-Litterman model is a sophisticated asset allocation method developed in 1990 by Fischer Black and Robert Litterman at Goldman Sachs. This method pools the concepts of Capital Asset Pricing Model (CAPM), Reverse optimization, mixed estimation, universal hedge ratio / Black's global CAPM and the Markowitz's mean-variance optimization model. The specialty of the method accommodates the investors/asset managers to determine the optimal portfolio weights by combining the subjective views regarding the expected return of one or more assets.
To incorporate the subjective views of investors/asset managers regarding the expected returns of one or more assets, the Bayesian method had been used in the

Black-Litterman model. [3]Bayesian method allowed the subjective views to be combined with the market equilibrium vector of expected returns (the prior distribution) to form a new, mixed estimate of expected returns. The resulting new vector of returns (the posterior distribution), leads to intuitive portfolios with sensible portfolio weights.

### 1.6 Significant of the Research

A revolution started on financial markets in 1952, when Harry Markowitz established the modern portfolio theory, applying mathematical concepts to finance. Later, the Capital asset pricing model (CAPM) was developed in early 1960s based on Markowitz work. Today, the results of the CAPM (and its extended versions) are widely used in describing the risks and returns of portfolios and for performance measurement.

Despite the fact, that CAPM has its unique useful applications during investment management, for instance, in explaining the benefits of diversification by presenting the idea of [4]systematic and unsystematic risk, as a tool for finding undervalued/ overvalued securities and in performance measurement, it's witnessed that the model describes capital markets returns only moderately.

It was observed that a small difference in the CAPM expected excess returns lead to huge changes in the resulted weights of portfolios. When managing investment funds which are sized in billions, the fund managers puzzled as the asset allocation percentages did swing to the roof and bottom for minor changes of the CAPM expected returns.

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[3]A Step-By-Step Guide to the Black-Litterman Model by Thomas M. Idzorek in 2005

[4]Systematic risk is the risk inherent to the entire market or an entire market segment. This is also known as "un diversifiable risk," "volatility" or "market risk," affects the overall market, not just a particular stock or industry. This type of risk is both unpredictable and impossible to completely avoid. It cannot be mitigated through diversification, only through hedging or by using the right asset allocation strategy.

Unsystematic risk, also known as "specific risk," "diversifiable risk" or "residual risk," is the type of uncertainty that comes with the company or industry you invest in. Unsystematic risk can be reduced through diversification. For example, news that is specific to a small number of stocks, such as a sudden strike by the employees of a company you have shares in, is considered to be unsystematic risk. [W3]

And also the CAPM returns are based on risk-free rate of return, the returns on the market (the equity risk premium) and the equity beta whereas the equity beta is an estimate that relies on past data. The estimate of beta may change over time. So using the historical estimates of betas may not be good measure for future expected returns. Hence there were lots of uncertainties inherent when calculating the estimate of expected excess returns using CAPM. The estimation uncertainties are not taken in to consideration when calculating CAPM expected returns, though it should be. Later these estimation errors in the expected excess returns are maximized when allocating portfolios.

Above all the CAPM can't accommodate opinions on the future space as the model did not have means for Investment managers to incorporate their ideas about particular assets' future performances and news. Hence there was an enormous necessity to reduce the estimation error maximization and for a forward looking measure that accommodates views of Investment Managers.

To address these issues, it was ascertained that the best measure of expected returns would be the market's estimate of expected returns for each asset. Market's estimate is not just one person's view but the entire market's estimate such as weighted average view of every person in the market. The two researchers Black and Litterman came up with an idea to find the market's estimate. They proposed a way based on Investor Utility Maximization problem and developed challenging procedures towards finding the solution. With the introduction of their model, all the limitations that were inherent to the derived methods of Modern portfolio theory had been practically wiped out. Black-Litterman asset allocation model is a sophisticated portfolio construction method that specially overcomes the limitations of the Markowitz model such as problem of unintuitive, highly concentrated portfolios, input sensitivity, and estimation error maximization.

The Black Litterman model is recognized by top quantitative investment managers as a very practical methodology towards asset allocation in investment management. But novice investors/asset managers yet find it difficult to understand the application of the model. This research work describes the stages one by one by applying model techniques to the stocks of emerging market.

## 1.7 Limitations

The Overall objective of the research is to study the application of Black - Litterman model which has been regarded as an instrument for portfolio optimization. The research could have been expanded by showing the derivation of the model by applying Bayesian Theory. That would have depicted more details about the model and it's by applications.  But due to time constraints the research has been restricted only to the application of Black - Litterman model.

The application of user specified confidence levels of views as done by Thomas M. Idzorek in his paper 'A step-by-step guide to the black-litterman model' could have been implemented to show the control over the views that is very much applicable to a real time research environment. Because in practice expressing the views over a range of confidence level is much comfortable, due to high fluxes in financial market. But having considered the complexity of such extension, it was decided to avoid discussing it in this paper as implementing plain vanilla Black Litterman model itself quite lengthy and dense. Hence learning the Black Litterman model with varied types of views have only been fingered to the extreme in this paper.

Also different asset classes such as treasury securities, bonds and currencies across different nations could have been added to the portfolio.This would have required a Bench mark index that would need to comprise all the financial asset classes of the portfolio constituents. But it's quite impractical to find such Bench mark index which closely follows the portfolio. And also it involves various extensions and assumption about the BL model which would be an out of focus of our objective of studying the application of Black- Litterman method in a way that can be used by novice practitioners of investment management industry.

## 1.8 Contents of Thesis

The research is based on Black Litterman asset allocation methodology which presents the way to incorporate asset manager's views in to the model. Thus the Chapter one has presented the preliminary background of the Black Litterman asset allocation methodology. The whole aim of this research is to observe the shifts of portfolio weightages that could closely follow the Asset managers' views. This would be clearly explained and the procedures would be presented in following chapters. To begin with the analysis, the study and review of the relevant researches were done. This has been listed under the chapter of literature reviews that would open up very next. Chapter three offers the Black Litterman model description and related methodologies while the chapter four demonstrates the model evaluation by using the statistics of equity securities from emerging market economies.

Finally, the Chapter five presents the expedient findings and discusses the scopes for future researches on this topic.

# CHAPTER 2: LITERATURE REVIEW

## 2.1 Introduction

The chapter concerns about the published researches relevant to Black Litterman Asset Allocation model. In addition, the research refers to the ideas such Bayesian theory, Modern portfolio theory, Risk return optimization methodologies, Capital Asset Pricing Models, Portfolio performance parameters Black Litterman methodology and its extensions.

Hence the literature review encompasses all the above mentioned areas.

## 2.2 Literature Review

A brief summary of the references to Black-Litterman model in literature is outlined here.

The initial paper by Black and Litterman (1991) did not comprise substantial details such as the required data set and examples to trial their invention, but had given some insight and introduction about their model. They introduced a parameter, weight on views scalar τ, which had been used by other researchers in their papers. But this had not been clearly defined on that paper. In their second paper on the model, Black and Litterman (1992) had delivered detailed discussion on the model along with the major assumptions. They presented both the data sets and results. Yet their assumptions were not documented in an easy-to –practice style. Hence it has been rather challenging to repeat the workings.

Bevan and Winkelmann (1998) provided detail on how they implemented Black-Litterman as part of their broader asset allocation process at Goldman Sachs, including some tunings of the model, which they perform. He and Litterman (1999) presented a clear and easy to follow discussion on Black-Litterman. Satchell and Scowcroft (2000) did interpret Black-Litterman model, but did not provide enough evidence to duplicate their findings. And also, they proposed a value for the parameter τ which had not beensimilar with other researchers' findings. There were no intuitive reasons given to back their proclamation that τ should be set to 1.

Nevertheless, they explained the derivation of the master formula of Black-Litterman model in detail.

Christadoulakis (2002) studied the details of Bayesian mechanisms, assumptions of the model and computed the key formulas for posterior returns. Herold (2003) looked at the problem in a different way by investigating optimizing alpha generation and specified that the sample distribution has zero mean. He provided some additional measures, which can be used to validate whether the views are reasonable. Krishnan and Manis (2005) provided an extension to the Black-Litterman model, adding another aspect which is uncorrelated with the market. They called it the two-factor Black-Litterman model and showed an example of extending Black-Litterman model with a recession factor. M. Idzorek (2005) consolidated insights from the relatively few works on the model and provided step-by-step instructions of the model. Also he introduced a new method for controlling the tilts and the final portfolio weights caused by views.

Mankert (2006) provided a rich literature on the Black-Litterman model and also presented a detailed transformation between the two specifications of Black-Litterman master formula for the estimated asset returns. She also provided a new approach for the value $\tau$ from the view of point of sampling theory. Meucci (2006) provided a method to use non-normal views of Black-Litteraman model. Barga and Natale (2007) described a method for attuning the uncertainty in the views by tracing error volatility. This metric became popular for its use in benchmark relative portfolio. Meucci (2009) extended the method of non-normal views in Black-Litterman model to any model parameter, which allowed both full distribution analysis and scenario analysis.
The following literatures have been closely followed to make this research which applies the Black- Litterman method to the emerging markets of economy.

### 2.2.1 Global Asset Allocation with Equities, Bonds, and Currencies

Global Asset Allocation with Equities, Bonds, and Currencies had been written by Fischer Black and Robert Litterman by October 1991 who worked for Fixed Income

Research, Goldman, Sachs & Company. This paper discussed an updated version of the Black-Litterman Model which incorporates equities as well as bonds and currencies. This was an extended work of the founder of the Black-Litterman Model. He suggests that this model is ideal for the portfolio managers who make global asset allocation decisions across equity and fixed income markets, whereas it would also have advantages for pure fixed income managers. He showed how an addition of the equity asset class to the model allows the analyst to use an equilibrium based on both bonds and equities.

This paper specifically helped to optimize a portfolio that is entirely composed of equities, incontrasting to the original Black-Litterman Model that discussed about optimizing the portfolio of fixed income securities.

### 2.2.2 A Step-By-Step Guide to the Black-Litterman Model: Incorporating user-specified confidence levels

A Step-By-Step Guide to the Black-Litterman Model was done by Thomas M. Idzorek in 2005.This paper consolidates insights from the relatively few works on the model and provides step-by-step instructions that enabled to implement this complex model. A new method for controlling the tilts and the final portfolio weights caused by views is introduced.

Despite implementing the idea in to the research, it has been learnt the user- specified confidence levels based on an intuitive 0% to 100% confidence. This is an intuitive technique for specifying one of most abstract mathematical parameters of the Black-Litterman model. But from this paper, the methods associate with determining the market weights when a set of assets are compared with another set of assets have been learnt.

### 2.2.3 Using the Black-Litterman Global Asset Allocation Model: Three Years of Practical Experience

Using the Black-Litterman Global Asset Allocation Model: Three Years of Practical Experience was published by Andrew Bevan and Kurt Winkelmann in June 1998 who worked for Fixed Income Research, Goldman, Sachs & Company.

This paper involved a discussion of how they used the model to observe the equilibrium returns in global capital markets and then blend the equilibrium returns with their own views to provide a set of expected returns by using three years portfolio performance. They explained how they determined the weight and confidence levels on their own views relative to the equilibrium. Also they discussed risk control and optimization.

In addition they described a process that they followed to set tracking error risk and Market Exposure (a statistical measure of a portfolio's sensitivity to market moves). At last, they discussed their portfolio performance over the three-year period and considered showing how the same frame work can be applied to other fund management issues.

From this research paper, the ideas for determining the market weights and portfolio performance have been learnt.

### 2.2.4 Global Portfolio Optimization

Global Portfolio Optimization was later published by Fisher Black and Robert Litterman in October 1992.The Investors of the Black Litterman model had here only delivered detailed discussion on the model along with the major assumptions. The method had been described with data set and results as well.

This paper has facilitated for setting up the model, to learn about the model assumption and to get clear idea about having an optimal portfolio with neutral views.

### 2.2.5 The Intuition behind Black-Litterman Model Portfolios

The Intuition behind Black-Litterman Model Portfolios by Guangliang He and Robert Litterman. This paper had illustrated the precise phenomenon and real intuitions behind the Black Litterman model. The Authors showed the conditions for the weight on a view portfolio to be positive, negative, or zero. And also they showed that the weight on a view increases when the investor becomes more bullish on the view and the magnitude of the weight increases when the investor becomes less uncertain about the view.

This paper has been a great resource to learn about having varied types of views to the presumed portfolio of eight stocks.

### 2.2.6 Universal Hedging: Optimizing Currency Risk and Reward in International Equity Portfolios

Universal Hedging: Optimizing Currency Risk and Reward in International Equity Portfolios by Fischer Black was published in Financial Analysts Journal in August 1989 which did depict the ways to hedge the portfolio of foreign currency and equities.

### 2.2.7 The Black Litterman Model: A Detailed Exploration

The BlackLitterman Model: A Detailed Exploration was done and released by Jay Walters in January 2008.The first section provides a quick overview of the Black Litterman model. The second section provides an introduction to the relevant portion of Bayesian theory. This paper helps to understand the approach of Bayesian theory towards the BL model by going through proves given by Jay Walters.

### 2.2.8 Portfolio Selection

Portfolio Selection was written by Harry Markowitz and published in The Journal of Finance in March 1952. This is known as Modern Portfolio Theory (MPT), a hypothesis put forth by Harry Markowitz in his paper "Portfolio Selection," is an investment theory based on the idea that risk-averse investors can construct portfolios to optimize or maximize expected return based on a given level of market risk, emphasizing that risk is an inherent part of higher reward. It is one of the most important and influential economic theories dealing with finance and investment.

This research uses his theories and by theories often as all portfolio allocation methods are all about risk return optimization.

### 2.2.9 Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk

Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk was published by William F. Sharpe in The Journal of Finance in September 1964. The Author had described the manner in which the price of risk results from the basic influences of investor preferences, the physical attributes of capital assets, etc. Also he had given detail description on individual investor behavior under conditions of risk. This paper has been the main text to learn about the equilibrium conditions for the capital market and about the capital market line.

# CHAPTER 3: METHODOLOGY

## 3.1 Introduction

The theoretical contextual of the research has been presented in this chapter. The Black- Litterman asset allocation methodology and how it applies to the research are discussed here. Also some sub subjects associated with portfolio dynamic and portfolio asset allocation have been added to give a complete picture about the theories relevant to investment management.

## 3.2 Research Methods

The intention of the research is to find more applicable vector of expected excess returns of a portfolio of financial assets which captivates the ideas and viewpoints of Asset Managers using Black Litterman method. Hence under this section, the BL method would be conferred profoundly and widely. Initially the Black Litterman model description would be clearly presented followed by the statements of assumptions of the model. Then the modelling of expected returns and the reverse optimization process which is the core concept of BL method would be elaborated.
The neutral staring point, which is the establishment of implied equilibrium excess returns, would be elucidated then to move in to the BL method. Incorporation of views in to the implied equilibrium excess returns is the highlight of the model, and the launch of View and Link matrices would be expounded thereafter. The Views also would have uncertainties and this is here measured by $\Omega$. Hence the derivation of $\Omega$ matrix would be exhibited later and the BL model would be set up. Also under this section, the fundamentals of portfolio dynamics would be discussed slightly.

## 3.3 Black-Litterman Model Description

The Black-Litterman asset allocation model was developed and conceived to overcome the problems of not being able to incorporate investor’s views, highly slanted portfolio weightages caused by insensible numbers of return and risks, input sensitivity and its consequence of estimation error maximization. Hence Black-Litterman model is regarded as an operative portfolio allocation method that allows the asset managers to include their unique opinions about the assets/markets to the model that would cause resulting in steady and distinctively engaging mean- variance efficient portfolios.

The Black Litterman model essentially combines information from two sources to generate an estimate of expected returns. The first source is the information about the expected excess returns that could be taken from the current market.This is the Implied Equilibrium Excess Returns. The Implied Equilibrium Excess Returns are more intuitively connected to market and reverse optimization of the same will generate a stable distribution of return estimations.

The second source is about the views that the investment managers have about particular stocks, sectors, asset classes, or countries. The BL model combines these different sources to produce estimates of expected excess returns. That is the BL expected excess return is the combination of predicted and implied returns. The formal equation of the BL model goes as follows.

$$E(r - r_f) = \left[(\tau S)^{-1} + P^T \Omega^{-1} P\right]^{-1} \left[({}^2S)^{-1} \Pi + P^T \Omega^{-1} Q\right] \quad (1)$$

Where

| | |
|---|---|
| $E\ (r - r_f)$ | is the resulting combined return vector (N x 1 column vector) |
| τ | is the Weight- on- views scalar (Assume τ = 1) |
| S | is the Variance - Covariance matrix of excess returns (N x N matrix) |
| Ω | is a diagonal covariance matrix of error terms from the expressed views representing the uncertainty in each view |
| Π | is the Implied Equilibrium Return Vector |
| Q | is the View |
| P | is a matrix that identifies the assets involved in the different views (Link matrix)(m x N matrix or 1 x N row vector in case of absolute views) |

The formula has been split in to two sections as follows to study and illustrate.

$$E(r - r_f) = \underbrace{\left[({}^2S)^{-1} + P^T \Omega^{-1} P\right]^{-1}}_{\mathbf{1}} \underbrace{\left[({}^2S)^{-1} \Pi + P^T \Omega^{-1} Q\right]}_{\mathbf{2}}$$

Then second part is analyzed first. The second part is:

$$\left[(\tau S)^{-1}\Pi + P^{T}\Omega^{-1}Q\right] \tag{2}$$

$\uparrow$ $\quad\quad$ $\uparrow$

Weights Weights

In this section, the implied equilibrium excess return is combined with the views on the expected excess return. Essentially this is an attempt to calculate the weighted averages of the implied returns and the views.

The parameters **Π** and **Q** are accompanied with some measures quantifying respectively to $\mathbf{(\tau S)^{-1}}$ and $\mathbf{(P^{T}\Omega^{-1})}$. These are the weights that specify how confident the investor is about his/her views relative to the implied excess returns. So these are the measures of confidence.The more confident the Investor/Asset Manager is about their views relative to the implied excess returns, more weights they could place on their views. This makes sense as that's what it's expected to happen.

For instance if an Asset Manager is uncertain about a value about an analyst's prediction, they can translate this in to the equation via these weights. It will mean that if there is lots of uncertainly about an analyst's prediction, then the elements of **Ω** will be large to reflect that uncertainty. That is when the **Ω** is large the elements of $\mathbf{\Omega^{-1}}$ will be small. So that the resulted expected excess returns would have only a small contribution or impact from the Analyst's prediction. This is what the Asset Manager required as he was not confident about the Analyst's forecast.

Hence the second part of this formula is just the weighted average combining implied equilibrium excess returns with the views about different assets.

The first part that goes as follows:

$$\left[(\tau S)^{-1} + P^{T}\Omega^{-1}P\right]^{-1} \tag{3}$$

This has been affixed there to ensure that the weights assigned to the implied excess returns and views add up to 1. Since the whole purpose of this formula is to confirm that weights assigned to the implied excess returns and views not exceeding 1, this is again a weighted average of implied excess returns which is the market's best guess of what is going to happen in future and the views about the future.

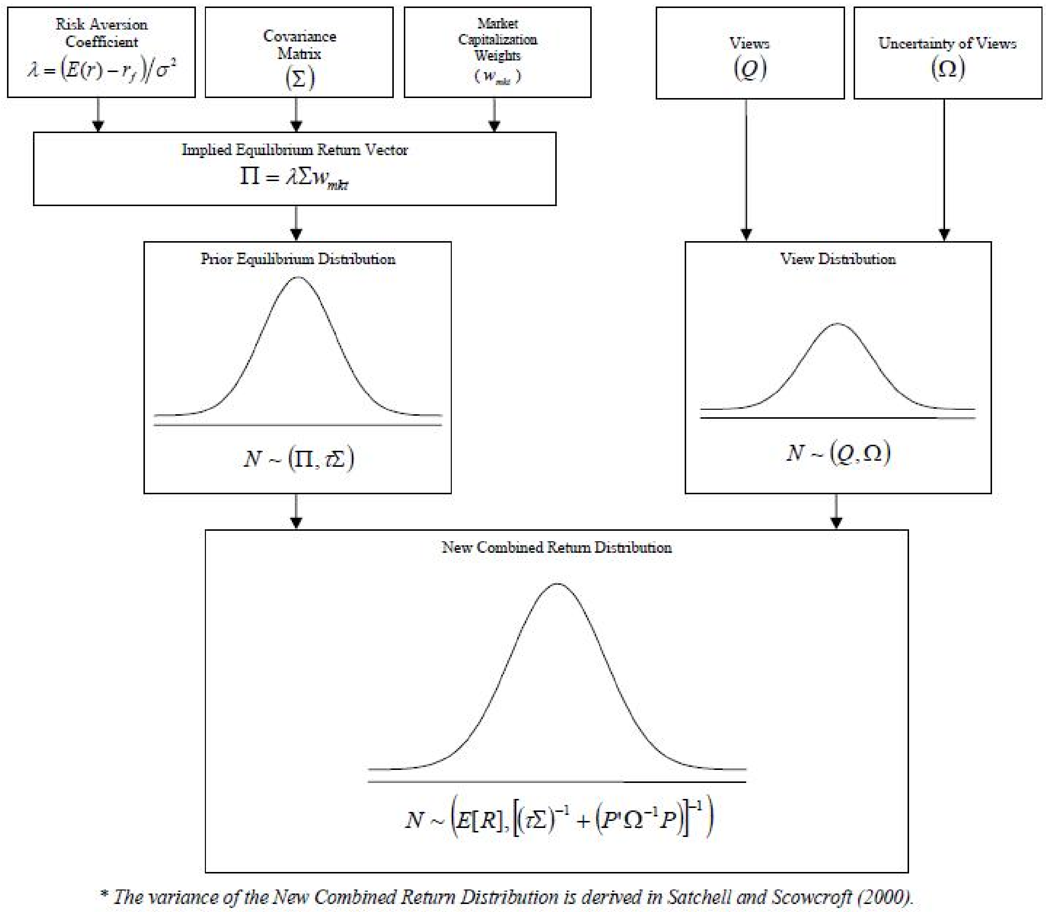


**Figure 3.1: Deriving the New Combined Return Vector $E[R]_{BL}$ (Idzorek, 2004)**

### 3.3.1 Assumptions of the model

This mean- variance optimization model was established on the background of two main assumptions. [5]At the outset it was assumed that all the asset returns follow the similar probability distribution. Generally it's expected that the asset returns follow normal distribution, but it's at the discretion of asset managers to decide and choose the distribution that all the selected assets in his portfolio fits and follows.

[5]The Black-Litterman Approach: Original Model and Extensions by Attilio Meucci

Also it's assumed that the variance of the prior and the conditional distribution about the true means of the assets and investor's subjective views are unknown. Hence to build the model in practice the following assumptions had to be adhered.

- Asset returns (Prior) are normally distributed
- Expected returns (Post) would be normal distributed
- Variance of the expected returns is smaller. (Expected returns would not lay distanced from equilibrium market returns)
- At any given time, in equilibrium, the market portfolio is mean-variance efficient
- The market weights can be observed

And also

- The views of investors are unique and uncorrelated (The views had to be expressed as there is no correlation among them)
- The covariance matrix can be determined using historical data and changes in it have no major impacts on optimal portfolio weights.

### 3.3.2 Expected returns

Vector of expected returns is most essential input to any mean- variance optimization techniques, and while setting up the BL model it was ascertained that the best measure of expected returns would be the market's estimate of expected returns for each asset. The market's estimate did mean to the weighted average of every person's view in the market about the expected returns. The Black and Litterman proposed a method that was based on the Investor Utility Maximization problem to find the market's estimate of expected returns.

In a market, all investors would try to maximize their expected utility.The expected utility is defined by the following Utility function.

$$Max_w : w^T\left(\text{«} - r_f\right) - \text{ª} w^T S w \qquad (4)$$

By solving this Utility function, portfolio weights could be calculated. That is the optimal portfolio weights that give the maximum utility for the Investors. The solution to the problem is:

$$Z = S^{-1}\left(\text{«} - r_f\right) \tag{5}$$

And $\quad Z = 2^{\text{a}} w$

Implies

$$w = \frac{S^{-1}\left(\text{«} - r_f\right)}{2^{\text{a}}} = \frac{Z}{1^T Z} \tag{6}$$

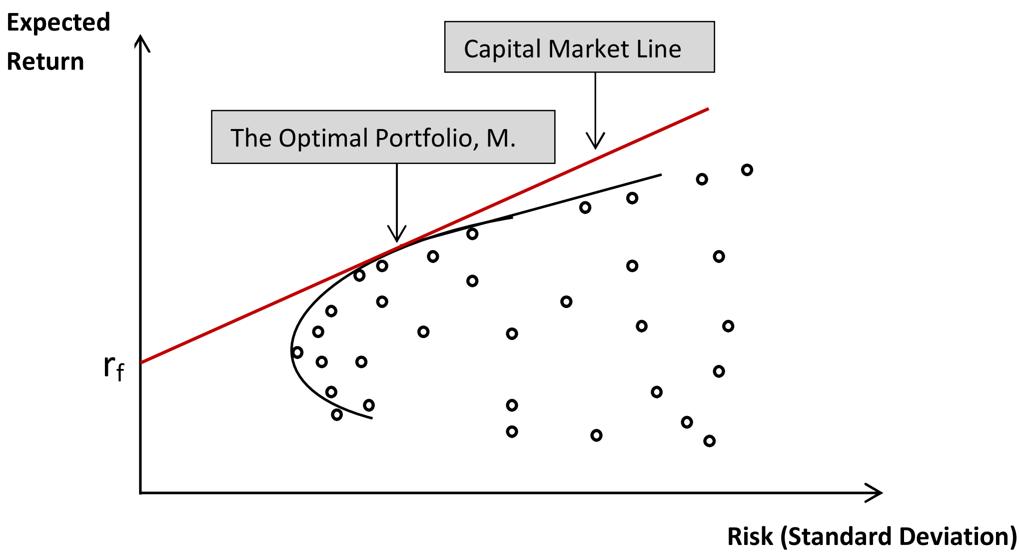


**Figure 3.2: The efficient market portfolio**

It was prudently assumed by Black and Litterman that at any given time, in equilibrium, the market portfolio is mean-variance efficient as all investors are maximizing their expected utility. That is all investors would hold some combination of risk free assets (Treasury Securities) and the optimal portfolio M. When investing in risky stocks, if all investors hold the same optimal portfolio, the market portfolio is going to be the optimal portfolio. And also the market portfolio would be the efficient portfolio as its lying in the efficient frontier.

That is the optimal portfolio would be the market portfolio as well as an efficient portfolio.

### 3.3.3 Reverse optimization

Based on above insight, The Black and Litterman proposed smart procedures for reversing the problem. They said that the weights were observed. That is the weights for the market were observed. Because of the assumption, that at any given time, in equilibrium, the market portfolio is mean-variance efficient, it can be recognized that how much it should be invested in each stock in proportion to the total market capitalization of the stocks.

Because they said that it was witnessed that the market capitalization of each stock can be considered as big chunks when comparing to the entire market portfolio.This was a very ingenious observation by Black and Litterman. They turned the problem around and said as the weights (W) were observed it could be reversed to find out what would be the expected excess returns for each stock. Also they said that not to estimate this expected excess returns as it would be again estimates. They suggested and proposed ways to solve the problem to find an expression for the expected excess returns.

So originally the Utility function was solved to calculate the portfolio weights. But now the market weights are observed and it’s known that all investors are investing in market portfolio (as it’s the optimal portfolio). Thus instead of unraveling for weights the utility function can be solved for expected excess returns.

$$Max_w : w^T\left(« - r_f\right) - ª w^T S w$$

The solution of the Utility maximization problem lies on the following equation

$$« - r_f - 2ªSw = 0$$

Hence the expression for expected excess return would be as follows.

$$\left(« - r_f\right) = 2ªSw \qquad (7)$$

Using this expression, the expected excess returns for every stock can be found. Here W, the market weights can be observed and are known. S is Variance- Covariance matrix and can be found using the historical data. Black and Litterman showed that portfolio weights are not very sensitive to the Variance- Covariance matrix. They found that the changes in the Variance-Covariance matrix did not lead to large changes in the optimal portfolio weights. So they recommended using historical data for finding the Variance- Covariance matrix. Finding the Lambda has been discussed in the next sub section.

### 3.3.4 Incorporation of risk and return

Lambda is a measure of risk aversion. This refers to the average market level of risk aversion that is calculated by considering all individuals across the market. This is done for reversing the procedure to find the market's best guess of expected returns for all stocks. So the risk aversion level would be the market's risk aversion level. Black and Litterman showed how to derive formulae to find the [6]Lambda.

Hence when the market is mean variance efficient we can estimate the price of risk, Lambda. A value for lambda can be extracted as follows.

$$a = \frac{E\left(r_m - r_f\right)}{2\pm {}^{2}{}_{m}} \qquad (8)$$

Here the average excess returns on the stock market and the risk of investing in the stock market are known. Higher the Lambda the higher the compensation it's required for taking the risk.

Next to calculate the Implied Equilibrium Excess Returns it's essential to have the market weights. In equilibrium, the market portfolio is optimal portfolio for investors and is mean-variance efficient; hence the market weights could be observed. They are the optimal weights. Market weights could be calculated by using the market capitalization of each stock. Hence by using the following formula, Implied Equilibrium Excess Returns can be calculated:

$$\Pi = « - r_f = 2ªSw \qquad (9)$$

---

[6]Satchell & Scowcroft (2000)

Here $\lambda$ is price for average market risk

**S** is Variance- Covariance matrix

**w** is Observed market weights

So that the Implied Equilibrium Excess Returns vector is the market capitalization weighted portfolio. If Investors do not have any views about the market or assets, then they would hold this market portfolio. In addition these equilibrium returns are the set of returns that clear the market if all investors have identical views. Implied returns are also known as CAPM returns, market returns, equilibrium return, consensus returns, neutral returns and reverse optimized returns.

### 3.3.5 Assimilation of Black Litterman parameters

It has been explicated the treatment of equilibrium market returns as the neutral beginning steps towards finding the Implied Equilibrium Excess Returns by using the technique of reverse optimization. But the state of the art principle of the Black – Litterman method is the ability of the model in allowing Investors/ Asset Managers to incorporate their own views about the assets that constitutes the portfolio. View is a particular opinion or judgment about the future performance and returns of financial assets. The views are subjective that varies Asset Managers to Asset managers and can be expressed in either absolute or relative terms.

The translation of the Black and Litterman's ideas in to the mathematical formula has been illustrated and explained in section 3.3 by the equation (1) where the input Q is the vector of views on expected excess returns for some or all assets. If the number of views is to be **m**, then the views vector Q would have the dimension of **m × 1**.

It has been exemplified by the equation (2), how the implied equilibrium excess return was combined with the views vector and related weight parameters on the expected excess return to produce the weighted averages of the implied returns and the views. Hence it's needed at this point to study the derivation of the other parameters such as:

$\tau$ is the weight- on- views scalar (Assume $\tau = 1$)

Ω is the diagonal matrix that identifies the uncertainty of views
P is the matrix that identifies the assets involved in the different views (Link matrix)

Uncertainties associated with the estimated equilibrium mean returns are explained by the scalar τ. Fixing a value for τ, the weight- on- views scalar is quite awkward part in the Black-Litterman model and it's been assumed to be 1 in the evaluation of the model in this context, despite earlier researches on this topic have different assessments of how this parameter should be set. Apparently it had been suggested by most of the researchers that τ must be assigned with a low value somewhere between 0 and 1.Black-Litterman (1990) also emphasized the same citing the reason that the uncertainty in the mean is much smaller than the uncertainty in the return itself.

When Asset managers do specify views about few or more assets, they would not be very sure, that means they would not be 100% positive that their view is correct. There would be uncertainties about the views. For instance it's very challenging to predict either the stock markets or debt markets. Black and Litterman stated that the uncertainty of views results in a random, unknown, independently, normally distributes error term vector (Ɛ) with mean 0 and Variance (Covariance matrix)Ω. Thus a view has the form Q+ Ɛ.

Hence the Uncertainty or the Variance in the views had been given by Ω.

$$Q + £ = \begin{pmatrix} Q_1 \\ Q_2 \\ . \\ . \\ Q_m \end{pmatrix} + \begin{pmatrix} £_1 \\ £_2 \\ . \\ . \\ £_m \end{pmatrix} \text{ Where } \begin{pmatrix} £_1 \\ £_2 \\ . \\ . \\ £_m \end{pmatrix} \sim N \left( \begin{bmatrix} 0 \\ 0 \\ . \end{bmatrix} , \begin{pmatrix} e_{11} & e_{12} & .. & e_{1m} \\ 0 & 0 & 0 & 0 \\ e_{m1} & .. & .. & e_{mm} \end{pmatrix} \right)$$

The larger the variance of the error term ($\mathcal{E}$ ), the greater the uncertainty of the view. In contrast, when the Investor is more confident about the views, he would get much smaller error term. If he is 100% confident about his views, there would be no error terms. Because he does perfectly predict what's going to happen.

Ω would have the dimensions equal to the no. of views. Suppose there are m views, then Ω would be an $m \times m$ matrix.

There had been no finest ways found to measure the uncertainty Ω. The financial mathematicians and practitioners tended to find it based on their levels of confidence about the views. Fundamentally the reasonable best way to calculate Ω would depend on how confident the Investor is about his prediction. He could adjust Ω to reflect his confidence. But as a base estimate of Ω, Black and Litterman recommended the following formula.

$$\Omega = {}^{2}PSP^{T} \tag{10}$$

τ is the weight- on- views scalar (Assume τ = 1)

P is the matrix that identifies the assets involved in the different views (Link matrix)

S is Variance- Covariance matrix

Since a focal assumption of the Black-Litterman model is that investors' views are distinctive and uncorrelated, the matrix Ω will be an m × m diagonal-matrix. In this case the matrix would looks alike,

$$\Omega = \begin{pmatrix} e_1 & 0 & 0 & 0 & 0 \\ 0 & . & 0 & 0 & 0 \\ 0 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & e_m \end{pmatrix}$$

where $e_m$ is the uncertainty in the m$^{th}$ view

The variances of the error terms ($\mathcal{E}$ ) form Ω , where Ω is a diagonal covariance matrix with 0's in all of the off-diagonal positions. The off-diagonal elements of Ω are 0's because the model assumes that the views are independent of one another (uncorrelated). If $e_m = 0$, that means that the investor is 100% confident about the view.

Then the views have to be incorporated in to the model and combined with the Implied Equilibrium Excess Returns. For that a Link matrix would have to be used and that matrix is called "P". Matrix P is constructed in the following way:

Each row represents a View and each column represents an Asset. There are two rules that articulate the calculation of the elements of P matrix.

The rule one is applied in the case of absolute views. The absolute view would involve only one asset. And the weightage would have to be shown by entering ‘1’ under the column of respective asset corresponding to the respective view’s row.

The rule two is that the relative views must sum up to Zero.So the elements of each view must sum up to 0 across all the assets in a portfolio. Positive components in the (View) link matrix for particular view must add up to 1 while the negative components must add up to -1. This would ensure the summation across all stocks in a view row to become 0. Positive numbers in a (View) link matrix shows that the Asset manager is positive about those companies and the negative numbers shows the companies that the Asset manager is negative about.

In case of relative views where a set of assets do outperform or underperform of another sets of assets, then the [7]market capitalization weighting scheme can be used. The terms “Outperforming” and “Underperforming” are relative. The number of outperforming assets is not required to match the number of assets underperforming.

More specifically, the relative weighting of each individual asset is proportional to the asset’s market capitalization divided by the total market capitalization of either the outperforming or underperforming assets of that particular view. Hence the relative market capitalization weights of the nominally outperforming assets and the relative market capitalization weights of the nominally underperforming assets has to be calculated and specified in such ways that the weightages of the relative view summing up to Zero.

So that, once the Link Matrix “P” is defined, the variance of each individual view portfolio can be calculated to create the covariance matrix Ω. It has been discussed above about the calculation of Ω where the diagonal elements could be calculated by using the formula $\tau P_m S P_m^T$.

[7]Thomas M. Idzorek in 2005

In this case Pm is a single 1 x N row vector from Matrix P that corresponds to the $m^{th}$ view and S is the covariance matrix of excess returns.

### 3.3.6 The portfolio allocation

By plugging all the inputs such as the Variance- Covariance matrix S, Link matrix P, View matrix Q, Implied Equilibrium Excess Return vector Π and the Weight- on-views scalar (Assumed τ to be 1), the Black- Litterman Expected Excess Return would be established. Having found the Black- Litterman Expected Excess Returns, by using the formulae (5) the corresponding portfolio weightages can be found.

$$w = \frac{S^{-1}\left(« - r_f\right)}{2ª} = \frac{Z}{1^T Z} \qquad (6)$$

The formulae for portfolio weights w, could be used with any modellings of mean variance as it is derived by solving the Utility function given by the equation (4)

$$Max_w : w^T\left(« - r_f\right) - ª w^T S w \qquad (4)$$

And that is the optimal portfolio weights that give the maximum utility for the Investors.

## 3.4 Fundamentals of Portfolio Dynamics

### 3.4.1 Parameters of risk and return

Simple stock returns over a period can be found using arithmetic averages. The arithmetic average rate of return over n time periods of equal length is defined as:

$$« = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad (11)$$

Risk is measured either by the standard deviation or variance. Standard deviation/ Variance measures how much return on an investment is deviating from the expected normal or average returns.

$$± = \sqrt{\frac{\sum_{i=1}^{n}\left(x_1 - \bar{x}\right)^2}{n}} \qquad (12)$$

Where

σ = the standard deviation

$x_i$= each value in the sample

$\overline{x}$ = the mean of the values

n = the number of values in the sample

$$s^2 = \frac{\sum\left(X_i - \bar{X}\right)^2}{n-1} \tag{13}$$

Where

$s^2$ = the variance

$x_i$= each value in the sample

$\overline{x}$ = the mean of the values

n = the number of values in the sample

Covariance measures how two variables move together. It measures whether the two move in the same direction (a positive covariance) or in opposite directions (a negative covariance). In this research, the variables will usually be stock prices, but they can be anything.

$$Cov(X,Y) = \frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)}{(n-1)} \tag{14}$$

Where

Cov (X, Y) = the co variance in between samples of X and samples of Y

$x_i$= each value in the independent sample X

$y_i$= each value in the dependent sampleY

$\overline{x}$ = the mean of the independent variable x

$\overline{y}$ = the mean of the dependent variable y

n = the number of data points in the sample

The capital asset pricing model (CAPM) is a model that calculates expected return based on expected rate of return on the market, the risk-free rate and the beta coefficient of the stock.

CAPM Return = Risk free rate + β × Risk premium rate (15)

Beta is the measure of a stock's sensitivity of returns to changes in the market. It is a measure of systematic risk.

$$\text{Beta } \beta = \frac{\text{Covariance of Stock to the market}}{\text{Variance of the market}} \tag{16}$$

Sharpe Ratio is an indicator of whether an investments' return is due to smart investing decisions or a result of excess risk.The Sharpe ratio is the average return earned in excess of the risk-free rate per unit of volatility or total risk. By subtracting the risk-free rate from the mean return, the performance associated with risk-taking activities can be isolated.

$$\text{Sharpe Ratio} = \frac{\left(\bar{r}_p - r_f\right)}{\pm_p} \tag{17}$$

Where

$\bar{r}_p$ = Expected portfolio return

$r_f$ = Risk free rate

$\sigma_p$ = Portfolio standard deviation

### 3.4.2 Parameters of portfolio risk and return

The return can be computed by using the probability distribution of expected returns for a portfolio. The risk is assumed to be measured by the variability around the expected value of the probability distribution of returns. Hence the risk will be measured by either variance or by standard deviation. The return has been calculated as follows:

$$E\left(r_p\right) = \sum_{i=1}^{n} w_i E\left(r_i\right) \tag{18}$$

Where

$$\sum_{i=1}^{n} w_i = 1.0;$$

n = the number of securities;

$w_i$ = the proportion of the funds invested in security i;

$r_i, r_p$ = the return on i[th] security and portfolio p; and

$E$ () = the expectation of the variable in the parentheses

The risk of a single security is the expected value of the sum of the squared deviations from the mean, whereas the variance of a portfolio combination of securities is equal to the weighted average covariance of the returns on its individual securities.

$$Var(r_P) = \pm_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(r_i, r_j)$$

Covariance can also be expressed in terms of the correlation coefficient as follows:

$$Cov(r_i, r_j) = {}^\circ_{ij} \pm_i \pm_j = \pm_{ij} \qquad (19)$$

Where $^\circ_{ij}$ , correlation coefficient between the rates of returns on security i, and the rates of return on security j,

With the assumption that the covariance is less than 1 (which is not a practical assumption), it is derived that the weighted average of the standard deviation of the expected returns of the securities shall be more. As such the theory proves that diversification of securities in a portfolio reduces risk.

$$Var(r_p) = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j {}^\circ_{ij} \pm_i \pm_j \qquad (20)$$

Overall, the estimate of the mean return for each security is its average value in the sample period; the estimate of variance is the average value of the squared deviations around the sample average; the estimate of the covariance is the average value of the cross-product of the deviations.

### 3.5 Chapter Summary

Under this section, the theoretical contextual of the research has been presented. The Black- Litterman model description with assumptions and how it applies to the research has been discussed extensively. Incorporation of Black Litterman model parameters was shown and related portfolio allocation methods were described. Further few traditional concepts associated with portfolio measurement and asset allocation was discussed to give a complete picture about the theories relevant to investment management that has been in practice since long time back.

# CHAPTER 4: EVALUATION OF THE MODEL

## 4.1 Introduction

The assessment of the Black- Litterman model would be done in this chapter. The data definition and presentation would be done primarily. Then under the Data manipulation section, the Stock return calculation, Portfolio return measures when applying Historical and Capital Asset Pricing Model methods, about inputs required for setting up the Black Litterman model and setting up the BL model have been described. Having set up the Black Litterman model the BL expected excess return has been found. Later the BL expected excess return would be compared with implied equilibrium excess returns and other equilibrium excess returns derived via historical and CAPM methods. The outcomes of all three methods would be analyzed and the grandeur of BL model would be illustrated. The computations have been done by using both the Microsoft Excel and Matlab.

## 4.2 Model Evaluation

The model evaluation has been done using the [8]equity securities of the corporations from the emerging market economy. It's expected that the emerging markets are [9]efficient and the Market Benchmark index is best described by the constituents of the index. Here the [10]MSCI Emerging Market Index has been considered as the Benchmark index and eight equity securities covering across the emerging market economies have been taken to constitute the portfolio.

The monthly closing prices which are dividend and share split adjusted of the following eight stocks were downloaded from the [11]Bloomberg L. P. trading platform.

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[8]Equity Security is an instrument that signifies an ownership position in a corporation, and represents a claim on its proportional share in the corporation's assets and profits. Ownership in the company is determined by the number of shares a person owns divided by the total number of shares outstanding. [W4]

[9]The efficient market hypothesis (EMH) is an investment theory that states it is impossible to "beat the market" because stock market efficiency causes existing share prices to always incorporate and reflect all relevant information. According to the EMH, stocks always trade at their fair value on stock exchanges, making it impossible for investors to either purchase undervalued stocks or sell stocks for inflated prices. As such, it should be impossible to outperform the overall market through expert stock selection or market timing, and that the only way an investor can possibly obtain higher returns is by purchasing riskier investments. [W5]

The Market capitalizations of these equity securities were recognized in respective domestic currencies from the Bloomberg L. P. website and converted to common US Dollar denominated market caps as shown later in the Table 4.10.

Also the securities were selected as to represent different business sectors reputed for Information Technology, Telecom Services, Consumer discretionary, Property Development, Diversified Holdings, Banking and Finance, Petro Chemicals and Construction Materials.

| Stock | Country | Sector | [12]Primary Listings | Currency |
|---|---|---|---|---|
| Samsung Electronics Co., Ltd | Korea | Information Technology | Korea Stock Exchange | South Korean Won |
| China Mobile Communications Corporation | China | Telecom Services | Shanghai Stock Exchange | Renminbi/ Yuan |
| Naspers Limited | South Africa | Consumer discretionary | Johannesburg Stock Exchange | South African rand |
| Emaar Properties | UAE | Property Development | United Arab Emirates Stock Market | UAE dirham |
| Koc Holding AS | Turkey | Diversified Holdings | Borsa İstanbul | Turkish lira |
| Akbank | Turkey | Banking and Finance | Borsa İstanbul | Turkish lira |
| Braskem SA | Brazil | Petro Chemicals | BM&F Bovespa exchange(BOVESPA) | Brazilian real |
| Taiwan Cement Corporation | Taiwan | Construction Materials | Taiwan Stock Exchange | New Taiwan dollar |

**Table 4.1 : The eight stocks and the respective sectors**

[10]The MSCI Emerging Markets Index was launched on Jan 01, 2001 and is designed to measure the equity market performance of the emerging markets. [W6]

[11]Bloomberg L.P. is privately held financial software, data, and Media Company headquartered in Midtown Manhattan, New York City. [W7]

[12]Primary Listing refers to the main stock exchange where a publicly traded company's stock is bought and sold. Having a prestigious primary listing, such as the New York Stock Exchange, lends company credibility and makes investors more likely to purchase its shares. In addition to its primary listing, a stock may also trade on other exchanges. A company might want to do this to increase its liquidity and ability to raise capital. [W8]

Moreover the monthly closings of the MSCI Emerging Markets Index were downloaded from the Bloomberg and the US Treasury rates have been obtained from the website of U.S. Department of the Treasury.These have been used respectively as the market benchmark index and market risk free rate.

### 4.3 Data

135 monthly closing prices of each stocks and the index, spanning over 11 years starting from July 2004 to September 2015, which are monthly closings of the eight stocks and the MSCI Emerging market index that has been used for the model evaluation is presented below in Table 4.2:

| MONTH | DATE | SAMSUNG | CHINA_MOB | NASPERS | EMAR | KCHOL | AK_BANK | BRKM5 | TAIWAN_CE | MSCI_INDEX |
|---|---|---|---|---|---|---|---|---|---|---|
| 134 | 09-30-15 | 1,134,000 | 59.50 | 173,066 | 6.46 | 11.80 | 6.78 | 16.67 | 33.35 | 792.05 |
| 133 | 08-31-15 | 1,089,000 | 59.82 | 171,900 | 6.75 | 11.35 | 6.82 | 14.07 | 34.85 | 818.73 |
| 132 | 07-31-15 | 1,185,000 | 65.00 | 177,000 | 7.90 | 12.30 | 7.42 | 12.54 | 34.20 | 901.68 |
| 131 | 06-30-15 | 1,268,000 | 64.09 | 189,500 | 7.88 | 12.40 | 7.75 | 13.62 | 38.95 | 972.25 |
| 130 | 05-29-15 | 1,307,000 | 65.74 | 178,505 | 7.75 | 11.90 | 7.93 | 12.97 | 41.70 | 1004.22 |
| : | : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : | : |
| 5 | 12-31-04 | 450,500 | 17.16 | 7,500 | 5.23 | 2.97 | 3.42 | 33.50 | 18.97 | 542.17 |
| 4 | 11-30-04 | 434,500 | 16.32 | 6,690 | 4.12 | 2.65 | 2.78 | 29.60 | 17.61 | 517.95 |
| 3 | 10-29-04 | 439,500 | 14.55 | 5,620 | 3.19 | 3.02 | 2.72 | 26.25 | 16.43 | 474.27 |
| 2 | 09-30-04 | 458,000 | 15.30 | 5,121 | 3.24 | 2.97 | 2.78 | 23.80 | 17.07 | 464.15 |
| 1 | 08-31-04 | 451,000 | 14.62 | 4,896 | 2.49 | 2.75 | 2.62 | 21.20 | 15.43 | 439.75 |
| 0 | 07-30-04 | 417,000 | 14.52 | 4,500 | 2.38 | 2.55 | 2.48 | 15.30 | 13.11 | 423.14 |

**Table 4.2 : Monthly closings of the eight stocks and the MSCI Emerging market index**

The complete listings of data in table 4.2 can be found in the Appendix A1. Figure 4.1 shows the monthly closing indices of MSCI Emerging market from July 2004 to September 2015. It’s witnessed in the chart, that an uptrend during the period of July 2004 to October 2007. During this period the index had been continuously progressing and recorded a height of 1,337. This indicates that emerging market economies had been performing well and would have been flourishing during this period. Thereafter the index started to drop and was falling deep down till December

2008. The index had been kept on falling to record the lowest points of 529. Since then to date, a flat fluctuation has been observed where the index swings between the levels of 800 - 1,100.

The MSCI Emerging market index had undergone the steep drop during late 2007 and 2008 due to the financial crisis that happened in U.S.A and had spread to the other parts of the world. This instigated due to the [13]subprime mortgage crisis happened in U.S.A.

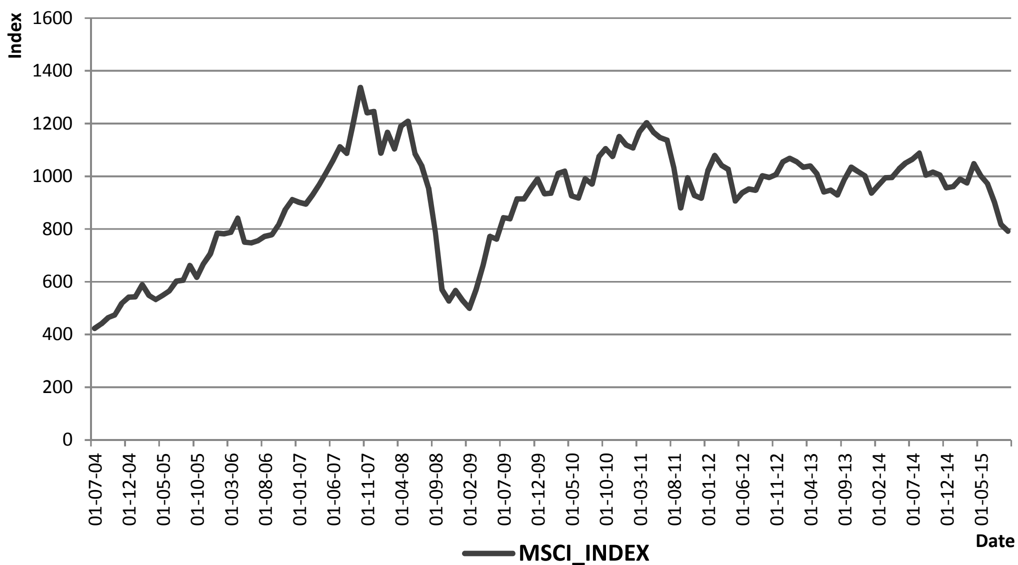


**Figure 4.1: The monthly closing indices of MSCI Emerging market from July 2004 to September2015**

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[13]The United States (U.S.) subprime mortgage crisis was a nationwide banking emergency that coincided with the U.S. recession of December 2007 – June 2009. It was triggered by a large decline in home prices after the collapse of a housing bubble, leading to mortgage delinquencies and foreclosures and the devaluation of housing-related securities. Declines in residential investment preceded the recession and were followed by reductions in household spending and then business investment. Spending reductions were more significant in areas with a combination of high household debt and larger housing price declines. [W9]

A subprime mortgage is a type of mortgage that is normally made out to borrowers with lower credit ratings. As a result of the borrower's lowered credit rating, a conventional mortgage is not offered because the lender views the borrower as having a larger-than-average risk of defaulting on the loan. Lending institutions often charge interest on subprime mortgages at a rate that is higher than a conventional mortgage in order to compensate them for carrying more risk. [W10]

From the chart given by the Figure 4.2, it's clear that the data set of these eight stocks do not have outliers. That means that these stocks' price fluctuations are steady despite depicting a [14]stochastic oscillation that is general for asset price movements.

Hence this can be correlated to the efficiency of the emerging market economy and say that there are no serious [15]market anomalies perceived.

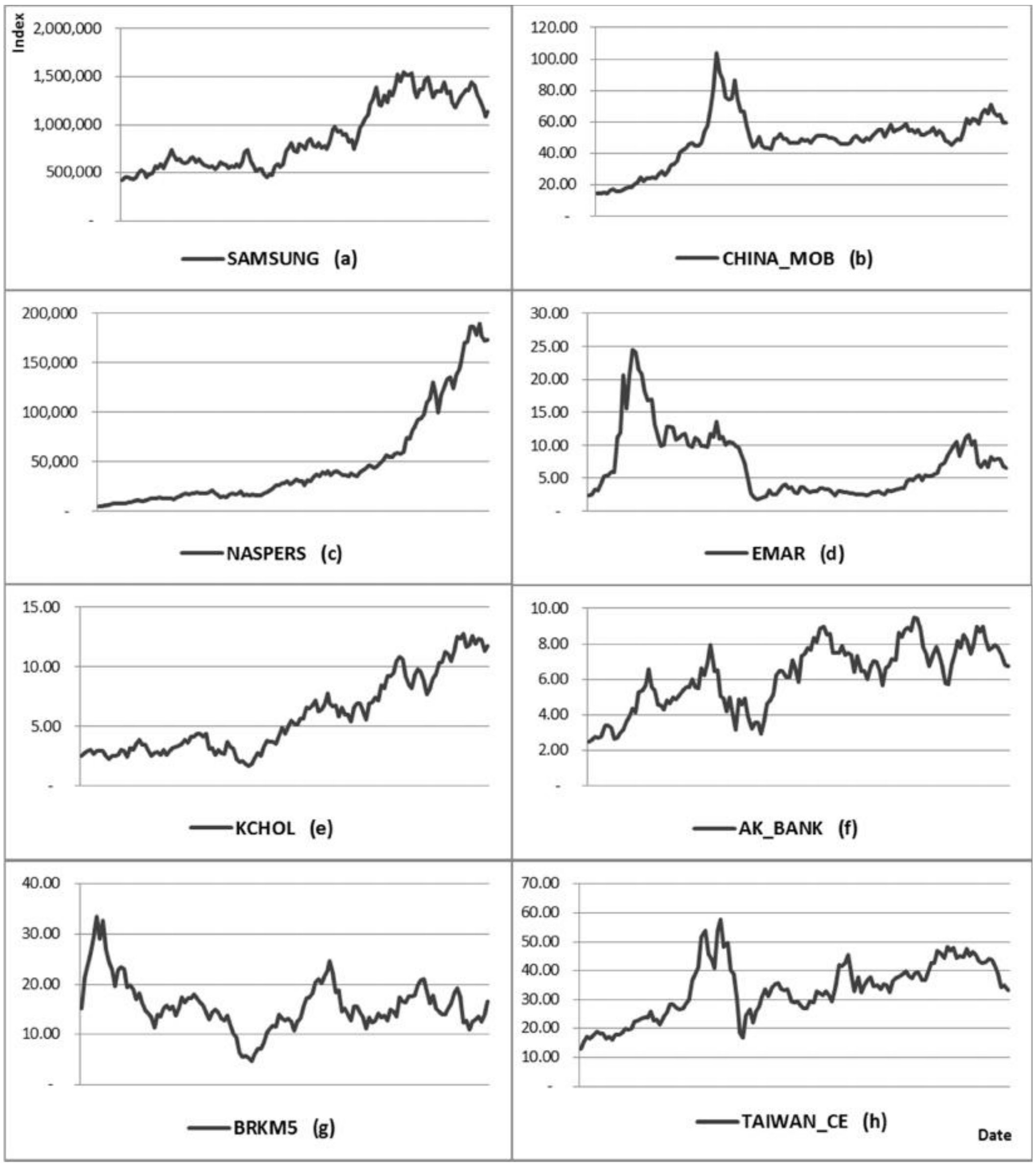


**Figure 4.2: The monthly closing prices of the portfolio constituents from July 2004 to September 2015**

From the stochastic process of the eight equities shown by the chart in Figure 4.2, the following can be noticed. The Stocks NASPERS and KCHOL have displayed a continued uptrend whereas all other Stocks had plunged during the period of late 2007 and 2008.

But the stock SAMSUNG which falls under the Information Technology sector had exhibited only a small drop during above mentioned period compared to other stocks. Also it's noticed that the stock BRKM5 falling under the Petro chemicals sector, had been gradually dropping in points till early 2009 and since then it started to pick up which is not an identical pattern with the other stocks. Hence it can be assumed that the Company BraskemSA could have carried some stresses that belong to either the Company itself or to that particular sector and not due to external (systematic) pressures.

## 4.4 Data Manipulation

### 4.4.1 Monthly returns

The markets' and stocks' monthly returns have been calculated by using the following simple formula:

$$R_i(t) = \frac{S_i(t) - S_i(t-1)}{S_j(t-1)} \qquad (21)$$

Where $R_j$ (t) is the monthly return for the investment j at time t

$S_j$ (t) is the monthly closing price for investment j at time t

$S_j$ (t-1) is the monthly closing price for investment j at time t-1

134 monthly return numbers of both the portfolio constituents and the Benchmark from July 2004 to September 2015 has been calculated by using the above equation (21) and is presented below in Table 4.3.

[14]In probability theory, a stochastic process, or often random process, is a collection of random variables representing the evolution of some system of random values over time. [W11]

[15]In financial markets, anomalies refer to situations when a security or group of securities performs contrary to the notion of efficient markets, where security prices are said to reflect all available information at any point in time. With the constant release and rapid dissemination of new information, sometimes efficient markets are hard to achieve and even more difficult to maintain. There are many market anomalies; some occur once and disappear, while others are continuously observed. [W12]

The complete listings of returns in table 4.3 can be found in the Appendix A2. Figure 4.3 shows the monthly returns of MSCI Emerging market index from July 2004 to September 2015. From the chart it can be seen that the highest and lowest monthly returns of MSCI Emerging market index which are 16.6% and negative 27.5% was recorded respectively on May 2009 and October 2008. The plunge of 27.5% in year 2008 was due to the financial crisis happened in most parts of the world that had been started with the Subprime mortgage crisis in U.S.A.

| MONTH | DATE | RETSAM | RETCHINA | RETNASP | RETEMAR | RETKCHOL | RETAKBANK | RETBRKM | RETTAIWAN | RETINDEX |
|---|---|---|---|---|---|---|---|---|---|---|
| 134 | 09-30-15 | 0.0413 | -0.0053 | 0.0068 | -0.0430 | 0.0396 | -0.0059 | 0.1848 | -0.0430 | -0.0326 |
| 133 | 08-31-15 | -0.0810 | -0.0797 | -0.0288 | -0.1456 | -0.0772 | -0.0809 | 0.1220 | 0.0190 | -0.0920 |
| 132 | 07-31-15 | -0.0655 | 0.0142 | -0.0660 | 0.0025 | -0.0081 | -0.0426 | -0.0793 | -0.1220 | -0.0726 |
| 131 | 06-30-15 | -0.0298 | -0.0251 | 0.0616 | 0.0168 | 0.0420 | -0.0227 | 0.0501 | -0.0659 | -0.0318 |
| 130 | 05-29-15 | -0.0730 | -0.0797 | -0.0460 | -0.0595 | -0.0593 | 0.0180 | 0.0294 | -0.0436 | -0.0416 |
| : | : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : | : |
| 5 | 12-31-04 | 0.0368 | 0.0515 | 0.1211 | 0.2703 | 0.1208 | 0.2279 | 0.1318 | 0.0772 | 0.0468 |
| 4 | 11-30-04 | -0.0114 | 0.1216 | 0.1904 | 0.2913 | -0.1229 | 0.0224 | 0.1276 | 0.0718 | 0.0921 |
| 3 | 10-29-04 | -0.0404 | -0.0490 | 0.0974 | -0.0154 | 0.0172 | -0.0219 | 0.1029 | -0.0375 | 0.0218 |
| 2 | 09-30-04 | 0.0155 | 0.0465 | 0.0460 | 0.2987 | 0.0797 | 0.0626 | 0.1226 | 0.1063 | 0.0555 |
| 1 | 08-31-04 | 0.0815 | 0.0069 | 0.0880 | 0.0492 | 0.0793 | 0.0578 | 0.3856 | 0.1770 | 0.0393 |
| 0 | 07-30-04 | | | | | | | | | |

**Table 4.3 : Monthly returns of the eight stocks and the MSCI Emerging market index**

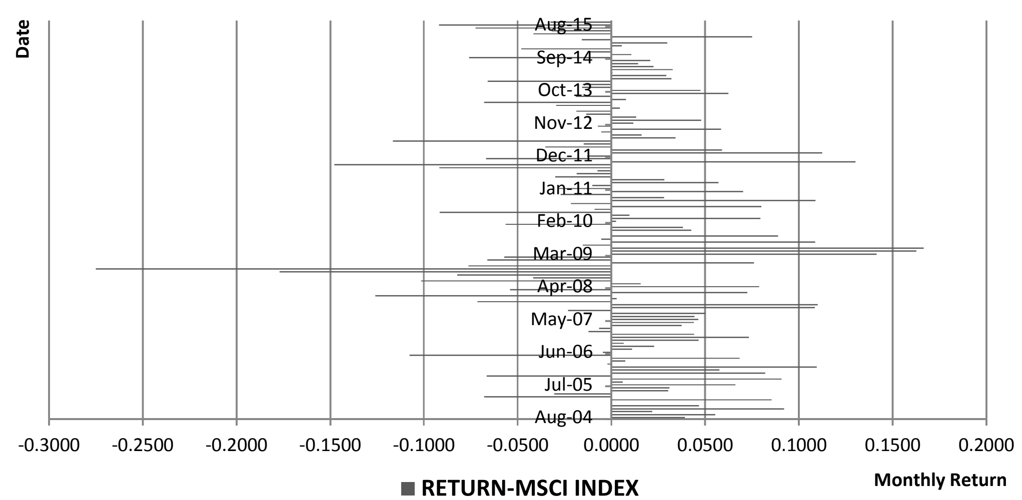


**Figure 4.3: The monthly returns of MSCI Emerging market index from July 2004 to September 2015**

From the return spikes bar chart in Figure 4.4, the followings can be noticed. All Stocks across the board have given the least monthly returns in the second half of year 2008. In contrast to this, the stocks KCHOL, AK BANK and TAIWAN CE's highest monthly returns also had been recorded in the second half of 2008. The Property development stock EMAR had given a fabulous monthly return of 87.5% for April 2005 and have depreciated by the highest 45.1% over a month for November 2008. It should be noted that by looking only at the points of monthly returns, the decisions for long term investments cannot be made.

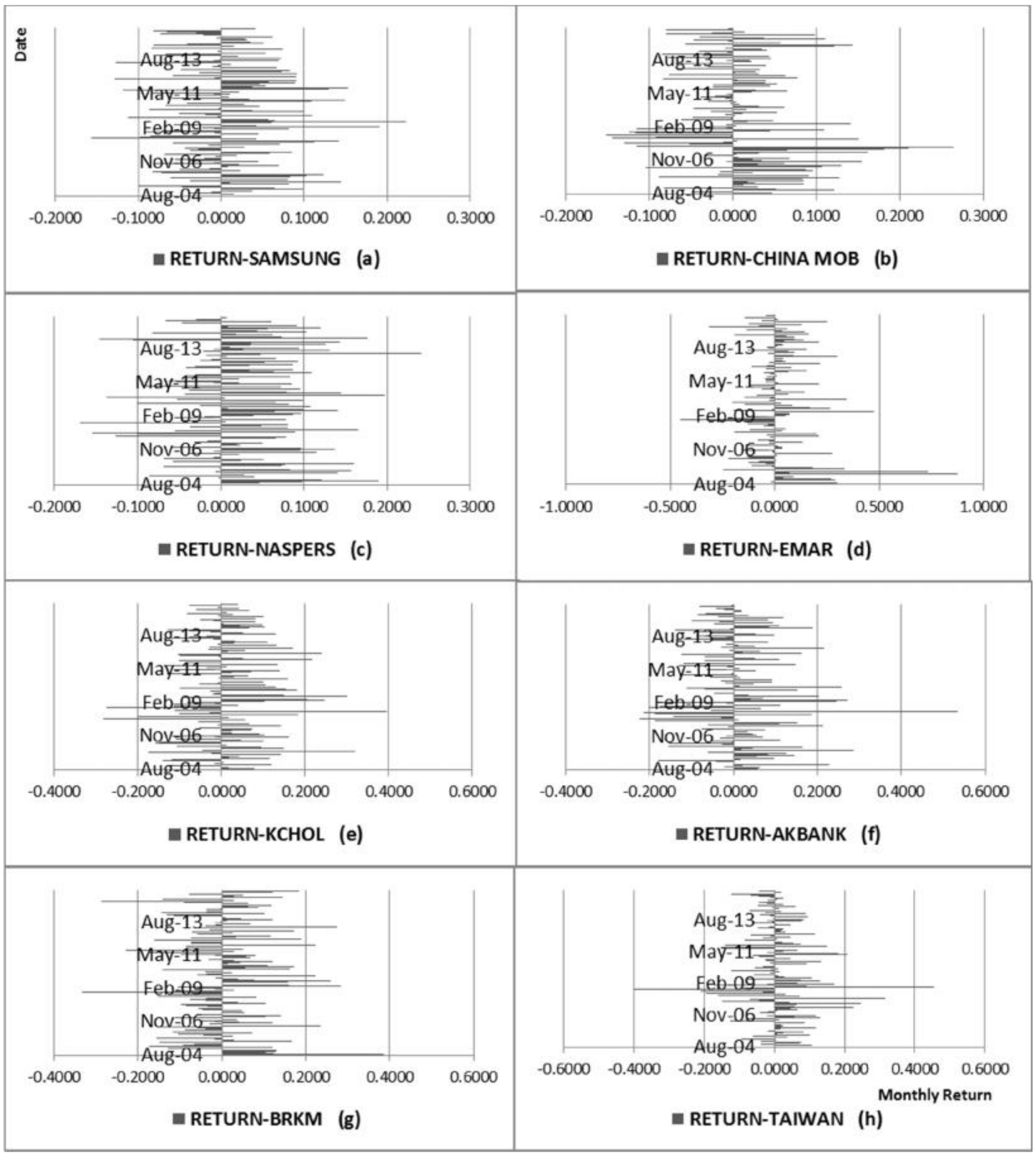


**Figure 4.4: The monthly returns of the portfolio constituents from July 2004 to September 2015**

### 4.4.2 Asset returns

It's intended to seek the portfolio's return performances and allocations using two conventional methods such as Historical method and CAPM method to compare with the Black Litterman method. Hence under this section the Portfolio measures when using the Historical method and CAPM method have been demonstrated.

| STATS | SAMS | CHN | NASPR | EMAR | KCHOL | AK_BN | BKM5 | TW_CE | MSCI |
|---|---|---|---|---|---|---|---|---|---|
| Avg Retns | 1.00% | 1.31% | 3.06% | 2.07% | 1.77% | 1.38% | 0.75% | 1.21% | 0.70% |
| Variance | 0.52% | 0.52% | 0.61% | 2.94% | 1.27% | 1.31% | 1.38% | 1.01% | 0.45% |
| Stnd Dev. | 7.18% | 7.23% | 7.83% | 17.15% | 11.3% | 11.5% | 11.7% | 10.1% | 6.7% |
| [16]Beta (β) | 38.3% | 56.9% | 59.9% | 91.43% | 89.2% | 84.1% | 68.0% | 75.5% | 100.0% |

**Table 4.4 : Statistical summary of returns of the eight stocks and the MSCI Emerging market index**

The Table 4.4 tabulates the Historical average returns, Variances, Standard deviation and Beta values for the eight stocks and for the Benchmark index. These have been calculated based on 134 monthly return figures using the equations specified by (11), (13), (12) and (16) respectively.

It's witnessed that the stock NASPERS had yielded the highest historical average monthly return of 3.06% with the volatility of 7.83%. That means NASPERS' returns had varied within the range of 10.89% to negative 4.77% per month. The stock EMAR's performance is very volatile with the monthly standard deviation of 17.15%. Interestingly there is no stock which is more volatile than the market as the betas of all eight securities are lesser than 100%.

| | Annual | Monthly |
|---|---|---|
| Risk Free Rate | 8.00% | 0.6667% |
| Market Risk Premium Rate | 9.00% | 0.7500% |

**Table 4.5 : Inputs for the CAPM required rate of return**

[16]Beta is a measure of the volatility, or systematic risk, of a security or a portfolio in comparison to the market as a whole. Beta is calculated by using the regression analysis and this is the tendency of a security's returns to respond to swings in the market. A beta of 1 indicates that the security's price will move with the market. A beta of less than1 means that the security will be less volatile than the market. A beta of greater than 1 indicates that the security's price will be more volatile than the market. [W13]

When comparing to other stocks, the stock EMAR has the highest beta of 91.43%. That means whenever the market swings either up or down by a certain scale and if that swing is given a weightage of 100% then EMAR could swing up or down only by 91.43% which is below the market's swing. The stock SAMSUNG would fluctuate only by little 38.31% whenever the market does by 100%.

The CAPM returns are calculated using the equation (15). It requires the Risk free rate, Market risk premium rate and beta values of all the portfolio constituents. The beta values for all the stocks have already been calculated and tabulated in Table 4.4.

The Table 4.5 shows the assumed values of Risk free rate (8.00% p.a.) and Market risk premium rate per annum (9.00% p.a.). Further it shows the monthly rates that have been obtained by dividing the annual rates by 12.

These rates are higher when comparing to the actual risk free rate and risk premium rates that currently prevails in the countries of Emerging economy. Despite the fact, these higher rates have been assumed in order to result noteworthy return numbers that would help for easy learning.

| STATISTICS | SAMSU | CHNA_M | NASP | EMA | KCHO | AK_B | BRM5 | TWN_C |
|---|---|---|---|---|---|---|---|---|
| Historical Returns ($\mu_{hist}$) | 1.00% | 1.31% | 3.06% | 2.07% | 1.77% | 1.38% | 0.75% | 1.21% |
| CAPM Returns ($\mu_{CAPM}$) | 0.95% | 1.09% | 1.12% | 1.35% | 1.34% | 1.30% | 1.18% | 1.23% |

**Table 4.6 : Traditional return calculation**

The Table 4.6 tabulates both the Historical returns and the CAPM returns for single glance. It's observed there that the stock EMAR had generated the highest CAPM return of 1.35% per month. It can be further noted that with most of the stocks the Historical returns and the CAPM returns differ much in values.

### 4.4.3 Portfolio returns

Under this section, the portfolio performance measures would be calculated by using both Historical returns and the CAPM returns. The Portfolio expected returns, Portfolio variance, and the Expected sharp ratio would be calculated by using the equations (18), (20) and (17).

To find these measures, in addition to returns the portfolio weightages and the Covariance – Variance of the Excess returns are required to be input. In this case the portfolio is equally allocated with the eight stocks.Thus each would carry 12.50% in the portfolio.

Then to find the matrix of Covariance – Variance of the Excess returns (S), the Excess returns (Demeaned returns) are needed to be found. This has been found by subtracting the Historical average returns of every stock from the Monthly closings of the stocks and tabulated in Table 4.7. The complete listings of data in Table 4.7 can be found in the Appendix A3.

While constructing the matrix of Covariance – Variance of the Excess returns (S), the number of observations were taken as 133 as it's the sample data only with 8 stocks which covers a short portion of data since the time the stocks got listed in an Exchange. The calculated matrix is given in the Figure 4.5.

| MONTH | DATE | $EXC.RET_{SAM}$ | $EXC.RET_{CHI}$ | $EXC.RET_{NAS}$ | $EXC.RET_{EMAR}$ | $EXC.RET_{KCHO}$ | $EXC.RET_{AK}$ | $EXC.RET_{BRM}$ | $EXC.RET_{TAIW}$ |
|---|---|---|---|---|---|---|---|---|---|
| 134 | 30-09-15 | 0.0313 | -0.0185 | -0.0238 | -0.0637 | 0.0219 | -0.0197 | 0.1773 | -0.0551 |
| 133 | 31-08-15 | -0.0910 | -0.0928 | -0.0594 | -0.1663 | -0.0950 | -0.0947 | 0.1145 | 0.0069 |
| 132 | 31-07-15 | -0.0755 | 0.0011 | -0.0966 | -0.0182 | -0.0258 | -0.0564 | -0.0868 | -0.1340 |
| 131 | 30-06-15 | -0.0399 | -0.0382 | 0.0310 | -0.0040 | 0.0243 | -0.0365 | 0.0426 | -0.0780 |
| 130 | 29-05-15 | -0.0831 | -0.0928 | -0.0766 | -0.0802 | -0.0770 | 0.0042 | 0.0219 | -0.0556 |
| : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : |
| : | : | : | : | : | : | : | : | : | : |
| 5 | 31-12-04 | 0.0313 | -0.0185 | -0.0238 | -0.0637 | 0.0219 | -0.0197 | 0.1773 | -0.0551 |
| 4 | 30-11-04 | -0.0910 | -0.0928 | -0.0594 | -0.1663 | -0.0950 | -0.0947 | 0.1145 | 0.0069 |
| 3 | 29-10-04 | -0.0755 | 0.0011 | -0.0966 | -0.0182 | -0.0258 | -0.0564 | -0.0868 | -0.1340 |
| 2 | 30-09-04 | -0.0399 | -0.0382 | 0.0310 | -0.0040 | 0.0243 | -0.0365 | 0.0426 | -0.0780 |
| 1 | 31-08-04 | -0.0831 | -0.0928 | -0.0766 | -0.0802 | -0.0770 | 0.0042 | 0.0219 | -0.0556 |
| 0 | 30-07-04 | | | | | | | | |

**Table 4.7 : Excess Returns(Demeaned Returns) of the eight stocks**

| | SAMSUG | CHIA_MOB | NASPRS | EMAR | KCHOL | AK_BAK | BRM5 | TAIN_CE |
|---|---|---|---|---|---|---|---|---|
| **SAMSUNG** | 0.0052 | 0.0002 | 0.0010 | -0.0002 | 0.0019 | 0.0024 | 0.0012 | 0.0005 |
| **CHINA_MO** | 0.0002 | 0.0053 | 0.0014 | 0.0028 | 0.0024 | 0.0027 | 0.0007 | 0.0025 |
| **NASPERS** | 0.0010 | 0.0014 | 0.0062 | 0.0027 | 0.0030 | 0.0028 | 0.0022 | 0.0018 |
| **EMAR** | -0.0002 | 0.0028 | 0.0027 | 0.0296 | 0.0039 | 0.0043 | 0.0045 | 0.0023 |
| **KCHOL** | 0.0019 | 0.0024 | 0.0030 | 0.0039 | 0.0128 | 0.0103 | 0.0045 | 0.0015 |
| **AK_BANK** | 0.0024 | 0.0027 | 0.0028 | 0.0043 | 0.0103 | 0.0132 | 0.0038 | 0.0008 |
| **BRKM5** | 0.0012 | 0.0007 | 0.0022 | 0.0045 | 0.0045 | 0.0038 | 0.0139 | 0.0021 |
| **TAIW_CE** | 0.0005 | 0.0025 | 0.0018 | 0.0023 | 0.0015 | 0.0008 | 0.0021 | 0.0102 |

**Figure 4.5 : Variance - Covariance matrix of excess returns ( S )**

| PORTFOLIO STATISTICS | | |
|---|---|---|
| | $\mu_{hist}$ | $\mu_{CAPM}$ |
| Portfolio Expected Return | 1.57% | 1.19% |
| Portfolio Variance | 0.37% | 0.37% |
| Portfolio Standard deviation | 6.07% | 6.07% |
| Sharpe Ratio | 25.86% | 19.68% |

**Table 4.8 : Statistics of equally weighted portfolio**

Hence the Table 4.8 depicts the equally weighted portfolio's return measures such as the Portfolio expected returns, Portfolio variance, Portfolio standard deviation and the Sharp ratio in both the cases of using Historical method and CAPM method. It is noted that the Sharp ratio is higher in Historical method than in CAPM method such as 25.86% and 19.68% respectively.

### 4.4.4 Establishment of Black- Litterman model inputs

The master formulae of the Black -Litterman method has been given by the equation (1) under the Section 3.3. It requires the parameters τ, S, P, Ω, Π and Q to be input. Then to find Π, it's required to have the value for λ.

Under this section the calculation of all these parameters,except τ and S would be done and then the Black -Litterman equilibrium excess returns would be established. Here τ has been assumed to be 1 and the reasoning for this has been discussed under the section 3.3.5. "S" is the Matrix of Covariance – Variance of the excess returns and it has been calculated above in Section 4.4.3.

### 4.4.5 Lambda

171 data points spanning over 14 years, starting from July 2001 to September 2015, which are monthly closings of MSCI Emerging market index and 1 month US Treasury rates have been considered to find the λ. The monthly Market returns and the Excess market returns have been derived and presented below along with the raw data in Table 4.9. The complete listings of data in Table 4.9 can be found in the Appendix A4.

| Date | MSCI Emerging Markets Index | MSCI_Return (Market return - $r_m$) | US Treasury rates - rf | Excess return |
|---|---|---|---|---|
| 30-09-15 | 792.05 | -0.0326 | 0.0000 | -0.0326 |
| 31-08-15 | 818.73 | -0.0920 | 0.0000 | -0.0920 |
| 31-07-15 | 901.68 | -0.0726 | 0.0400 | -0.1126 |
| 30-06-15 | 972.25 | -0.0318 | 0.0200 | -0.0518 |
| 29-05-15 | 1004.22 | -0.0416 | 0.0100 | -0.0516 |
| 30-04-15 | 1047.78 | 0.0751 | 0.0000 | 0.0751 |
| 31-03-15 | 974.57 | -0.0159 | 0.0500 | -0.0659 |
| 27-02-15 | 990.28 | 0.0298 | 0.0200 | 0.0098 |
| 30-01-15 | 961.61 | 0.0055 | 0.0100 | -0.0045 |
| 31-12-14 | 956.31 | -0.0482 | 0.0300 | -0.0782 |
| 28-11-14 | 1004.72 | -0.0112 | 0.0400 | -0.0512 |
| : | : | : | : | : |
| : | : | : | : | : |
| : | : | : | : | : |
| : | : | : | : | : |
| : | : | : | : | : |
| : | : | : | : | : |
| 31-01-02 | 327.75 | 0.0326 | 1.6900 | -1.6574 |
| 31-12-01 | 317.40 | 0.0780 | 1.6800 | -1.6020 |
| 30-11-01 | 294.43 | 0.1033 | 1.8700 | -1.7667 |
| 31-10-01 | 266.86 | 0.0615 | 2.1500 | -2.0885 |
| 28-09-01 | 251.40 | -0.1569 | 2.2800 | -2.4369 |
| 31-08-01 | 298.17 | -0.0116 | 3.4000 | -3.4116 |
| 31-07-01 | 301.67 | | 3.6700 | |

**Table 4.9 : Inputs for Lambda calculation**

The λ can be calculated by using the equation (8) given under the section 3.3.4. The evaluation of λ has resulted in a value of -0.252. Lambda is a measure of risk aversion. That is the estimation of price of risk. Here it's the average market level of risk aversion that is calculated by considering all individuals across the market. This is the marginal return an investor needs to earn for accepting an extra unit of risk. This would be a positive number in an ideal market condition. To arrive at the value for λ of -0.252, the data was taken over the years starting from July 2001 to September 2015. This comprises the period of 2008 where the financial markets plunged heavily giving deadly negative returns. Hence a negative figure has been obtained for λ.

Thus in this research of Black -Litterman method, it's been assumed a value of 2 for λ to make easy the study and understandings of the results.

### 4.4.6 Market weights

It was sagely prescribed by Black and Litterman that at any given time, in equilibrium, the market portfolio is mean-variance efficient as all investors are maximizing their expected utility. Based on above insight, the Black and Litterman proposed smart procedures for reversing the problem and said that the weights for the market were observed. Because of the assumption it can be recognized that how much it should be invested in each stock in proportion to the total market capitalization of the stocks as it's observed how big a chunk it is the market capitalization of the entire market portfolio. Hence the market capitalizations of the eight stocks were observed from the Stock exchanges where they had got primary listings.

| **Stock** | **Market Capitalization (Bn) USD (As at 31.10.2015)** | **Market Weights** |
|---|---:|---:|
| SAMSUNG | 118.71 | 42.11% |
| CHINA MOB | 69.35 | 24.60% |
| NASPERS | 54.52 | 19.34% |
| EMAR | 13.19 | 4.68% |
| KCHOL | 10.46 | 3.71% |
| AK BANK | 9.63 | 3.42% |
| BRKM5 | 3.11 | 1.10% |
| TAIWAN CE | 2.9614 | 1.05% |
| **Total Market Cap** | **281.9314** | **100%** |

**Table 4.10 : Observed market weights**

To elude the dissimilarities in currencies, all market capitalization chunks had been converted in to the common currency US Dollars and the market weights were calculated. The calculated weights have been laid out in Table 4.10. Here it was assumed that the market portfolio is made only up with these eight stocks.

### 4.4.7 Equilibrium returns

As Black Litterman proposed, having observed the weights (w) it could be reversed to find out the expected excess returns for each stock. According to their proposal, as clearly discussed in section 3.3.4, an expression for the expected excess returns can be found by solving the utility function. Hence the Implied equilibrium excess returns have been found by using the expression given by the equation (9) and have been presented below in Table 4.11.

| Stock | Implied Equilibrium Excess Returns |
|---|---:|
| SAMSUNG | 1.0357% |
| CHINA MOB | 0.8030% |
| NASPERS | 0.9249% |
| EMAR | 1.1571% |
| KCHOL | 1.2222% |
| AK BANK | 1.3098% |
| BRKM5 | 0.7066% |
| TAIWAN CE | 0.5951% |

**Table 4.11 : Implied equilibrium excess return vector**

Thus this is the set of equilibrium returns that clear the market if all investors have identical views. Implied returns are also known as CAPM returns, market returns, equilibrium return, consensus returns, neutral returns and reverse optimized returns.

### 4.4.8 The views

| | |
|---|---|
| **View 1 :** | **SAMSUNG ELECTRONICS will record an absolute excess return of 2.50% per month** |
| **Rationale:** | The competitive Information Technology & Electronics Industry would maintain its superfluous gains in the coming quarters |

| | |
|---|---|
| **View 2 :** | **NASPERS will outperform CHINA MOBILE only by 0.25% per month** |
| **Rationale:** | Economic contraction in China could limit the purchasing power of people, curtail the revenues of Utility companies in China whereas the increasing demand for internet, media group, e-commerce, video entertainment and print would continue to boost Consumer Discretionary sectors all over the world |
| **View 3 :** | **Taiwan Cement Corporation will outperform EMAAR by 1.50% per month** |
| **Rationale:** | An economic slowdown is expected for the upcoming two quarters and this would first hit the Consumption Sector and later the Construction raw material sectors. In addition to this the over supplied real estate market would face struggles in the backdrop of increased Federal Reserve's rates and eventually other countries' interest rate |
| **View 4 :** | **Turkey stocks AKBNK and KCHOL to outperform petro chemical stock BRKM5 and EMAAR by 0.5% per month** |
| **Rationale:** | Turkey undergoes an economic growth due to increased private consumption over the arrival of refugees from Siriya. This would cause a higher credit growth and result the top lines of the Turkish companies to grow. The falling Oil prices would affect the pricing of petro chemical products and would negatively hit in their margins |

**Table 4.12: The views and rationales**

The above column depicts four views that an Asset manager expressed about these eight stocks. He had also articulated the reasoning for his views that is tabulated in the figure.

### 4.4.9 View and Link matrices

The expressed views have to be converted in to numbers which are acceptable to pluck in to the Black –Litterman model. Under this section the metamorphosis of views in to View and Link matrices have been done.

| No. | View Matrix, Q |
|---|---|
| 1 | 2.50% |
| 2 | 0.25% |
| 3 | 1.50% |
| 4 | 0.50% |

**Figure 4.6 : Q Matrix**

| Link Matrix, P | | | | | | | |
|---|---|---|---|---|---|---|---|
| SAMSUNG | CHINA MOB | NASPERS | EMAR | KCHOL | AK BANK | BRKM5 | TAIWAN CE |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | -1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | -1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | -0.81 | 0.52 | 0.48 | -0.19 | 0 |

**Figure 4.7 : P Matrix**

The Asset manager expressed four views about the stocks of emerging market economy. So that the views vector Q has the dimension of 4 × 1contains the monthly expected return figures in each view. Column matrix Q has been given above in Figure 4.6.

Then to incorporate the Views in to the model, a Link matrix P has been constructed in a way discussed under the section 3.3.4.

Here the View 1 is an absolute view that involves only Samsung Electronics Co., Ltd. Hence the weightage have been shown by entering '1' under the column of SAMSUNG corresponding to the row of View 1.

All other views are relative views that involve two or more stocks. In case of relative views the elements of each view must sum up to 0 across all the stocks. View 2 involves two stocks such as Naspers Limited and China Mobile Communications Corporation.Thus Positive 1 has been entered under NASPERS and Negative 1 has been entered under CHINA MOB as the Asset manager is positive about NASPERS and negative about CHINA MOB where the elements across that row sum up to 0. View 3 also works the same as View 2. Hence Positive 1 has been entered under TAIWAN CE and Negative 1 has been entered under EMAR.

View 4 involves sets of two stocks, so that the market capitalization weighting scheme has been applied as discussed under the section 3.3.5 to arrive at the weightages of the stocks KCHOL, AK BANK, EMAR and BRKM5. The Asset manager does have positive views about the stocks KCHOL and AK BANK (Outperforming assets) and he is negative on both EMAR and BRKM5 (Underperforming assets). Consequently the weightages of 52%, 48%, -81% and -19% has been found using the respective market capitalizations as shown in Figures

4.8 and 4.9. It is noted that the positive components of the particular view adds up to 1 while the negative components to -1 allowing all the elements across that row to become 0.

| **View 4 – Nominally "Outperforming" Assets** | | | | |
|---|---|---|---|---|
| **Stock** | **Market Cap. (Bn)** | **Relative Weight** | **Implied Equilibrium Excess Return** | **Weighted Excess Return** |
| **AKBNK** | 9.630 | 47.93% | 1.310% | 0.63% |
| **KCHOL** | 10.460 | 52.07% | 1.222% | 0.64% |
| **TOTAL** | **20.090** | | | **1.2642%** |

**Figure 4.8 : Weighted excess returns of the Outperforming assets**

| **View 4 – Nominally "Underperforming" Assets** | | | | |
|---|---|---|---|---|
| **Stock** | **Market Cap. (Bn)** | **Relative Weight** | **Implied Equilibrium Excess Return** | **Weighted Excess Return** |
| **BRKM5** | 3.11 | 19.08% | 0.7066% | 0.13% |
| **EMAAR** | 13.19 | 80.92% | 1.1571% | 0.94% |
| **TOTAL** | **16.30** | | | **1.0712%** |

**Figure 4.9 : Weighted excess returns of the Underperforming assets**

Further with respect to the relative weightages, by using the implied equilibrium excess returns the individual weighted excess returns of each stock and then the weighted excess returns as the sets of Outperforming and Underperforming assets has been set up. This also has been raised in the Figures 4.8 and 4.9. It's noted that the set of Outperforming stocks yields a weighted excess return of 1.26% while the Underperforming stocks yields only 1.07%.

### 4.4.10 Omega

Having established the structures of P and S, under this section the Ω, uncertainty of views has been derived using the equation (10). Ω would have the dimensions equal to the no. of views and in this case the Ω would be a 4 × 4 matrix as shown in Figure 4.11.

$$\Omega = {}^{2}PSP^{T} \qquad (10)$$

| | | | |
|---|---|---|---|
| 0.519% | 0.073% | 0.065% | 0.205% |
| 0.073% | 0.864% | -0.057% | 0.020% |
| 0.065% | -0.057% | 3.523% | 1.962% |
| 0.205% | 0.020% | 1.962% | 2.477% |

**Figure 4.10 : Ω Matrix**

| | | | |
|---|---|---|---|
| 0.519% | 0.000% | 0.000% | 0.000% |
| 0.000% | 0.864% | 0.000% | 0.000% |
| 0.000% | 0.000% | 3.523% | 0.000% |
| 0.000% | 0.000% | 0.000% | 2.477% |

**Figure 4.11 : Ω Diagonal Matrix**

Since the focal assumption of the Black-Litterman model is that investors' views are distinctive and uncorrelated, except the diagonal elements all other elements of the matrix of Ω would have to be made to 0. Hence a 4 × 4 diagonal-matrix Ω has been obtained as shown in Figure 4.11.

### 4.4.11 Setting up Black- Litterman model

All the Inputs to the master formula of Black Litterman method as given and discussed under the section 3.3 by the equation (1) have been equipped in previous sections. Here the product components of the Black Litterman results have been derived in by two parts as shown by the equation (22).

$$E(r - r_f) = \underbrace{\left[({}^{2}S)^{-1} + P^T\Omega^{-1}P\right]^{-1}}_{1} \underbrace{\left[({}^{2}S)^{-1}\Pi + P^T\Omega^{-1}Q\right]}_{2} \tag{22}$$

| | SAMSUN | CHAMO | NASPES | EMAR | KCHOL | AK BA | BRKM5 | TWN CE |
|---|---|---|---|---|---|---|---|---|
| **SAMSUN** | 0.2566% | 0.0193% | 0.0374% | 0.0339% | 0.0799% | 0.1036% | 0.0596% | 0.0302% |
| **CHIA MO** | 0.0193% | 0.4384% | 0.2433% | 0.2662% | 0.2412% | 0.2525% | 0.1003% | 0.2381% |
| **NASPERS** | 0.0374% | 0.2433% | 0.4789% | 0.2508% | 0.2604% | 0.2461% | 0.1684% | 0.2046% |
| **EMAR** | 0.0339% | 0.2662% | 0.2508% | 1.5062% | 0.5310% | 0.5358% | 0.3077% | 0.4330% |
| **KCHOL** | 0.0799% | 0.2412% | 0.2604% | 0.5310% | 1.0889% | 0.8280% | 0.4426% | 0.2276% |
| **AK BANK** | 0.1036% | 0.2525% | 0.2461% | 0.5358% | 0.8280% | 1.1015% | 0.3718% | 0.1717% |
| **BRKM5** | 0.0596% | 0.1003% | 0.1684% | 0.3077% | 0.4426% | 0.3718% | 1.3486% | 0.2245% |
| **TAIWN C** | 0.0302% | 0.2381% | 0.2046% | 0.4330% | 0.2276% | 0.1717% | 0.2245% | 0.9020% |

**Figure 4.12 : First part of the Black Littterman results**

Figure 4.12 shows the results of section 1 of the Black Litterman master formula. It should be noted that Figure 4.12 is a matrix output with the dimension of 8 × 8.

| | |
|---|---|
| SAMSUNG | 650.10% |
| CHINA MOB | 69.47% |
| NASPERS | 106.28% |
| EMAR | -40.21% |
| KCHOL | 25.34% |
| AK BANK | 23.35% |
| BRKM5 | 0.58% |
| TAIWAN CE | 46.78% |

**Figure 4.13 : Second part of the Black Littterman results**

Figure 4.13 shows the results of section 2 of the Black Litterman master formula. It should be noted that Figure 4.13 is a column matrix output with the dimension of $8 \times 1$.

| **Stocks** | **Black- Litterman Expected Excess Return** |
|---|---|
| SAMSUNG | 1.767% |
| CHINA MOB | 0.814% |
| NASPERS | 1.041% |
| EMAR | 0.530% |
| KCHOL | 1.329% |
| AK BANK | 1.445% |
| BRKM5 | 0.824% |
| TAIWAN CE | 0.926% |

**Figure 4.14 : BL Expected excess return**

Finding the Black Litterman excess return is the whole purpose of this research which is the amalgamation of views in to the implied equilibrium excess return matrix $\pi$, has been derived by doing a matrix multiplication of the results shown by the figures 4.12 and 4.13. The BL Expected excess return is shown in Figure 4.14 above.

| **BL Returns – Nominally "Outperforming" Assets** | | | | |
|---|---|---|---|---|
| **Stock** | **BL portfolio weights** | **Relative Weight** | **BL Expected Excess Return** | **Weighted BL Excess Return** |
| **AKBNK** | 0.014550 | 47.89% | 1.445% | 0.692% |
| **KCHOL** | 0.015834 | 52.11% | 1.329% | 0.692% |
| **TOTAL** | **0.030384** | | | **1.3842%** |

**Figure 4.15 : Weighted BL excess returns of the Outperforming assets**

| BL Returns – Nominally "Underperforming" Assets | | | | |
|---|---|---|---|---|
| **Stock** | **BL portfolio weights** | **Relative Weight** | **BL Expected Excess Return** | **Weighted BL Excess Return** |
| **BRKM5** | 0.01238 | 174.08% | 0.824% | 1.435% |
| **EMAAR** | (0.00527) | -74.08% | 0.530% | -0.392% |
| **TOTAL** | **0.00711** | | | **1.0421%** |

**Figure 4.16 : Weighted BL excess returns of the Underperforming assets**

Figures 4.15 and 4.16 show the weighted BL Excess returns of Outperforming and Underperforming assets' respectively.

## 4.5 Evaluation of Results

| The expected excess return ($\mu$ - $r_f$) | | | | |
|---|---|---|---|---|
| **Stocks** | **Historical** | **CAPM** | **Implied Equilibrium** | **Black- Litterman** |
| SAMSUNG | 0.337% | 0.287% | 1.036% | 1.767% |
| CHINA MOB | 0.647% | 0.427% | 0.803% | 0.814% |
| NASPERS | 2.396% | 0.449% | 0.925% | 1.041% |
| EMAR | 1.407% | 0.686% | 1.157% | 0.530% |
| KCHOL | 1.106% | 0.669% | 1.222% | 1.329% |
| AK BANK | 0.714% | 0.631% | 1.310% | 1.445% |
| BRKM5 | 0.082% | 0.510% | 0.707% | 0.824% |
| TAIWAN CE | 0.540% | 0.567% | 0.595% | 0.926% |

**Table 4.13 : The List of Expected excess returns**

Table 4.13 lists the expected excess returns that have been derived by using different methodologies. Using these excess returns, portfolio allocations have been done and presented in Table 4.15. To arrive at the portfolio weightages, it's required to find the Zs by using the equation (5) as discussed under the section 3.3.2 those are the by solutions of the Utility function. Table 4.14 presents the values of Zs for all three methods such as Historical method, CAPM method and Black Litterman method. It's explicitly known that the market capitalization weights were used to find the implied equilibrium excess returns. Hence market capitalization weights are the corresponding weights for implied equilibrium excess returns.

| Z | | | |
|---|---|---|---|
| **Stocks** | **Historical** | **CAPM** | **BL** |
| SAMSUNG | 0.1833 | 0.3367 | 3.0970 |
| CHINA MOB | 0.3143 | 0.3923 | 0.9574 |
| NASPERS | 4.0010 | 0.3034 | 0.8000 |
| EMAR | 0.2294 | 0.0952 | -0.0285 |
| KCHOL | 0.6426 | 0.1833 | 0.0857 |
| AK BANK | -0.7732 | 0.0464 | 0.0788 |
| BRKM5 | -0.6395 | 0.1216 | 0.0670 |
| TAIWAN CE | -0.2032 | 0.3134 | 0.3553 |
| **Sum** | **3.7548** | **1.7923** | **5.4128** |

**Table 4.14 : The List of Zs**

View 1 is an absolute view that involves the stock SAMSUNG. It can be seen from Table 4.13 that the Implied Equilibrium excess return of SAMSUNG is 1.036% per month, which is 1.464% lower than the Asset managers' expectation of 2.500% per month. Hence after the inclusion of view 1 in to the Implied Equilibrium excess return, the begetting Black Litterman excess return should increase and go towards 2.500%. It's witnessed from the same Table 4.13, that the derived BL return for SAMSUNG is 1.767% per month. The modeled has worked. The excess return has increased. The inclusion of the view caused the portfolio weightage of the stock SAMSUNG to get increase from the neutral market weightage of 42.11% to 57.22% prompting to give higher yield getting closer to Asset managers' view.

View 2 states that the return of NASPERS will be 0.25% greater than the return of CHINA MOBILE per month. In order to gauge whether View 2 will have a positive or negative effect on NASPERS relative to CHINA MOBILE, it is necessary to evaluate the respective implied equilibrium excess returns of the two assets. From Table 4.13, it can be witnessed that the implied equilibrium excess returns for NASPERS and CHINA MOBILE are 0.925% and 0.803%, respectively, recording a difference of 0.122%. The Asset managers' view of 0.25%, (from View 2) is higher than this gap of 0.122%, by which the return of NASPERS exceeds the return of CHINA MOBILE in equilibrium; Thus after the inclusion of view 2 in to the implied equilibrium excess return, the model should tilt the portfolio further away from

CHINA MOBILE in favor of NASPERS to satiate the view. Because the return expectation in view 2 is higher than the difference of the two implied equilibrium excess returns. The BL model works brilliantly by cutting down the portfolio weightage of the underperforming asset CHINA MOBILE from 24.6% to 17.69% (It's witnessed in Table 4.15) persuading the outperforming asset NASPERS to give higher yield of 1.041% compared to its neutral return of 0.925%, adhering to the Asset managers' view.

Since the return expectation in the view is greater than the difference between the two implied equilibrium excess returns, the model erodes the portfolio weightages of the underperforming asset in order to implicitly upsurge the weightage of outperforming asset.

| **UNRESTRICTED PORTFOLIO WEIGHTS** | | | | |
|---|---|---|---|---|
| **Stocks** | **Historical** | **CAPM** | **Market weights** | **BL** |
| SAMSUNG | 4.88% | 18.79% | 42.11% | 57.22% |
| CHINA MOB | 8.37% | 21.89% | 24.60% | 17.69% |
| NASPERS | 106.56% | 16.93% | 19.34% | 14.78% |
| EMAR | 6.11% | 5.31% | 4.68% | -0.53% |
| KCHOL | 17.11% | 10.23% | 3.71% | 1.58% |
| AK BANK | -20.59% | 2.59% | 3.42% | 1.45% |
| BRKM5 | -17.03% | 6.78% | 1.10% | 1.24% |
| TAIWAN CE | -5.41% | 17.49% | 1.05% | 6.56% |
| **Total Weights** | **100.00%** | **100.00%** | **100.00%** | **100.00%** |

**Table 4.15 : The Unrestricted portfolio weights**

Further the View 3 pronounces that the return of TAIWAN CE will be 1.50% greater than the return of EMAR per month. From Table 4.13, it can be observed that the implied equilibrium excess returns for TAIWAN CE and EMAR are 0.595% and 1.157%, respectively, for a difference of -0.562%. The view of 1.50% of outperformance by Taiwan Cement Corporation is greater than 0.562% by which EMAR exceeds the return of Taiwan Cement Corporation so far; thus, it's expected the model to move the portfolio weightage away from EMAR in favor of Taiwan Cement Corporation. The Black Litterman model worked absolutely well to raise portfolio weightage of the outperforming asset TAIWAN CE from 1.05% to 6.56% as depicted in Table 4.15 persuading the TAIWAN CE to give higher yield of

0.926% compared to its equilibrium return of 0.595%, observing the Asset managers' view.

Since the return expectation in the view is greater than the difference between the two implied equilibrium excess returns, the model tilts the portfolio toward the outperforming asset Taiwan Cement Corporation.

It's noted that the BL model cuts down the portfolio weightage of the underperforming asset EMAR from 4.68% to the short selling point of -0.53%. This research does allow the short selling situation.  If it's not permitted in real environment, the portfolio has to be optimized by using the Markowitz theory by counting a non-negativity condition to weightages.

View 4 demonstrates a view involving multiple assets and that the terms "outperforming" and "underperforming" are relative. From View 4 the nominally "outperforming" assets are AKBANK and KCHOL and the nominally "underperforming" assets are BRKM5 and EMAR. From Figure 4.8, the weighted average Implied Equilibrium return of the mini-portfolio formed from AKBANK and KCHOL is 1.264%. And, from Figure 4.9, the weighted average Implied Equilibrium return of the mini-portfolio formed from BRKM5 and EMAR is 1.071% per month. The weighted average Implied Equilibrium return differential is 0.1930%. Since View 4 states that AKBANK and KCHOL will outperform BRKM5 and EMAR by 0.5%, which is higher than the current weighted average Implied Equilibrium differential of 0.1930%, the view appears to actually represent an increment in the performance of AKBANK and KCHOL relative to BRKM5 and EMAR. This point is illustrated in Figures 4.15 and 4.16, where the nominally outperforming assets of View 4 – AKBANK and KCHOL– receive rise in their allocations and the nominally underperforming assets – BRKM5 and EMAR– receive a reduction in their allocations persuading the outperforming assets to give further higher weighted average yield of 1.3842% compared to their equilibrium return of 1.2642%, observing the Asset managers' view. It's also noted that the weighted average yield of the underperforming assets stands down at 1.0421% compared to their equilibrium return of 1.0712% per month.

| PORTFOLIO STATISTICS | | | |
|---|---|---|---|
| | Historical | CAPM | BL |
| Portfolio Expected Return | 2.71% | 0.48% | 1.42% |
| Portfolio Variance | 0.72% | 0.27% | 0.26% |
| Portfolio Standard deviation | 8.49% | 5.17% | 5.12% |
| Sharpe Ratio | 31.89% | 9.26% | 27.71% |

**Table 4.16 : Statistics of Black Litterman weighted portfolio**

Table 4.16 lists the monthly portfolio expected return, related portfolio variance, portfolio standard deviation and sharp ratio of the portfolio of eight stocks. The BL portfolio has recorded a least variance of 5.12% per month when comparing to other two portfolios. Also it has given a moderate return of 1.42% per month that is far better when comparing to the CAPM return of 0.48%. The portfolio constructed through historical method possesses high risk of 8.49%. So that it gives a high return as well. But having an excellent sharp ratio of 27.7% in Black Litterman portfolio is absolutely well above the industry expectation which can be easily achieved in a comfort zone by a novice investment practitioner.

## 4.6 Summary of Findings

| Stocks | BL Expected Excess return BL E[R] | Implied equilibrium Return vector π | Difference BL E[R] - π | BL Weight $W_{BL}$ | Market Capitalization weight Wmkt | Difference $W_{BL}$ -Wmkt |
|---|---|---|---|---|---|---|
| SAMSUNG | 1.767% | 1.036% | 0.731% | 57.22% | 42.11% | 15.11% |
| CHINA MO | 0.814% | 0.803% | 0.011% | 17.69% | 24.60% | -6.91% |
| NASPERS | 1.041% | 0.925% | 0.116% | 14.78% | 19.34% | -4.56% |
| EMAR | 0.530% | 1.157% | -0.627% | -0.53% | 4.68% | -5.21% |
| KCHOL | 1.329% | 1.222% | 0.107% | 1.58% | 3.71% | -2.13% |
| AK BANK | 1.445% | 1.310% | 0.135% | 1.45% | 3.42% | -1.96% |
| BRKM5 | 0.824% | 0.707% | 0.117% | 1.24% | 1.10% | 0.14% |
| TAIWAN CE | 0.926% | 0.595% | 0.331% | 6.56% | 1.05% | 5.51% |

**Table 4.17: Return Vectors and Resulting Portfolio Weights**

The whole aim of implementing the BL method was to find the BL expected excess returns which is the blend of Neutral market returns with Asset managers' views. Then based on these return calculations, it was expected to restructure the portfolio weights persuading to give better yield to the Investors. Thus the BL portfolio has got adjusted itself to carry SAMSUNG, CHINA MOBILE, NASPERS, EMAAR, KCHOL, AK BANK, BRKM5, and TAIWAN CEMENT 57.22%, 17.69%, 14.78%, -0.53%, 1.58%, 1.45%, 1.24% and 6.56% respectively as summarized in Table 4.17. The stock EMAAR had to be cleared out and undergone a short selling by 0.53% out of new BL portfolio. The BL portfolio had given an adequate return of 1.42% per month that is far better when comparing to the CAPM return of 0.48%.

Looking carefully at the numbers of BL expected excess returns and BL weights by comparing to implied equilibrium return and neutral market weights respectively, it had been proved that the Asset manager's views were well absorbed in to the model to reflect the revised portfolio structure, which verified the validity of the Black Litterman method.

## 4.7 Chapter Summary

Under this chapter, the procedures to assess BL method have been approached and setting up the model was done intensely and extensively. Initially the data required for the Black Litterman model evaluation was defined and presented. Then the data manipulation was instigated and primarily the monthly stock return calculations have been carried out. Then the individual stocks' return calculations were done using both historical and CAPM methods. There after the respective portfolio risk return calculations were tried out and the required parameters for this such as the excess returns of the eight stocks and the Variance- Covariance matrix of excess returns have been derived.

Then the inputs required for the Black Litterman method such as $\tau$, S, P, $\Omega$, $\Pi$, $\lambda$ and Q have been found one by one presenting the steps clearly. Later the Views have been well defined with reasoning and the View and Link matrices were established. At last the uncertainties of Views, the diagonal $\Omega$ matrix has been found and all the parameters required for the BL method were plucked in to the model. The most wanted BL expected excess return vector has been resulted and then the new portfolio weights based on BL return vector was allotted. Shifts of portfolio weightages which were caused by the BL method obeying the Asset managers' views have been discussed profoundly. Finally the portfolio parameters in all three cases were studied.

In order to find precise solutions to the required level accuracy and to handle large size of data, it's required to use scientific software. For this purpose the BL model was implemented using the software Matlab. Also some analytical algorithms cannot be used in Ms Excel due to the large size of the data. Thus overcome this problem the inbuilt numerical methods in Matlab were utilized. The complete works in Matlab both the results and coding have been attached in the Appendix A5.

# CHAPTER 5: CONCLUSION AND FURTHER RESEARCH

## 5.1 Introduction

This chapter discusses the ultimate findings of the research comprising the trace of methodology, the data processing procedures, and application of the method, broad view of the method, pros & cons of the methodology and about the possible related researches.

In contrast of presenting the core of the research here, all other previous chapters are slightly referred.

## 5.2 Conclusion

The whole research has been focusing on the easily graspable implementation phases of Black Litterman methodology. Also the paper could well portray the invincible features of the Black Litterman method that includes the incorporation of the investment management practitioner's views or opinions in to the model. In addition to this, the indeterminate natures of the conventional asset allocation practices such as the historical and CAPM methods have been paralleled with the findings of BL methodology. The Black Litterman model evaluation was done by using the monthly closings of eight stocks from emerging market economies by incorporating four views of an Asset manager. The Implementation has been experimented both in Ms Excel 2010 and Matlab. 9 inputs such as the arrays of monthly closing prices of eight stocks, MSCI Emerging market index, 1 month US treasury rates, Market risk free rate, Market risk premium rate, a value for lambda, market capitalization of the stocks, Views vector and link matrix were fed in to the model and executed to arrive at results. Both the obtained Ms Excel and MATLAB results were similar and resulted to the accuracy of up to sixteen decimal points. It is noted that the MATLAB results can be obtained for a superior accuracy of up to the 32 decimal places.

Meritoriously implemented Black Litterman model had displayed the shifts of portfolio masses smartly across the eight stocks,closely following the views of Asset manager. Table 4.17 clearly summarized these classy shifts of portfolio weights. It was clearly professed that, in the absence of constraints and additional observations, if the view is higher than the difference between the two Implied equilibrium returns, the model tilts the portfolio toward the outperforming asset, where as if the view is lesser than the difference between the two Implied equilibrium returns, then the model tilts the portfolio towards the underperforming asset.

The Black Litterman method has many advantages such as allowing asset managers to include their own views about assets, having controls over the confidence level of views etc. over the conservative portfolio allocation theories and its extensions. Apart from that the BL method itself has flexibilities such as including any number of assets in a portfolio, room to include any number of views and inserting portfolio constraints for instance non negativity constraint etc. In contrast the Black-Litterman model would not require giving the best possible portfolio. It purely gives the best portfolio based on the views stated. Hence the stated views should have to be spot-on to well achieve a best yielding portfolio. Also as with any other models, the Black Litterman method is sensitive to assumptions. For instance the model assumes that the views of investors are uncorrelated and the covariance matrix calculated using the historical data have no major impacts on optimal portfolio weights.

## 5.3 Further Research

In line with time constraints and unavailability of data, this paper is restricted to consider a portfolio only with stocks from emerging market economies. But future researches on Black Litterman method can be extended by including assets such as risk free government securities, corporates debts, derivatives of stocks and bonds, currencies, gold ETFs etc. across many countries. This would require the implementation of BL method with specific extensions giving in-depth ideas about the method. Also this paper discussed views for all eight stocks. But doing a study by leaving few assets without having any views could proof the solidarity of the BL model. This paper allowed room for short selling and having non-negativity constraints to stop short selling would lead to a diverse portfolio allocation.

Further a research of building a BL portfolio in a way to observe its performance year-on-year by comparing to the performance of Bench mark index fund year-on-year could be done, which could illustrate the supremacy of BL model.

Prominently extending researches by having asset manager specified confidence levels over views would be worthy as it’s the timely requirement in the investment management industry. A method for incorporating user-specified confidence levels was introduced by Thomas M. Idzorek. This could be applied and extended by financial mathematicians to explore more with Black Litterman methodologies.

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# APPENDICES

## Appendix A1 - Monthly closings of Portfolio constituents and Bench mark index

| MONTH | DATE | SAMSUNG | CHINA_MOB | NASPERS | EMAR | KCHOL | AK_BANK | BRKM5 | TAIWAN_CE | MSCI_INDEX |
|---|---|---|---|---|---|---|---|---|---|---|
| 134 | 9/30/2015 | 1,134,000 | 59.50 | 173,066 | 6.46 | 11.80 | 6.78 | 16.67 | 33.35 | 792.05 |
| 133 | 8/31/2015 | 1,089,000 | 59.82 | 171,900 | 6.75 | 11.35 | 6.82 | 14.07 | 34.85 | 818.73 |
| 132 | 7/31/2015 | 1,185,000 | 65.00 | 177,000 | 7.90 | 12.30 | 7.42 | 12.54 | 34.20 | 901.68 |
| 131 | 6/30/2015 | 1,268,000 | 64.09 | 189,500 | 7.88 | 12.40 | 7.75 | 13.62 | 38.95 | 972.25 |
| 130 | 5/29/2015 | 1,307,000 | 65.74 | 178,505 | 7.75 | 11.90 | 7.93 | 12.97 | 41.70 | 1004.22 |
| 129 | 4/30/2015 | 1,410,000 | 71.43 | 187,103 | 8.24 | 12.65 | 7.79 | 12.60 | 43.60 | 1047.78 |
| 128 | 3/31/2015 | 1,441,000 | 65.03 | 187,000 | 6.60 | 11.85 | 7.65 | 11.00 | 44.10 | 974.57 |
| 127 | 2/27/2015 | 1,357,000 | 67.75 | 171,225 | 7.55 | 11.70 | 8.20 | 12.80 | 42.95 | 990.28 |
| 126 | 1/30/2015 | 1,365,000 | 65.32 | 169,785 | 6.70 | 12.75 | 8.96 | 12.45 | 42.60 | 961.61 |
| 125 | 12/31/2014 | 1,327,000 | 58.82 | 151,512 | 7.26 | 12.40 | 8.66 | 17.50 | 43.40 | 956.31 |
| 124 | 11/28/2014 | 1,287,000 | 61.72 | 143,396 | 10.60 | 12.50 | 8.99 | 19.25 | 45.45 | 1004.72 |
| 123 | 10/31/2014 | 1,244,000 | 62.09 | 137,264 | 10.00 | 11.35 | 8.03 | 18.13 | 46.45 | 1016.07 |
| 122 | 9/30/2014 | 1,184,000 | 58.75 | 124,500 | 11.55 | 10.50 | 7.42 | 16.20 | 45.30 | 1005.33 |
| 121 | 8/29/2014 | 1,234,000 | 62.28 | 135,552 | 11.15 | 11.05 | 8.24 | 15.25 | 47.50 | 1087.88 |
| 120 | 7/31/2014 | 1,343,000 | 54.49 | 133,068 | 9.77 | 11.25 | 8.53 | 14.03 | 44.85 | 1065.77 |
| 119 | 6/30/2014 | 1,322,000 | 48.61 | 125,200 | 8.41 | 10.40 | 7.79 | 14.09 | 45.20 | 1050.78 |
| 118 | 5/30/2014 | 1,443,000 | 49.06 | 116,673 | 10.45 | 10.35 | 8.18 | 14.64 | 44.60 | 1027.69 |
| 117 | 4/30/2014 | 1,343,000 | 47.43 | 99,200 | 9.91 | 9.43 | 7.37 | 15.22 | 47.90 | 995.28 |
| 116 | 3/31/2014 | 1,343,000 | 45.59 | 116,134 | 9.05 | 9.02 | 6.80 | 17.79 | 47.00 | 994.65 |
| 115 | 2/28/2014 | 1,349,000 | 47.55 | 129,878 | 8.27 | 8.18 | 5.72 | 16.15 | 48.30 | 966.42 |
| 114 | 1/31/2014 | 1,280,000 | 47.85 | 113,567 | 7.27 | 7.66 | 5.76 | 18.58 | 44.30 | 936.53 |
| 113 | 12/31/2013 | 1,372,000 | 52.29 | 109,601 | 6.95 | 8.80 | 6.70 | 21.00 | 46.25 | 1002.69 |
| 112 | 11/29/2013 | 1,494,000 | 54.24 | 97,288 | 5.73 | 9.54 | 7.40 | 20.80 | 46.85 | 1018.28 |
| 111 | 10/31/2013 | 1,465,000 | 52.02 | 93,900 | 5.53 | 9.80 | 7.84 | 19.88 | 42.75 | 1034.42 |
| 110 | 9/30/2013 | 1,367,000 | 56.43 | 92,844 | 5.30 | 9.32 | 7.44 | 17.75 | 42.80 | 987.46 |
| 109 | 8/30/2013 | 1,368,000 | 53.97 | 84,837 | 5.23 | 8.24 | 6.78 | 17.55 | 39.50 | 929.54 |
| 108 | 7/31/2013 | 1,280,000 | 52.92 | 82,590 | 5.43 | 8.54 | 7.44 | 17.58 | 36.65 | 947.55 |
| 107 | 6/28/2013 | 1,342,000 | 51.77 | 73,000 | 4.71 | 9.26 | 7.86 | 16.47 | 36.90 | 940.33 |
| 106 | 5/31/2013 | 1,538,000 | 51.80 | 74,513 | 5.43 | 10.65 | 8.94 | 16.71 | 39.45 | 1008.88 |

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|---|---|---|---|---|---|---|---|---|---|---|
| 105 | 4/30/2013 | 1,520,000 | 55.24 | 60,050 | 5.10 | 10.85 | 9.42 | 17.40 | 39.20 | 1039.45 |
| 104 | 3/29/2013 | 1,527,000 | 53.13 | 57,285 | 4.66 | 10.50 | 9.46 | 13.65 | 37.50 | 1034.9 |
| 103 | 2/28/2013 | 1,544,000 | 54.80 | 58,282 | 4.82 | 9.46 | 8.74 | 14.58 | 38.05 | 1054.62 |
| 102 | 1/31/2013 | 1,448,000 | 54.70 | 57,883 | 4.43 | 9.18 | 8.90 | 15.01 | 39.90 | 1069.01 |
| 101 | 12/31/2012 | 1,522,000 | 58.72 | 54,320 | 3.41 | 9.26 | 8.80 | 12.80 | 38.90 | 1055.2 |
| 100 | 11/30/2012 | 1,406,000 | 56.92 | 54,900 | 3.42 | 8.18 | 8.38 | 13.78 | 38.05 | 1007.02 |
| 99 | 10/31/2012 | 1,310,000 | 55.39 | 56,291 | 3.28 | 8.42 | 8.64 | 13.42 | 37.45 | 995.33 |
| 98 | 9/28/2012 | 1,346,000 | 55.36 | 51,497 | 3.16 | 7.18 | 7.10 | 14.27 | 36.30 | 1002.66 |
| 97 | 8/31/2012 | 1,233,000 | 53.69 | 48,900 | 3.01 | 7.40 | 7.18 | 12.80 | 32.55 | 947.33 |
| 96 | 7/31/2012 | 1,309,000 | 58.12 | 44,990 | 3.09 | 7.00 | 6.76 | 12.39 | 34.90 | 952.49 |
| 95 | 6/29/2012 | 1,201,000 | 54.67 | 43,499 | 2.55 | 6.90 | 6.62 | 13.38 | 35.35 | 937.35 |
| 94 | 5/31/2012 | 1,211,000 | 50.73 | 44,889 | 2.65 | 5.56 | 5.70 | 11.25 | 33.75 | 906.3 |
| 93 | 4/30/2012 | 1,390,000 | 55.34 | 46,845 | 2.97 | 6.21 | 6.52 | 13.40 | 34.85 | 1026.02 |
| 92 | 3/30/2012 | 1,275,000 | 55.08 | 43,100 | 2.76 | 6.90 | 7.00 | 14.45 | 34.55 | 1041.45 |
| 91 | 2/29/2012 | 1,206,000 | 53.01 | 41,692 | 2.87 | 6.93 | 7.02 | 15.58 | 37.75 | 1079.44 |
| 90 | 1/31/2012 | 1,107,000 | 51.08 | 39,178 | 2.49 | 6.59 | 6.68 | 15.64 | 36.90 | 1019.39 |
| 89 | 12/30/2011 | 1,058,000 | 48.49 | 35,319 | 2.34 | 5.41 | 6.02 | 12.80 | 35.00 | 916.39 |
| 88 | 11/30/2011 | 1,004,000 | 49.67 | 36,435 | 2.46 | 6.02 | 6.46 | 13.99 | 32.55 | 928.32 |
| 87 | 10/31/2011 | 968,000 | 47.56 | 37,999 | 2.53 | 6.02 | 6.46 | 15.30 | 37.85 | 995 |
| 86 | 9/30/2011 | 840,000 | 48.71 | 35,097 | 2.50 | 6.59 | 7.34 | 14.58 | 32.90 | 880.43 |
| 85 | 8/31/2011 | 744,000 | 51.17 | 36,601 | 2.60 | 5.81 | 6.40 | 18.90 | 38.75 | 1033.15 |
| 84 | 7/29/2011 | 844,000 | 49.83 | 35,820 | 2.62 | 6.74 | 7.38 | 18.39 | 45.60 | 1137.73 |
| 83 | 6/30/2011 | 826,000 | 46.78 | 38,195 | 2.75 | 6.65 | 7.50 | 22.29 | 42.80 | 1146.22 |
| 82 | 5/31/2011 | 902,000 | 45.77 | 40,375 | 2.86 | 6.90 | 7.40 | 24.70 | 41.70 | 1167.97 |
| 81 | 4/29/2011 | 893,000 | 46.09 | 39,500 | 2.99 | 7.79 | 7.90 | 22.90 | 41.95 | 1204.03 |
| 80 | 3/31/2011 | 932,000 | 46.25 | 36,400 | 2.94 | 6.84 | 7.50 | 21.64 | 35.50 | 1170.87 |
| 79 | 2/28/2011 | 923,000 | 47.27 | 39,995 | 2.43 | 6.38 | 7.48 | 20.28 | 29.40 | 1107.77 |
| 78 | 1/31/2011 | 981,000 | 49.14 | 37,300 | 2.77 | 6.23 | 7.52 | 20.97 | 31.35 | 1119.08 |
| 77 | 12/31/2010 | 949,000 | 49.62 | 38,795 | 3.23 | 7.16 | 8.58 | 20.37 | 32.80 | 1151.38 |
| 76 | 11/30/2010 | 826,000 | 49.85 | 35,400 | 3.29 | 6.72 | 8.50 | 18.19 | 31.35 | 1075.85 |
| 75 | 10/29/2010 | 745,000 | 51.37 | 36,782 | 3.50 | 6.52 | 9.00 | 17.44 | 32.19 | 1105.75 |
| 74 | 9/30/2010 | 777,000 | 51.13 | 34,080 | 3.39 | 6.57 | 8.85 | 17.18 | 32.98 | 1075.53 |

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|---|---|---|---|---|---|---|---|---|---|---|
| 73 | 8/31/2010 | 756,000 | 51.32 | 29,778 | 2.96 | 5.67 | 8.10 | 15.48 | 29.08 | 970.05 |
| 72 | 7/30/2010 | 810,000 | 50.94 | 31,100 | 2.96 | 5.67 | 8.35 | 13.21 | 29.43 | 991.41 |
| 71 | 6/30/2010 | 774,000 | 49.41 | 25,990 | 2.78 | 5.14 | 7.65 | 12.53 | 26.96 | 917.99 |
| 70 | 5/31/2010 | 776,000 | 46.57 | 30,130 | 3.05 | 5.19 | 7.80 | 10.79 | 27.01 | 926.4 |
| 69 | 4/30/2010 | 849,000 | 48.90 | 30,000 | 3.56 | 5.48 | 7.45 | 12.57 | 27.95 | 1020.03 |
| 68 | 3/31/2010 | 818,000 | 48.12 | 31,650 | 3.64 | 4.95 | 7.35 | 13.09 | 29.57 | 1010.33 |
| 67 | 2/26/2010 | 744,000 | 49.43 | 28,800 | 2.71 | 4.38 | 5.85 | 12.78 | 29.23 | 935.93 |
| 66 | 1/29/2010 | 784,000 | 46.95 | 27,000 | 2.79 | 4.86 | 6.60 | 13.26 | 29.48 | 933.59 |
| 65 | 12/31/2009 | 799,000 | 46.43 | 30,000 | 3.51 | 4.21 | 7.09 | 14.08 | 33.57 | 989.47 |
| 64 | 11/30/2009 | 720,000 | 46.87 | 27,731 | 3.41 | 3.56 | 6.15 | 11.52 | 33.13 | 953.13 |
| 63 | 10/30/2009 | 723,000 | 46.73 | 28,444 | 3.99 | 3.66 | 6.15 | 11.70 | 33.57 | 914.26 |
| 62 | 9/30/2009 | 815,000 | 49.11 | 25,667 | 3.68 | 3.71 | 6.45 | 11.26 | 35.65 | 914.05 |
| 61 | 8/31/2009 | 771,000 | 49.22 | 25,450 | 3.16 | 3.79 | 6.45 | 10.46 | 35.25 | 839.46 |
| 60 | 7/31/2009 | 724,000 | 52.47 | 23,150 | 2.49 | 3.30 | 6.23 | 8.31 | 34.31 | 844.02 |
| 59 | 6/30/2009 | 592,000 | 50.08 | 20,300 | 2.53 | 2.53 | 5.18 | 7.18 | 31.01 | 761.3 |
| 58 | 5/29/2009 | 558,000 | 49.21 | 19,061 | 3.16 | 2.79 | 4.84 | 7.16 | 33.57 | 773.12 |
| 57 | 4/30/2009 | 592,000 | 43.16 | 17,382 | 2.14 | 2.32 | 4.65 | 6.17 | 31.20 | 662.73 |
| 56 | 3/31/2009 | 568,000 | 43.52 | 16,000 | 2.00 | 1.86 | 3.66 | 4.81 | 27.60 | 569.97 |
| 55 | 2/27/2009 | 477,000 | 43.35 | 15,400 | 1.87 | 1.68 | 2.94 | 5.38 | 25.87 | 499.3 |
| 54 | 1/30/2009 | 488,000 | 44.97 | 15,705 | 1.78 | 1.83 | 3.59 | 5.71 | 22.07 | 529.53 |
| 53 | 12/31/2008 | 451,000 | 50.85 | 16,625 | 2.06 | 2.08 | 3.59 | 5.55 | 26.61 | 567.04 |
| 52 | 11/28/2008 | 486,000 | 45.83 | 15,420 | 2.66 | 2.00 | 3.23 | 6.26 | 24.39 | 526.97 |
| 51 | 10/31/2008 | 535,000 | 43.89 | 16,200 | 4.86 | 2.25 | 3.92 | 9.41 | 16.74 | 570.52 |
| 50 | 9/30/2008 | 539,000 | 50.08 | 16,100 | 7.05 | 3.11 | 4.91 | 10.17 | 18.42 | 786.92 |
| 49 | 8/29/2008 | 516,000 | 56.72 | 19,375 | 8.28 | 3.29 | 4.61 | 11.68 | 30.71 | 956.25 |
| 48 | 7/31/2008 | 564,000 | 66.85 | 17,929 | 9.55 | 3.70 | 4.88 | 13.80 | 38.81 | 1041.86 |
| 47 | 6/30/2008 | 625,000 | 66.95 | 17,100 | 9.86 | 2.65 | 3.18 | 12.76 | 40.09 | 1087.12 |
| 46 | 5/30/2008 | 741,000 | 73.79 | 17,750 | 10.41 | 2.73 | 4.05 | 13.24 | 49.86 | 1210.04 |
| 45 | 4/30/2008 | 711,000 | 86.32 | 16,426 | 10.46 | 3.04 | 4.99 | 14.32 | 48.40 | 1191.53 |
| 44 | 3/31/2008 | 623,000 | 75.01 | 14,100 | 9.96 | 2.57 | 4.20 | 14.90 | 57.59 | 1104.58 |
| 43 | 2/29/2008 | 560,000 | 74.62 | 14,919 | 11.32 | 3.20 | 4.91 | 14.34 | 53.77 | 1167.66 |
| 42 | 1/31/2008 | 595,000 | 75.52 | 13,700 | 10.91 | 3.15 | 5.06 | 13.00 | 40.87 | 1088.72 |

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|---|---|---|---|---|---|---|---|---|---|---|
| 41 | 12/31/2007 | 556,000 | 86.87 | 16,200 | 13.55 | 4.38 | 6.53 | 14.40 | 44.00 | 1245.59 |
| 40 | 11/30/2007 | 565,000 | 91.66 | 18,000 | 11.27 | 4.14 | 6.45 | 15.75 | 46.00 | 1242.06 |
| 39 | 10/31/2007 | 550,000 | 103.68 | 20,600 | 11.73 | 4.38 | 7.95 | 16.31 | 53.97 | 1337.63 |
| 38 | 9/28/2007 | 575,000 | 82.04 | 19,102 | 9.68 | 4.38 | 6.90 | 17.27 | 51.72 | 1204.9 |
| 37 | 8/31/2007 | 591,000 | 67.79 | 17,910 | 9.91 | 4.11 | 6.23 | 18.10 | 41.45 | 1086.98 |
| 36 | 7/31/2007 | 614,000 | 57.39 | 17,900 | 9.91 | 4.14 | 6.64 | 17.27 | 38.96 | 1112.77 |
| 35 | 6/29/2007 | 566,000 | 53.90 | 18,200 | 10.77 | 3.62 | 5.48 | 17.26 | 36.88 | 1059.69 |
| 34 | 5/31/2007 | 535,000 | 46.42 | 18,795 | 11.09 | 3.83 | 5.55 | 16.40 | 30.11 | 1014.78 |
| 33 | 4/30/2007 | 574,000 | 45.01 | 17,900 | 9.77 | 3.56 | 6.00 | 17.35 | 28.80 | 969.93 |
| 32 | 3/30/2007 | 563,000 | 44.85 | 17,550 | 10.05 | 3.33 | 5.58 | 15.21 | 27.01 | 929.03 |
| 31 | 2/28/2007 | 567,000 | 46.47 | 17,150 | 11.68 | 3.25 | 5.58 | 13.78 | 26.77 | 895.54 |
| 30 | 1/31/2007 | 579,000 | 46.15 | 18,209 | 11.50 | 3.17 | 5.40 | 15.53 | 27.30 | 901.48 |
| 29 | 12/29/2006 | 613,000 | 43.22 | 16,600 | 11.09 | 2.90 | 5.16 | 15.00 | 28.51 | 912.65 |
| 28 | 11/30/2006 | 638,000 | 42.20 | 14,600 | 10.73 | 2.63 | 4.89 | 15.78 | 28.41 | 874.08 |
| 27 | 10/31/2006 | 611,000 | 40.78 | 13,330 | 12.59 | 2.98 | 4.98 | 15.15 | 25.41 | 814.44 |
| 26 | 9/29/2006 | 664,000 | 35.35 | 11,960 | 12.73 | 2.57 | 4.65 | 13.53 | 24.01 | 778.16 |
| 25 | 8/31/2006 | 650,000 | 33.31 | 12,450 | 12.82 | 2.88 | 4.80 | 13.95 | 21.25 | 773.12 |
| 24 | 7/31/2006 | 608,000 | 32.33 | 12,185 | 10.05 | 2.75 | 4.32 | 11.30 | 23.02 | 755.84 |
| 23 | 6/30/2006 | 603,000 | 28.61 | 12,187 | 9.91 | 2.49 | 4.56 | 13.29 | 22.68 | 747.54 |
| 22 | 5/31/2006 | 612,000 | 25.85 | 12,300 | 11.46 | 2.93 | 4.57 | 14.12 | 25.92 | 751 |
| 21 | 4/28/2006 | 644,000 | 28.86 | 13,200 | 13.09 | 3.48 | 5.40 | 14.70 | 23.83 | 841.58 |
| 20 | 3/31/2006 | 630,000 | 26.54 | 12,550 | 16.86 | 3.43 | 5.55 | 16.12 | 23.83 | 787.8 |
| 19 | 2/28/2006 | 687,000 | 24.23 | 12,255 | 16.73 | 3.84 | 6.58 | 18.24 | 23.35 | 782.11 |
| 18 | 1/31/2006 | 740,000 | 24.60 | 13,000 | 18.14 | 3.50 | 5.65 | 17.02 | 22.78 | 783.77 |
| 17 | 12/30/2005 | 659,000 | 24.04 | 11,201 | 20.73 | 3.05 | 5.40 | 19.00 | 22.35 | 706.48 |
| 16 | 11/30/2005 | 598,000 | 24.49 | 10,402 | 21.59 | 3.19 | 5.30 | 19.90 | 19.97 | 667.99 |
| 15 | 10/31/2005 | 552,000 | 22.45 | 9,685 | 24.23 | 2.42 | 4.12 | 19.40 | 19.63 | 617.41 |
| 14 | 9/30/2005 | 588,000 | 24.64 | 10,399 | 24.50 | 2.93 | 4.39 | 22.99 | 19.97 | 661.32 |
| 13 | 8/31/2005 | 544,000 | 21.85 | 10,400 | 20.68 | 3.00 | 3.90 | 23.43 | 18.43 | 606.23 |
| 12 | 7/29/2005 | 565,000 | 20.17 | 9,605 | 15.50 | 2.63 | 3.61 | 22.80 | 17.97 | 602.56 |
| 11 | 6/30/2005 | 494,000 | 18.59 | 8,300 | 20.60 | 2.52 | 3.15 | 19.55 | 17.84 | 565.17 |
| 10 | 5/31/2005 | 489,000 | 18.26 | 8,350 | 11.87 | 2.48 | 2.99 | 22.96 | 16.20 | 548.15 |

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|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 4/29/2005 | 452,000 | 17.80 | 7,318 | 11.08 | 2.22 | 2.72 | 24.57 | 17.25 | 531.99 |
| 8 | 3/31/2005 | 502,000 | 16.41 | 7,121 | 5.91 | 2.52 | 2.68 | 27.07 | 16.61 | 548.69 |
| 7 | 2/28/2005 | 527,000 | 16.20 | 7,786 | 5.92 | 2.94 | 3.27 | 32.75 | 18.34 | 588.68 |
| 6 | 1/31/2005 | 495,000 | 15.74 | 7,485 | 5.44 | 2.92 | 3.42 | 29.19 | 18.25 | 542.28 |
| 5 | 12/31/2004 | 450,500 | 17.16 | 7,500 | 5.23 | 2.97 | 3.42 | 33.50 | 18.97 | 542.17 |
| 4 | 11/30/2004 | 434,500 | 16.32 | 6,690 | 4.12 | 2.65 | 2.78 | 29.60 | 17.61 | 517.95 |
| 3 | 10/29/2004 | 439,500 | 14.55 | 5,620 | 3.19 | 3.02 | 2.72 | 26.25 | 16.43 | 474.27 |
| 2 | 9/30/2004 | 458,000 | 15.30 | 5,121 | 3.24 | 2.97 | 2.78 | 23.80 | 17.07 | 464.15 |
| 1 | 8/31/2004 | 451,000 | 14.62 | 4,896 | 2.49 | 2.75 | 2.62 | 21.20 | 15.43 | 439.75 |
| 0 | 7/30/2004 | 417,000 | 14.52 | 4,500 | 2.38 | 2.55 | 2.48 | 15.30 | 13.11 | 423.14 |

**Table 4.2 : Monthly closings of the eight stocks and the MSCI Emerging market index**

## Appendix A2 - Monthly returns of Portfolio constituents and Bench mark index

| MONTH | DATE | RETSAM | RETCHINA | RETNASP | RETEMAR | RETKCHOL | RETAKBANK | RETBRKM | RETTAIWAN | RETINDEX |
|---|---|---|---|---|---|---|---|---|---|---|
| 134 | 09/30/15 | 0.0413 | -0.0053 | 0.0068 | -0.0430 | 0.0396 | -0.0059 | 0.1848 | -0.0430 | -0.0326 |
| 133 | 08/31/15 | -0.0810 | -0.0797 | -0.0288 | -0.1456 | -0.0772 | -0.0809 | 0.1220 | 0.0190 | -0.0920 |
| 132 | 07/31/15 | -0.0655 | 0.0142 | -0.0660 | 0.0025 | -0.0081 | -0.0426 | -0.0793 | -0.1220 | -0.0726 |
| 131 | 06/30/15 | -0.0298 | -0.0251 | 0.0616 | 0.0168 | 0.0420 | -0.0227 | 0.0501 | -0.0659 | -0.0318 |
| 130 | 05/29/15 | -0.0730 | -0.0797 | -0.0460 | -0.0595 | -0.0593 | 0.0180 | 0.0294 | -0.0436 | -0.0416 |
| 129 | 04/30/15 | -0.0215 | 0.0984 | 0.0006 | 0.2485 | 0.0675 | 0.0183 | 0.1455 | -0.0113 | 0.0751 |
| 128 | 03/31/15 | 0.0619 | -0.0401 | 0.0921 | -0.1258 | 0.0128 | -0.0671 | -0.1406 | 0.0268 | -0.0159 |
| 127 | 02/27/15 | -0.0059 | 0.0372 | 0.0085 | 0.1269 | -0.0824 | -0.0848 | 0.0281 | 0.0082 | 0.0298 |
| 126 | 01/30/15 | 0.0286 | 0.1105 | 0.1206 | -0.0771 | 0.0282 | 0.0346 | -0.2886 | -0.0184 | 0.0055 |
| 125 | 12/31/14 | 0.0311 | -0.0470 | 0.0566 | -0.3151 | -0.0080 | -0.0367 | -0.0909 | -0.0451 | -0.0482 |
| 124 | 11/28/14 | 0.0346 | -0.0060 | 0.0447 | 0.0600 | 0.1013 | 0.1196 | 0.0618 | -0.0215 | -0.0112 |
| 123 | 10/31/14 | 0.0507 | 0.0569 | 0.1025 | -0.1342 | 0.0810 | 0.0822 | 0.1191 | 0.0254 | 0.0107 |
| 122 | 09/30/14 | -0.0405 | -0.0567 | -0.0815 | 0.0359 | -0.0498 | -0.0995 | 0.0623 | -0.0463 | -0.0759 |
| 121 | 08/29/14 | -0.0812 | 0.1430 | 0.0187 | 0.1412 | -0.0178 | -0.0340 | 0.0870 | 0.0591 | 0.0207 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 120 | 07/31/14 | 0.0159 | 0.1210 | 0.0628 | 0.1617 | 0.0817 | 0.0950 | -0.0043 | -0.0077 | 0.0143 |
| 119 | 06/30/14 | -0.0839 | -0.0092 | 0.0731 | -0.1952 | 0.0048 | -0.0477 | -0.0376 | 0.0135 | 0.0225 |
| 118 | 05/30/14 | 0.0745 | 0.0344 | 0.1761 | 0.0546 | 0.0976 | 0.1099 | -0.0381 | -0.0689 | 0.0326 |
| 117 | 04/30/14 | 0.0000 | 0.0404 | -0.1458 | 0.0955 | 0.0455 | 0.0838 | -0.1445 | 0.0191 | 0.0006 |
| 116 | 03/31/14 | -0.0044 | -0.0412 | -0.1058 | 0.0933 | 0.1027 | 0.1888 | 0.1015 | -0.0269 | 0.0292 |
| 115 | 02/28/14 | 0.0539 | -0.0063 | 0.1436 | 0.1375 | 0.0679 | -0.0069 | -0.1308 | 0.0903 | 0.0319 |
| 114 | 01/31/14 | -0.0671 | -0.0849 | 0.0362 | 0.0472 | -0.1295 | -0.1403 | -0.1152 | -0.0422 | -0.0660 |
| 113 | 12/31/13 | -0.0817 | -0.0360 | 0.1266 | 0.2127 | -0.0776 | -0.0946 | 0.0096 | -0.0128 | -0.0153 |
| 112 | 11/29/13 | 0.0198 | 0.0427 | 0.0361 | 0.0362 | -0.0265 | -0.0561 | 0.0463 | 0.0959 | -0.0156 |
| 111 | 10/31/13 | 0.0717 | -0.0781 | 0.0114 | 0.0428 | 0.0515 | 0.0538 | 0.1200 | -0.0012 | 0.0476 |
| 110 | 09/30/13 | -0.0007 | 0.0456 | 0.0944 | 0.0140 | 0.1311 | 0.0973 | 0.0114 | 0.0835 | 0.0623 |
| 109 | 08/30/13 | 0.0688 | 0.0198 | 0.0272 | -0.0369 | -0.0351 | -0.0887 | -0.0017 | 0.0778 | -0.0190 |
| 108 | 07/31/13 | -0.0462 | 0.0222 | 0.1314 | 0.1525 | -0.0778 | -0.0534 | 0.0674 | -0.0068 | 0.0077 |
| 107 | 06/28/13 | -0.1274 | -0.0006 | -0.0203 | -0.1323 | -0.1305 | -0.1208 | -0.0144 | -0.0646 | -0.0679 |
| 106 | 05/31/13 | 0.0118 | -0.0623 | 0.2408 | 0.0641 | -0.0184 | -0.0510 | -0.0397 | 0.0064 | -0.0294 |
| 105 | 04/30/13 | -0.0046 | 0.0397 | 0.0483 | 0.0956 | 0.0333 | -0.0042 | 0.2747 | 0.0453 | 0.0044 |
| 104 | 03/29/13 | -0.0110 | -0.0305 | -0.0171 | -0.0338 | 0.1099 | 0.0824 | -0.0638 | -0.0145 | -0.0187 |
| 103 | 02/28/13 | 0.0663 | 0.0018 | 0.0069 | 0.0883 | 0.0305 | -0.0180 | -0.0286 | -0.0464 | -0.0135 |
| 102 | 01/31/13 | -0.0486 | -0.0685 | 0.0656 | 0.2986 | -0.0086 | 0.0114 | 0.1727 | 0.0257 | 0.0131 |
| 101 | 12/31/12 | 0.0825 | 0.0316 | -0.0106 | -0.0026 | 0.1320 | 0.0501 | -0.0711 | 0.0223 | 0.0478 |
| 100 | 11/30/12 | 0.0733 | 0.0276 | -0.0247 | 0.0414 | -0.0285 | -0.0301 | 0.0268 | 0.0160 | 0.0117 |
| 99 | 10/31/12 | -0.0267 | 0.0005 | 0.0931 | 0.0373 | 0.1727 | 0.2169 | -0.0596 | 0.0317 | -0.0073 |
| 98 | 09/28/12 | 0.0916 | 0.0311 | 0.0531 | 0.0515 | -0.0297 | -0.0111 | 0.1148 | 0.1152 | 0.0584 |
| 97 | 08/31/12 | -0.0581 | -0.0762 | 0.0869 | -0.0265 | 0.0571 | 0.0621 | 0.0331 | -0.0673 | -0.0054 |
| 96 | 07/31/12 | 0.0899 | 0.0631 | 0.0343 | 0.2145 | 0.0145 | 0.0211 | -0.0740 | -0.0127 | 0.0162 |
| 95 | 06/29/12 | -0.0083 | 0.0777 | -0.0310 | -0.0378 | 0.2406 | 0.1614 | 0.1893 | 0.0474 | 0.0343 |
| 94 | 05/31/12 | -0.1288 | -0.0833 | -0.0418 | -0.1103 | -0.1043 | -0.1258 | -0.1604 | -0.0316 | -0.1167 |
| 93 | 04/30/12 | 0.0902 | 0.0047 | 0.0869 | 0.0791 | -0.0993 | -0.0686 | -0.0727 | 0.0087 | -0.0148 |
| 92 | 03/30/12 | 0.0572 | 0.0390 | 0.0338 | -0.0411 | -0.0055 | -0.0028 | -0.0725 | -0.0848 | -0.0352 |
| 91 | 02/29/12 | 0.0894 | 0.0378 | 0.0642 | 0.1534 | 0.0520 | 0.0509 | -0.0038 | 0.0230 | 0.0589 |
| 90 | 01/31/12 | 0.0463 | 0.0534 | 0.1093 | 0.0664 | 0.2181 | 0.1096 | 0.2219 | 0.0543 | 0.1124 |
| 89 | 12/30/11 | 0.0538 | -0.0238 | -0.0306 | -0.0519 | -0.1012 | -0.0681 | -0.0851 | 0.0753 | -0.0129 |

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|---|---|---|---|---|---|---|---|---|---|---|
| 88 | 11/30/11 | 0.0372 | 0.0444 | -0.0412 | -0.0249 | 0.0000 | 0.0000 | -0.0856 | -0.1400 | -0.0670 |
| 87 | 10/31/11 | 0.1524 | -0.0236 | 0.0827 | 0.0108 | -0.0866 | -0.1199 | 0.0494 | 0.1505 | 0.1301 |
| 86 | 09/30/11 | 0.1290 | -0.0481 | -0.0411 | -0.0385 | 0.1343 | 0.1469 | -0.2286 | -0.1510 | -0.1478 |
| 85 | 08/31/11 | -0.1185 | 0.0269 | 0.0218 | -0.0069 | -0.1384 | -0.1328 | 0.0277 | -0.1502 | -0.0919 |
| 84 | 07/29/11 | 0.0218 | 0.0652 | -0.0622 | -0.0463 | 0.0143 | -0.0160 | -0.1750 | 0.0654 | -0.0074 |
| 83 | 06/30/11 | -0.0843 | 0.0221 | -0.0540 | -0.0416 | -0.0358 | 0.0135 | -0.0976 | 0.0264 | -0.0186 |
| 82 | 05/31/11 | 0.0101 | -0.0069 | 0.0222 | -0.0425 | -0.1149 | -0.0633 | 0.0786 | -0.0060 | -0.0299 |
| 81 | 04/29/11 | -0.0418 | -0.0035 | 0.0852 | 0.0187 | 0.1392 | 0.0533 | 0.0582 | 0.1817 | 0.0283 |
| 80 | 03/31/11 | 0.0098 | -0.0216 | -0.0899 | 0.2097 | 0.0716 | 0.0027 | 0.0671 | 0.2075 | 0.0570 |
| 79 | 02/28/11 | -0.0591 | -0.0381 | 0.0723 | -0.1248 | 0.0244 | -0.0053 | -0.0329 | -0.0622 | -0.0101 |
| 78 | 01/31/11 | 0.0337 | -0.0097 | -0.0385 | -0.1407 | -0.1303 | -0.1235 | 0.0295 | -0.0442 | -0.0281 |
| 77 | 12/31/10 | 0.1489 | -0.0046 | 0.0959 | -0.0194 | 0.0651 | 0.0094 | 0.1198 | 0.0463 | 0.0702 |
| 76 | 11/30/10 | 0.1087 | -0.0296 | -0.0376 | -0.0597 | 0.0307 | -0.0556 | 0.0430 | -0.0261 | -0.0270 |
| 75 | 10/29/10 | -0.0412 | 0.0047 | 0.0793 | 0.0321 | -0.0072 | 0.0169 | 0.0151 | -0.0240 | 0.0281 |
| 74 | 09/30/10 | 0.0278 | -0.0037 | 0.1445 | 0.1441 | 0.1595 | 0.0926 | 0.1098 | 0.1341 | 0.1087 |
| 73 | 08/31/10 | -0.0667 | 0.0075 | -0.0425 | 0.0000 | 0.0000 | -0.0299 | 0.1718 | -0.0119 | -0.0215 |
| 72 | 07/30/10 | 0.0465 | 0.0310 | 0.1966 | 0.0654 | 0.1019 | 0.0915 | 0.0543 | 0.0916 | 0.0800 |
| 71 | 06/30/10 | -0.0026 | 0.0610 | -0.1374 | -0.0864 | -0.0091 | -0.0192 | 0.1613 | -0.0019 | -0.0091 |
| 70 | 05/31/10 | -0.0860 | -0.0476 | 0.0043 | -0.1435 | -0.0522 | 0.0470 | -0.1416 | -0.0336 | -0.0918 |
| 69 | 04/30/10 | 0.0379 | 0.0162 | -0.0521 | -0.0223 | 0.1058 | 0.0136 | -0.0397 | -0.0548 | 0.0096 |
| 68 | 03/31/10 | 0.0995 | -0.0265 | 0.0990 | 0.3422 | 0.1303 | 0.2564 | 0.0243 | 0.0116 | 0.0795 |
| 67 | 02/26/10 | -0.0510 | 0.0528 | 0.0667 | -0.0294 | -0.0980 | -0.1136 | -0.0362 | -0.0085 | 0.0025 |
| 66 | 01/29/10 | -0.0188 | 0.0112 | -0.1000 | -0.2046 | 0.1537 | -0.0688 | -0.0582 | -0.1218 | -0.0565 |
| 65 | 12/31/09 | 0.1097 | -0.0094 | 0.0818 | 0.0293 | 0.1819 | 0.1525 | 0.2222 | 0.0133 | 0.0381 |
| 64 | 11/30/09 | -0.0041 | 0.0030 | -0.0251 | -0.1458 | -0.0260 | 0.0000 | -0.0154 | -0.0131 | 0.0425 |
| 63 | 10/30/09 | -0.1129 | -0.0485 | 0.1082 | 0.0839 | -0.0153 | -0.0465 | 0.0391 | -0.0583 | 0.0002 |
| 62 | 09/30/09 | 0.0571 | -0.0022 | 0.0085 | 0.1670 | -0.0201 | 0.0000 | 0.0765 | 0.0113 | 0.0889 |
| 61 | 08/31/09 | 0.0649 | -0.0619 | 0.0994 | 0.2666 | 0.1502 | 0.0361 | 0.2587 | 0.0274 | -0.0054 |
| 60 | 07/31/09 | 0.2230 | 0.0477 | 0.1404 | -0.0142 | 0.3008 | 0.2029 | 0.1574 | 0.1064 | 0.1087 |
| 59 | 06/30/09 | 0.0609 | 0.0177 | 0.0650 | -0.1990 | -0.0934 | 0.0697 | 0.0028 | -0.0763 | -0.0153 |
| 58 | 05/29/09 | -0.0574 | 0.1402 | 0.0966 | 0.4771 | 0.2059 | 0.0404 | 0.1605 | 0.0760 | 0.1666 |
| 57 | 04/30/09 | 0.0423 | -0.0083 | 0.0864 | 0.0680 | 0.2477 | 0.2705 | 0.2827 | 0.1304 | 0.1627 |

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|---|---|---|---|---|---|---|---|---|---|---|
| 56 | 03/31/09 | 0.1908 | 0.0039 | 0.0390 | 0.0678 | 0.1034 | 0.2449 | -0.1059 | 0.0669 | 0.1415 |
| 55 | 02/27/09 | -0.0225 | -0.0360 | -0.0194 | 0.0511 | -0.0818 | -0.1799 | -0.0578 | 0.1722 | -0.0571 |
| 54 | 01/30/09 | 0.0820 | -0.1156 | -0.0553 | -0.1328 | -0.1183 | 0.0000 | 0.0288 | -0.1706 | -0.0662 |
| 53 | 12/31/08 | -0.0720 | 0.1095 | 0.0781 | -0.2286 | 0.0395 | 0.1116 | -0.1134 | 0.0910 | 0.0760 |
| 52 | 11/28/08 | -0.0916 | 0.0442 | -0.0481 | -0.4513 | -0.1127 | -0.1762 | -0.3348 | 0.4570 | -0.0763 |
| 51 | 10/31/08 | -0.0074 | -0.1236 | 0.0062 | -0.3109 | -0.2755 | -0.2031 | -0.0747 | -0.0912 | -0.2750 |
| 50 | 09/30/08 | 0.0446 | -0.1171 | -0.1690 | -0.1494 | -0.0533 | 0.0650 | -0.1293 | -0.4002 | -0.1771 |
| 49 | 08/29/08 | -0.0851 | -0.1515 | 0.0807 | -0.1323 | -0.1114 | -0.0537 | -0.1536 | -0.2087 | -0.0822 |
| 48 | 07/31/08 | -0.0976 | -0.0015 | 0.0485 | -0.0323 | 0.3949 | 0.5330 | 0.0815 | -0.0319 | -0.0416 |
| 47 | 06/30/08 | -0.1565 | -0.0927 | -0.0366 | -0.0524 | -0.0289 | -0.2148 | -0.0363 | -0.1959 | -0.1016 |
| 46 | 05/30/08 | 0.0422 | -0.1452 | 0.0806 | -0.0044 | -0.1011 | -0.1881 | -0.0754 | 0.0302 | 0.0155 |
| 45 | 04/30/08 | 0.1413 | 0.1508 | 0.1650 | 0.0502 | 0.1831 | 0.1876 | -0.0389 | -0.1596 | 0.0787 |
| 44 | 03/31/08 | 0.1125 | 0.0052 | -0.0549 | -0.1204 | -0.1983 | -0.1451 | 0.0391 | 0.0710 | -0.0540 |
| 43 | 02/29/08 | -0.0588 | -0.0119 | 0.0890 | 0.0375 | 0.0175 | -0.0296 | 0.1031 | 0.3156 | 0.0725 |
| 42 | 01/31/08 | 0.0701 | -0.1307 | -0.1543 | -0.1946 | -0.2818 | -0.2241 | -0.0972 | -0.0711 | -0.1259 |
| 41 | 12/31/07 | -0.0159 | -0.0523 | -0.1000 | 0.2015 | 0.0582 | 0.0116 | -0.0857 | -0.0435 | 0.0028 |
| 40 | 11/30/07 | 0.0273 | -0.1159 | -0.1262 | -0.0387 | -0.0550 | -0.1887 | -0.0343 | -0.1477 | -0.0714 |
| 39 | 10/31/07 | -0.0435 | 0.2638 | 0.0784 | 0.2112 | 0.0000 | 0.1522 | -0.0556 | 0.0435 | 0.1102 |
| 38 | 09/28/07 | -0.0271 | 0.2102 | 0.0666 | -0.0229 | 0.0672 | 0.1084 | -0.0459 | 0.2478 | 0.1085 |
| 37 | 08/31/07 | -0.0375 | 0.1812 | 0.0006 | 0.0000 | -0.0085 | -0.0622 | 0.0481 | 0.0639 | -0.0232 |
| 36 | 07/31/07 | 0.0848 | 0.0647 | -0.0165 | -0.0802 | 0.1430 | 0.2124 | 0.0006 | 0.0564 | 0.0501 |
| 35 | 06/29/07 | 0.0579 | 0.1611 | -0.0317 | -0.0287 | -0.0533 | -0.0135 | 0.0524 | 0.2248 | 0.0443 |
| 34 | 05/31/07 | -0.0679 | 0.0313 | 0.0500 | 0.1349 | 0.0741 | -0.0750 | -0.0548 | 0.0455 | 0.0462 |
| 33 | 04/30/07 | 0.0195 | 0.0036 | 0.0199 | -0.0271 | 0.0716 | 0.0753 | 0.1407 | 0.0663 | 0.0440 |
| 32 | 03/30/07 | -0.0071 | -0.0349 | 0.0233 | -0.1401 | 0.0243 | 0.0000 | 0.1038 | 0.0090 | 0.0374 |
| 31 | 02/28/07 | -0.0207 | 0.0069 | -0.0582 | 0.0158 | 0.0249 | 0.0333 | -0.1127 | -0.0194 | -0.0066 |
| 30 | 01/31/07 | -0.0555 | 0.0678 | 0.0969 | 0.0369 | 0.0909 | 0.0465 | 0.0353 | -0.0424 | -0.0122 |
| 29 | 12/29/06 | -0.0392 | 0.0242 | 0.1370 | 0.0339 | 0.1042 | 0.0552 | -0.0494 | 0.0035 | 0.0441 |
| 28 | 11/30/06 | 0.0442 | 0.0348 | 0.0953 | -0.1480 | -0.1184 | -0.0181 | 0.0416 | 0.1181 | 0.0732 |
| 27 | 10/31/06 | -0.0798 | 0.1536 | 0.1145 | -0.0107 | 0.1626 | 0.0710 | 0.1197 | 0.0583 | 0.0466 |
| 26 | 09/29/06 | 0.0215 | 0.0612 | -0.0394 | -0.0071 | -0.1084 | -0.0312 | -0.0301 | 0.1299 | 0.0065 |
| 25 | 08/31/06 | 0.0691 | 0.0303 | 0.0217 | 0.2761 | 0.0481 | 0.1111 | 0.2345 | -0.0769 | 0.0229 |

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|---|---|---|---|---|---|---|---|---|---|---|
| 24 | 07/31/06 | 0.0083 | 0.1300 | -0.0002 | 0.0137 | 0.1020 | -0.0526 | -0.1497 | 0.0150 | 0.0111 |
| 23 | 06/30/06 | -0.0147 | 0.1068 | -0.0092 | -0.1350 | -0.1490 | -0.0013 | -0.0588 | -0.1250 | -0.0046 |
| 22 | 05/31/06 | -0.0497 | -0.1043 | -0.0682 | -0.1250 | -0.1587 | -0.1544 | -0.0395 | 0.0877 | -0.1076 |
| 21 | 04/28/06 | 0.0222 | 0.0874 | 0.0518 | -0.2237 | 0.0140 | -0.0265 | -0.0881 | 0.0000 | 0.0683 |
| 20 | 03/31/06 | -0.0830 | 0.0953 | 0.0241 | 0.0082 | -0.1063 | -0.1567 | -0.1162 | 0.0206 | 0.0073 |
| 19 | 02/28/06 | -0.0716 | -0.0150 | -0.0573 | -0.0777 | 0.0959 | 0.1651 | 0.0717 | 0.0250 | -0.0021 |
| 18 | 01/31/06 | 0.1229 | 0.0233 | 0.1606 | -0.1250 | 0.1497 | 0.0456 | -0.1042 | 0.0192 | 0.1094 |
| 17 | 12/30/05 | 0.1020 | -0.0184 | 0.0768 | -0.0400 | -0.0451 | 0.0185 | -0.0452 | 0.1192 | 0.0576 |
| 16 | 11/30/05 | 0.0833 | 0.0909 | 0.0740 | -0.1088 | 0.3197 | 0.2856 | 0.0258 | 0.0173 | 0.0819 |
| 15 | 10/31/05 | -0.0612 | -0.0889 | -0.0687 | -0.0111 | -0.1739 | -0.0614 | -0.1562 | -0.0170 | -0.0664 |
| 14 | 09/30/05 | 0.0809 | 0.1277 | -0.0001 | 0.1846 | -0.0240 | 0.1258 | -0.0188 | 0.0836 | 0.0909 |
| 13 | 08/31/05 | -0.0372 | 0.0833 | 0.0828 | 0.3343 | 0.1416 | 0.0818 | 0.0276 | 0.0256 | 0.0061 |
| 12 | 07/29/05 | 0.1437 | 0.0850 | 0.1572 | -0.2474 | 0.0425 | 0.1454 | 0.1662 | 0.0073 | 0.0662 |
| 11 | 06/30/05 | 0.0102 | 0.0181 | -0.0060 | 0.7359 | 0.0174 | 0.0549 | -0.1485 | 0.1012 | 0.0310 |
| 10 | 05/31/05 | 0.0819 | 0.0258 | 0.1410 | 0.0707 | 0.1153 | 0.0974 | -0.0653 | -0.0609 | 0.0304 |
| 9 | 04/29/05 | -0.0996 | 0.0847 | 0.0277 | 0.8751 | -0.1187 | 0.0153 | -0.0926 | 0.0385 | -0.0304 |
| 8 | 03/31/05 | -0.0474 | 0.0130 | -0.0854 | -0.0019 | -0.1414 | -0.1812 | -0.1733 | -0.0943 | -0.0679 |
| 7 | 02/28/05 | 0.0646 | 0.0292 | 0.0402 | 0.0884 | 0.0058 | -0.0419 | 0.1220 | 0.0049 | 0.0856 |
| 6 | 01/31/05 | 0.0988 | -0.0828 | -0.0020 | 0.0400 | -0.0168 | 0.0000 | -0.1287 | -0.0380 | 0.0002 |
| 5 | 12/31/04 | 0.0368 | 0.0515 | 0.1211 | 0.2703 | 0.1208 | 0.2279 | 0.1318 | 0.0772 | 0.0468 |
| 4 | 11/30/04 | -0.0114 | 0.1216 | 0.1904 | 0.2913 | -0.1229 | 0.0224 | 0.1276 | 0.0718 | 0.0921 |
| 3 | 10/29/04 | -0.0404 | -0.0490 | 0.0974 | -0.0154 | 0.0172 | -0.0219 | 0.1029 | -0.0375 | 0.0218 |
| 2 | 09/30/04 | 0.0155 | 0.0465 | 0.0460 | 0.2987 | 0.0797 | 0.0626 | 0.1226 | 0.1063 | 0.0555 |
| 1 | 08/31/04 | 0.0815 | 0.0069 | 0.0880 | 0.0492 | 0.0793 | 0.0578 | 0.3856 | 0.1770 | 0.0393 |
| 0 | 07/30/04 | | | | | | | | | |

**Table 4.3 : Monthly returns of the eight stocks and the MSCI Emerging market index**

## Appendix A3 - Excess Returns of Portfolio constituents

| MONTH | DATE | EXC.RETSAM | EXC.RETCHI | EXC.RETNAS | EXC.RETEMAR | EXC.RETKCHO | EXC.RETAK | EXC.RETBRM | EXC.RETTAIW |
|---|---|---|---|---|---|---|---|---|---|
| 134 | 9/30/2015 | 0.0313 | -0.0185 | -0.0238 | -0.0637 | 0.0219 | -0.0197 | 0.1773 | -0.0551 |
| 133 | 8/31/2015 | -0.0910 | -0.0928 | -0.0594 | -0.1663 | -0.0950 | -0.0947 | 0.1145 | 0.0069 |
| 132 | 7/31/2015 | -0.0755 | 0.0011 | -0.0966 | -0.0182 | -0.0258 | -0.0564 | -0.0868 | -0.1340 |
| 131 | 6/30/2015 | -0.0399 | -0.0382 | 0.0310 | -0.0040 | 0.0243 | -0.0365 | 0.0426 | -0.0780 |
| 130 | 5/29/2015 | -0.0831 | -0.0928 | -0.0766 | -0.0802 | -0.0770 | 0.0042 | 0.0219 | -0.0556 |
| 129 | 4/30/2015 | -0.0315 | 0.0853 | -0.0301 | 0.2277 | 0.0498 | 0.0045 | 0.1380 | -0.0234 |
| 128 | 3/31/2015 | 0.0519 | -0.0533 | 0.0615 | -0.1466 | -0.0049 | -0.0809 | -0.1481 | 0.0147 |
| 127 | 2/27/2015 | -0.0159 | 0.0241 | -0.0221 | 0.1061 | -0.1001 | -0.0986 | 0.0206 | -0.0038 |
| 126 | 1/30/2015 | 0.0186 | 0.0974 | 0.0900 | -0.0979 | 0.0105 | 0.0208 | -0.2961 | -0.0305 |
| 125 | 12/31/2014 | 0.0210 | -0.0601 | 0.0260 | -0.3358 | -0.0257 | -0.0505 | -0.0984 | -0.0572 |
| 124 | 11/28/2014 | 0.0245 | -0.0191 | 0.0141 | 0.0393 | 0.0836 | 0.1057 | 0.0543 | -0.0336 |
| 123 | 10/31/2014 | 0.0406 | 0.0437 | 0.0719 | -0.1549 | 0.0632 | 0.0684 | 0.1116 | 0.0133 |
| 122 | 9/30/2014 | -0.0506 | -0.0698 | -0.1122 | 0.0151 | -0.0675 | -0.1133 | 0.0548 | -0.0584 |
| 121 | 8/29/2014 | -0.0912 | 0.1298 | -0.0120 | 0.1205 | -0.0355 | -0.0478 | 0.0795 | 0.0470 |
| 120 | 7/31/2014 | 0.0059 | 0.1078 | 0.0322 | 0.1410 | 0.0640 | 0.0812 | -0.0117 | -0.0198 |
| 119 | 6/30/2014 | -0.0939 | -0.0223 | 0.0425 | -0.2160 | -0.0129 | -0.0615 | -0.0451 | 0.0014 |
| 118 | 5/30/2014 | 0.0644 | 0.0212 | 0.1455 | 0.0339 | 0.0798 | 0.0961 | -0.0456 | -0.0810 |
| 117 | 4/30/2014 | -0.0100 | 0.0272 | -0.1764 | 0.0748 | 0.0277 | 0.0700 | -0.1520 | 0.0071 |
| 116 | 3/31/2014 | -0.0145 | -0.0544 | -0.1364 | 0.0726 | 0.0850 | 0.1750 | 0.0941 | -0.0390 |
| 115 | 2/28/2014 | 0.0439 | -0.0194 | 0.1130 | 0.1168 | 0.0502 | -0.0208 | -0.1383 | 0.0782 |
| 114 | 1/31/2014 | -0.0771 | -0.0980 | 0.0056 | 0.0265 | -0.1473 | -0.1541 | -0.1227 | -0.0542 |
| 113 | 12/31/2013 | -0.0917 | -0.0491 | 0.0959 | 0.1919 | -0.0953 | -0.1084 | 0.0021 | -0.0249 |
| 112 | 11/29/2013 | 0.0098 | 0.0295 | 0.0055 | 0.0154 | -0.0443 | -0.0699 | 0.0388 | 0.0838 |
| 111 | 10/31/2013 | 0.0617 | -0.0913 | -0.0192 | 0.0221 | 0.0338 | 0.0400 | 0.1125 | -0.0132 |
| 110 | 9/30/2013 | -0.0108 | 0.0324 | 0.0638 | -0.0068 | 0.1133 | 0.0835 | 0.0039 | 0.0715 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 109 | 8/30/2013 | 0.0587 | 0.0067 | -0.0034 | -0.0576 | -0.0529 | -0.1025 | -0.0092 | 0.0657 |
| 108 | 7/31/2013 | -0.0562 | 0.0091 | 0.1007 | 0.1317 | -0.0955 | -0.0672 | 0.0599 | -0.0188 |
| 107 | 6/28/2013 | -0.1375 | -0.0137 | -0.0509 | -0.1530 | -0.1482 | -0.1346 | -0.0219 | -0.0767 |
| 106 | 5/31/2013 | 0.0018 | -0.0754 | 0.2102 | 0.0434 | -0.0362 | -0.0648 | -0.0471 | -0.0057 |
| 105 | 4/30/2013 | -0.0146 | 0.0266 | 0.0176 | 0.0749 | 0.0156 | -0.0180 | 0.2672 | 0.0333 |
| 104 | 3/29/2013 | -0.0210 | -0.0436 | -0.0477 | -0.0546 | 0.0922 | 0.0686 | -0.0713 | -0.0265 |
| 103 | 2/28/2013 | 0.0563 | -0.0113 | -0.0237 | 0.0676 | 0.0128 | -0.0318 | -0.0361 | -0.0584 |
| 102 | 1/31/2013 | -0.0587 | -0.0816 | 0.0350 | 0.2779 | -0.0264 | -0.0024 | 0.1652 | 0.0136 |
| 101 | 12/31/2012 | 0.0725 | 0.0185 | -0.0412 | -0.0234 | 0.1143 | 0.0363 | -0.0786 | 0.0103 |
| 100 | 11/30/2012 | 0.0632 | 0.0145 | -0.0553 | 0.0207 | -0.0462 | -0.0439 | 0.0193 | 0.0040 |
| 99 | 10/31/2012 | -0.0368 | -0.0126 | 0.0625 | 0.0166 | 0.1550 | 0.2031 | -0.0671 | 0.0196 |
| 98 | 9/28/2012 | 0.0816 | 0.0180 | 0.0225 | 0.0308 | -0.0475 | -0.0249 | 0.1074 | 0.1031 |
| 97 | 8/31/2012 | -0.0681 | -0.0894 | 0.0563 | -0.0473 | 0.0394 | 0.0483 | 0.0256 | -0.0794 |
| 96 | 7/31/2012 | 0.0799 | 0.0500 | 0.0037 | 0.1938 | -0.0032 | 0.0073 | -0.0815 | -0.0248 |
| 95 | 6/29/2012 | -0.0183 | 0.0645 | -0.0616 | -0.0585 | 0.2228 | 0.1476 | 0.1818 | 0.0353 |
| 94 | 5/31/2012 | -0.1388 | -0.0964 | -0.0724 | -0.1311 | -0.1221 | -0.1396 | -0.1679 | -0.0436 |
| 93 | 4/30/2012 | 0.0802 | -0.0084 | 0.0563 | 0.0584 | -0.1171 | -0.0824 | -0.0802 | -0.0034 |
| 92 | 3/30/2012 | 0.0472 | 0.0259 | 0.0031 | -0.0618 | -0.0232 | -0.0167 | -0.0800 | -0.0968 |
| 91 | 2/29/2012 | 0.0794 | 0.0246 | 0.0335 | 0.1326 | 0.0343 | 0.0371 | -0.0113 | 0.0110 |
| 90 | 1/31/2012 | 0.0363 | 0.0403 | 0.0786 | 0.0456 | 0.2004 | 0.0958 | 0.2144 | 0.0422 |
| 89 | 12/30/2011 | 0.0438 | -0.0369 | -0.0613 | -0.0727 | -0.1189 | -0.0819 | -0.0925 | 0.0632 |
| 88 | 11/30/2011 | 0.0272 | 0.0312 | -0.0718 | -0.0457 | -0.0177 | -0.0138 | -0.0931 | -0.1521 |
| 87 | 10/31/2011 | 0.1423 | -0.0367 | 0.0521 | -0.0099 | -0.1044 | -0.1337 | 0.0419 | 0.1384 |
| 86 | 9/30/2011 | 0.1190 | -0.0612 | -0.0717 | -0.0592 | 0.1165 | 0.1331 | -0.2361 | -0.1630 |
| 85 | 8/31/2011 | -0.1285 | 0.0138 | -0.0088 | -0.0276 | -0.1561 | -0.1466 | 0.0202 | -0.1623 |
| 84 | 7/29/2011 | 0.0118 | 0.0521 | -0.0928 | -0.0670 | -0.0034 | -0.0298 | -0.1825 | 0.0534 |
| 83 | 6/30/2011 | -0.0943 | 0.0089 | -0.0846 | -0.0623 | -0.0536 | -0.0003 | -0.1051 | 0.0143 |

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|---|---|---|---|---|---|---|---|---|---|
| 82 | 5/31/2011 | 0.0000 | -0.0201 | -0.0085 | -0.0632 | -0.1326 | -0.0771 | 0.0711 | -0.0180 |
| 81 | 4/29/2011 | -0.0519 | -0.0166 | 0.0545 | -0.0020 | 0.1215 | 0.0395 | 0.0507 | 0.1696 |
| 80 | 3/31/2011 | -0.0003 | -0.0347 | -0.1205 | 0.1890 | 0.0539 | -0.0111 | 0.0596 | 0.1954 |
| 79 | 2/28/2011 | -0.0692 | -0.0512 | 0.0416 | -0.1455 | 0.0067 | -0.0191 | -0.0404 | -0.0743 |
| 78 | 1/31/2011 | 0.0237 | -0.0228 | -0.0692 | -0.1614 | -0.1480 | -0.1374 | 0.0220 | -0.0563 |
| 77 | 12/31/2010 | 0.1389 | -0.0177 | 0.0653 | -0.0402 | 0.0474 | -0.0044 | 0.1124 | 0.0342 |
| 76 | 11/30/2010 | 0.0987 | -0.0427 | -0.0682 | -0.0805 | 0.0129 | -0.0694 | 0.0355 | -0.0382 |
| 75 | 10/29/2010 | -0.0512 | -0.0084 | 0.0487 | 0.0114 | -0.0249 | 0.0031 | 0.0076 | -0.0360 |
| 74 | 9/30/2010 | 0.0177 | -0.0168 | 0.1138 | 0.1233 | 0.1418 | 0.0788 | 0.1023 | 0.1220 |
| 73 | 8/31/2010 | -0.0767 | -0.0057 | -0.0731 | -0.0207 | -0.0177 | -0.0437 | 0.1644 | -0.0240 |
| 72 | 7/30/2010 | 0.0365 | 0.0178 | 0.1660 | 0.0447 | 0.0842 | 0.0777 | 0.0468 | 0.0796 |
| 71 | 6/30/2010 | -0.0126 | 0.0478 | -0.1680 | -0.1071 | -0.0268 | -0.0330 | 0.1538 | -0.0139 |
| 70 | 5/31/2010 | -0.0960 | -0.0608 | -0.0263 | -0.1642 | -0.0700 | 0.0332 | -0.1491 | -0.0457 |
| 69 | 4/30/2010 | 0.0279 | 0.0031 | -0.0828 | -0.0430 | 0.0881 | -0.0002 | -0.0472 | -0.0669 |
| 68 | 3/31/2010 | 0.0894 | -0.0396 | 0.0683 | 0.3215 | 0.1126 | 0.2426 | 0.0168 | -0.0004 |
| 67 | 2/26/2010 | -0.0611 | 0.0397 | 0.0360 | -0.0501 | -0.1157 | -0.1274 | -0.0437 | -0.0205 |
| 66 | 1/29/2010 | -0.0288 | -0.0019 | -0.1306 | -0.2254 | 0.1360 | -0.0827 | -0.0657 | -0.1339 |
| 65 | 12/31/2009 | 0.0997 | -0.0225 | 0.0512 | 0.0086 | 0.1642 | 0.1387 | 0.2147 | 0.0012 |
| 64 | 11/30/2009 | -0.0142 | -0.0101 | -0.0557 | -0.1666 | -0.0437 | -0.0138 | -0.0229 | -0.0252 |
| 63 | 10/30/2009 | -0.1229 | -0.0616 | 0.0776 | 0.0632 | -0.0331 | -0.0603 | 0.0316 | -0.0704 |
| 62 | 9/30/2009 | 0.0470 | -0.0154 | -0.0221 | 0.1463 | -0.0378 | -0.0138 | 0.0690 | -0.0007 |
| 61 | 8/31/2009 | 0.0549 | -0.0751 | 0.0687 | 0.2458 | 0.1325 | 0.0223 | 0.2512 | 0.0153 |
| 60 | 7/31/2009 | 0.2129 | 0.0346 | 0.1098 | -0.0350 | 0.2831 | 0.1891 | 0.1499 | 0.0944 |
| 59 | 6/30/2009 | 0.0509 | 0.0045 | 0.0344 | -0.2198 | -0.1111 | 0.0558 | -0.0047 | -0.0883 |
| 58 | 5/29/2009 | -0.0675 | 0.1270 | 0.0660 | 0.4563 | 0.1881 | 0.0266 | 0.1530 | 0.0639 |
| 57 | 4/30/2009 | 0.0322 | -0.0214 | 0.0558 | 0.0473 | 0.2300 | 0.2567 | 0.2753 | 0.1184 |
| 56 | 3/31/2009 | 0.1807 | -0.0092 | 0.0083 | 0.0471 | 0.0857 | 0.2311 | -0.1134 | 0.0548 |

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|---|---|---|---|---|---|---|---|---|---|
| 55 | 2/27/2009 | -0.0326 | -0.0492 | -0.0500 | 0.0303 | -0.0996 | -0.1937 | -0.0653 | 0.1601 |
| 54 | 1/30/2009 | 0.0720 | -0.1288 | -0.0860 | -0.1536 | -0.1361 | -0.0138 | 0.0213 | -0.1827 |
| 53 | 12/31/2008 | -0.0821 | 0.0964 | 0.0475 | -0.2493 | 0.0218 | 0.0978 | -0.1209 | 0.0790 |
| 52 | 11/28/2008 | -0.1016 | 0.0311 | -0.0788 | -0.4720 | -0.1304 | -0.1901 | -0.3422 | 0.4449 |
| 51 | 10/31/2008 | -0.0175 | -0.1367 | -0.0244 | -0.3316 | -0.2932 | -0.2169 | -0.0822 | -0.1033 |
| 50 | 9/30/2008 | 0.0345 | -0.1302 | -0.1997 | -0.1701 | -0.0710 | 0.0512 | -0.1368 | -0.4123 |
| 49 | 8/29/2008 | -0.0951 | -0.1647 | 0.0500 | -0.1531 | -0.1291 | -0.0676 | -0.1611 | -0.2208 |
| 48 | 7/31/2008 | -0.1076 | -0.0146 | 0.0179 | -0.0531 | 0.3772 | 0.5192 | 0.0740 | -0.0440 |
| 47 | 6/30/2008 | -0.1666 | -0.1058 | -0.0672 | -0.0731 | -0.0467 | -0.2286 | -0.0437 | -0.2080 |
| 46 | 5/30/2008 | 0.0322 | -0.1583 | 0.0500 | -0.0251 | -0.1188 | -0.2019 | -0.0829 | 0.0181 |
| 45 | 4/30/2008 | 0.1312 | 0.1376 | 0.1343 | 0.0295 | 0.1654 | 0.1738 | -0.0464 | -0.1716 |
| 44 | 3/31/2008 | 0.1025 | -0.0079 | -0.0855 | -0.1412 | -0.2160 | -0.1589 | 0.0316 | 0.0590 |
| 43 | 2/29/2008 | -0.0689 | -0.0251 | 0.0584 | 0.0168 | -0.0003 | -0.0434 | 0.0956 | 0.3036 |
| 42 | 1/31/2008 | 0.0601 | -0.1438 | -0.1849 | -0.2154 | -0.2996 | -0.2379 | -0.1047 | -0.0832 |
| 41 | 12/31/2007 | -0.0260 | -0.0654 | -0.1306 | 0.1808 | 0.0405 | -0.0022 | -0.0932 | -0.0555 |
| 40 | 11/30/2007 | 0.0172 | -0.1291 | -0.1568 | -0.0595 | -0.0727 | -0.2025 | -0.0418 | -0.1597 |
| 39 | 10/31/2007 | -0.0535 | 0.2506 | 0.0478 | 0.1905 | -0.0177 | 0.1384 | -0.0631 | 0.0314 |
| 38 | 9/28/2007 | -0.0371 | 0.1971 | 0.0359 | -0.0436 | 0.0495 | 0.0946 | -0.0533 | 0.2357 |
| 37 | 8/31/2007 | -0.0475 | 0.1681 | -0.0301 | -0.0207 | -0.0262 | -0.0760 | 0.0406 | 0.0518 |
| 36 | 7/31/2007 | 0.0748 | 0.0516 | -0.0471 | -0.1009 | 0.1252 | 0.1986 | -0.0069 | 0.0443 |
| 35 | 6/29/2007 | 0.0479 | 0.1480 | -0.0623 | -0.0494 | -0.0710 | -0.0273 | 0.0450 | 0.2128 |
| 34 | 5/31/2007 | -0.0780 | 0.0182 | 0.0194 | 0.1141 | 0.0564 | -0.0888 | -0.0622 | 0.0334 |
| 33 | 4/30/2007 | 0.0095 | -0.0096 | -0.0107 | -0.0478 | 0.0538 | 0.0615 | 0.1332 | 0.0542 |
| 32 | 3/30/2007 | -0.0171 | -0.0480 | -0.0073 | -0.1609 | 0.0066 | -0.0138 | 0.0963 | -0.0031 |
| 31 | 2/28/2007 | -0.0308 | -0.0062 | -0.0888 | -0.0049 | 0.0072 | 0.0195 | -0.1202 | -0.0315 |
| 30 | 1/31/2007 | -0.0655 | 0.0547 | 0.0663 | 0.0161 | 0.0732 | 0.0327 | 0.0278 | -0.0545 |
| 29 | 12/29/2006 | -0.0492 | 0.0110 | 0.1064 | 0.0132 | 0.0865 | 0.0414 | -0.0569 | -0.0085 |

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|---|---|---|---|---|---|---|---|---|---|
| 28 | 11/30/2006 | 0.0342 | 0.0217 | 0.0647 | -0.1688 | -0.1361 | -0.0319 | 0.0341 | 0.1060 |
| 27 | 10/31/2006 | -0.0899 | 0.1405 | 0.0839 | -0.0314 | 0.1448 | 0.0572 | 0.1122 | 0.0462 |
| 26 | 9/29/2006 | 0.0115 | 0.0481 | -0.0700 | -0.0278 | -0.1262 | -0.0451 | -0.0376 | 0.1178 |
| 25 | 8/31/2006 | 0.0590 | 0.0172 | -0.0089 | 0.2553 | 0.0304 | 0.0973 | 0.2270 | -0.0890 |
| 24 | 7/31/2006 | -0.0017 | 0.1169 | -0.0308 | -0.0070 | 0.0842 | -0.0664 | -0.1572 | 0.0029 |
| 23 | 6/30/2006 | -0.0247 | 0.0936 | -0.0398 | -0.1557 | -0.1667 | -0.0151 | -0.0663 | -0.1371 |
| 22 | 5/31/2006 | -0.0597 | -0.1174 | -0.0988 | -0.1457 | -0.1764 | -0.1683 | -0.0469 | 0.0756 |
| 21 | 4/28/2006 | 0.0122 | 0.0743 | 0.0212 | -0.2445 | -0.0037 | -0.0403 | -0.0956 | -0.0121 |
| 20 | 3/31/2006 | -0.0930 | 0.0822 | -0.0066 | -0.0125 | -0.1240 | -0.1705 | -0.1237 | 0.0085 |
| 19 | 2/28/2006 | -0.0817 | -0.0282 | -0.0879 | -0.0984 | 0.0782 | 0.1513 | 0.0642 | 0.0130 |
| 18 | 1/31/2006 | 0.1129 | 0.0102 | 0.1300 | -0.1457 | 0.1319 | 0.0317 | -0.1117 | 0.0072 |
| 17 | 12/30/2005 | 0.0920 | -0.0315 | 0.0462 | -0.0608 | -0.0629 | 0.0047 | -0.0527 | 0.1071 |
| 16 | 11/30/2005 | 0.0733 | 0.0777 | 0.0434 | -0.1295 | 0.3020 | 0.2718 | 0.0183 | 0.0053 |
| 15 | 10/31/2005 | -0.0713 | -0.1020 | -0.0993 | -0.0319 | -0.1916 | -0.0753 | -0.1636 | -0.0291 |
| 14 | 9/30/2005 | 0.0708 | 0.1146 | -0.0307 | 0.1639 | -0.0417 | 0.1120 | -0.0263 | 0.0715 |
| 13 | 8/31/2005 | -0.0472 | 0.0702 | 0.0521 | 0.3136 | 0.1239 | 0.0680 | 0.0201 | 0.0135 |
| 12 | 7/29/2005 | 0.1337 | 0.0719 | 0.1266 | -0.2682 | 0.0247 | 0.1316 | 0.1588 | -0.0048 |
| 11 | 6/30/2005 | 0.0002 | 0.0049 | -0.0366 | 0.7151 | -0.0004 | 0.0411 | -0.1560 | 0.0892 |
| 10 | 5/31/2005 | 0.0718 | 0.0127 | 0.1104 | 0.0499 | 0.0975 | 0.0836 | -0.0728 | -0.0729 |
| 9 | 4/29/2005 | -0.1096 | 0.0716 | -0.0030 | 0.8544 | -0.1364 | 0.0015 | -0.1001 | 0.0265 |
| 8 | 3/31/2005 | -0.0575 | -0.0002 | -0.1160 | -0.0226 | -0.1591 | -0.1950 | -0.1808 | -0.1064 |
| 7 | 2/28/2005 | 0.0546 | 0.0161 | 0.0096 | 0.0677 | -0.0119 | -0.0557 | 0.1145 | -0.0071 |
| 6 | 1/31/2005 | 0.0887 | -0.0959 | -0.0326 | 0.0192 | -0.0346 | -0.0138 | -0.1362 | -0.0500 |
| 5 | 12/31/2004 | 0.0268 | 0.0383 | 0.0905 | 0.2495 | 0.1031 | 0.2141 | 0.1243 | 0.0652 |
| 4 | 11/30/2004 | -0.0214 | 0.1085 | 0.1598 | 0.2706 | -0.1406 | 0.0086 | 0.1201 | 0.0598 |
| 3 | 10/29/2004 | -0.0504 | -0.0622 | 0.0668 | -0.0362 | -0.0005 | -0.0357 | 0.0955 | -0.0496 |
| 2 | 9/30/2004 | 0.0055 | 0.0334 | 0.0153 | 0.2780 | 0.0619 | 0.0488 | 0.1152 | 0.0942 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 8/31/2004 | 0.0715 | -0.0062 | 0.0574 | 0.0285 | 0.0616 | 0.0440 | 0.3781 | 0.1649 |
| 0 | 7/30/2004 | | | | | | | | |

**Table 4.7 : Excess Returns(Demeaned Returns) of the eight stocks**

## Appendix A4 – Data for Lambda calculation

| Date | MSCI Emerging Markets Index | MSCI_Return (Market return -$r_m$) | US Treasury rates - $r_f$ | Excess return ($r_m$- $r_f$) |
|---|---|---|---|---|
| 9/30/2015 | 792.05 | -0.0326 | 0.0000 | -0.0326 |
| 8/31/2015 | 818.73 | -0.0920 | 0.0000 | -0.0920 |
| 7/31/2015 | 901.68 | -0.0726 | 0.0400 | -0.1126 |
| 6/30/2015 | 972.25 | -0.0318 | 0.0200 | -0.0518 |
| 5/29/2015 | 1004.22 | -0.0416 | 0.0100 | -0.0516 |
| 4/30/2015 | 1047.78 | 0.0751 | 0.0000 | 0.0751 |
| 3/31/2015 | 974.57 | -0.0159 | 0.0500 | -0.0659 |
| 2/27/2015 | 990.28 | 0.0298 | 0.0200 | 0.0098 |
| 1/30/2015 | 961.61 | 0.0055 | 0.0100 | -0.0045 |
| 12/31/2014 | 956.31 | -0.0482 | 0.0300 | -0.0782 |
| 11/28/2014 | 1004.72 | -0.0112 | 0.0400 | -0.0512 |
| 10/31/2014 | 1016.07 | 0.0107 | 0.0100 | 0.0007 |
| 9/30/2014 | 1005.33 | -0.0759 | 0.0200 | -0.0959 |
| 8/29/2014 | 1087.88 | 0.0207 | 0.0200 | 0.0007 |
| 7/31/2014 | 1065.77 | 0.0143 | 0.0100 | 0.0043 |
| 6/30/2014 | 1050.78 | 0.0225 | 0.0200 | 0.0025 |
| 5/30/2014 | 1027.69 | 0.0326 | 0.0500 | -0.0174 |
| 4/30/2014 | 995.28 | 0.0006 | 0.0200 | -0.0194 |

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|---|---|---|---|---|
| 3/31/2014 | 994.65 | 0.0292 | 0.0300 | -0.0008 |
| 2/28/2014 | 966.42 | 0.0319 | 0.0400 | -0.0081 |
| 1/31/2014 | 936.53 | -0.0660 | 0.0300 | -0.0960 |
| 12/31/2013 | 1002.69 | -0.0153 | 0.0100 | -0.0253 |
| 11/29/2013 | 1018.28 | -0.0156 | 0.0500 | -0.0656 |
| 10/31/2013 | 1034.42 | 0.0476 | 0.0300 | 0.0176 |
| 9/30/2013 | 987.46 | 0.0623 | 0.0300 | 0.0323 |
| 8/30/2013 | 929.54 | -0.0190 | 0.0200 | -0.0390 |
| 7/31/2013 | 947.55 | 0.0077 | 0.0300 | -0.0223 |
| 6/28/2013 | 940.33 | -0.0679 | 0.0200 | -0.0879 |
| 5/31/2013 | 1008.88 | -0.0294 | 0.0300 | -0.0594 |
| 4/30/2013 | 1039.45 | 0.0044 | 0.0300 | -0.0256 |
| 3/29/2013 | 1034.90 | -0.0187 | 0.0400 | -0.0587 |
| 2/28/2013 | 1054.62 | -0.0135 | 0.0700 | -0.0835 |
| 1/31/2013 | 1069.01 | 0.0131 | 0.0400 | -0.0269 |
| 12/31/2012 | 1055.20 | 0.0478 | 0.0200 | 0.0278 |
| 11/30/2012 | 1007.02 | 0.0117 | 0.1100 | -0.0983 |
| 10/31/2012 | 995.33 | -0.0073 | 0.0900 | -0.0973 |
| 9/28/2012 | 1002.66 | 0.0584 | 0.0600 | -0.0016 |
| 8/31/2012 | 947.33 | -0.0054 | 0.0900 | -0.0954 |
| 7/31/2012 | 952.49 | 0.0162 | 0.0700 | -0.0538 |
| 6/29/2012 | 937.35 | 0.0343 | 0.0400 | -0.0057 |
| 5/31/2012 | 906.30 | -0.1167 | 0.0300 | -0.1467 |
| 4/30/2012 | 1026.02 | -0.0148 | 0.0700 | -0.0848 |
| 3/30/2012 | 1041.45 | -0.0352 | 0.0500 | -0.0852 |
| 2/29/2012 | 1079.44 | 0.0589 | 0.0800 | -0.0211 |
| 1/31/2012 | 1019.39 | 0.1124 | 0.0400 | 0.0724 |

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|---|---|---|---|---|
| 12/30/2011 | 916.39 | -0.0129 | 0.0100 | -0.0229 |
| 11/30/2011 | 928.32 | -0.0670 | 0.0200 | -0.0870 |
| 10/31/2011 | 995.00 | 0.1301 | 0.0200 | 0.1101 |
| 9/30/2011 | 880.43 | -0.1478 | 0.0200 | -0.1678 |
| 8/31/2011 | 1033.15 | -0.0919 | 0.0100 | -0.1019 |
| 7/29/2011 | 1137.73 | -0.0074 | 0.1600 | -0.1674 |
| 6/30/2011 | 1146.22 | -0.0186 | 0.0100 | -0.0286 |
| 5/31/2011 | 1167.97 | -0.0299 | 0.0400 | -0.0699 |
| 4/29/2011 | 1204.03 | 0.0283 | 0.0200 | 0.0083 |
| 3/31/2011 | 1170.87 | 0.0570 | 0.0500 | 0.0070 |
| 2/28/2011 | 1107.77 | -0.0101 | 0.1300 | -0.1401 |
| 1/31/2011 | 1119.08 | -0.0281 | 0.1500 | -0.1781 |
| 12/31/2010 | 1151.38 | 0.0702 | 0.0700 | 0.0002 |
| 11/30/2010 | 1075.85 | -0.0270 | 0.1800 | -0.2070 |
| 10/29/2010 | 1105.75 | 0.0281 | 0.1400 | -0.1119 |
| 9/30/2010 | 1075.53 | 0.1087 | 0.1400 | -0.0313 |
| 8/31/2010 | 970.05 | -0.0215 | 0.1600 | -0.1815 |
| 7/30/2010 | 991.41 | 0.0800 | 0.1400 | -0.0600 |
| 6/30/2010 | 917.99 | -0.0091 | 0.1700 | -0.1791 |
| 5/31/2010 | 926.40 | -0.0918 | 0.1500 | -0.2418 |
| 4/30/2010 | 1020.03 | 0.0096 | 0.1400 | -0.1304 |
| 3/31/2010 | 1010.33 | 0.0795 | 0.1500 | -0.0705 |
| 2/26/2010 | 935.93 | 0.0025 | 0.0900 | -0.0875 |
| 1/29/2010 | 933.59 | -0.0565 | 0.0200 | -0.0765 |
| 12/31/2009 | 989.47 | 0.0381 | 0.0400 | -0.0019 |
| 11/30/2009 | 953.13 | 0.0425 | 0.0800 | -0.0375 |
| 10/30/2009 | 914.26 | 0.0002 | 0.0100 | -0.0098 |

| | | | | |
|---|---|---|---|---|
| 9/30/2009 | 914.05 | 0.0889 | 0.0600 | 0.0289 |
| 8/31/2009 | 839.46 | -0.0054 | 0.1100 | -0.1154 |
| 7/31/2009 | 844.02 | 0.1087 | 0.1400 | -0.0313 |
| 6/30/2009 | 761.30 | -0.0153 | 0.1700 | -0.1853 |
| 5/29/2009 | 773.12 | 0.1666 | 0.1400 | 0.0266 |
| 4/30/2009 | 662.73 | 0.1627 | 0.0400 | 0.1227 |
| 3/31/2009 | 569.97 | 0.1415 | 0.1700 | -0.0285 |
| 2/27/2009 | 499.30 | -0.0571 | 0.1600 | -0.2171 |
| 1/30/2009 | 529.53 | -0.0662 | 0.1500 | -0.2162 |
| 12/31/2008 | 567.04 | 0.0760 | 0.1100 | -0.0340 |
| 11/28/2008 | 526.97 | -0.0763 | 0.0200 | -0.0963 |
| 10/31/2008 | 570.52 | -0.2750 | 0.1200 | -0.3950 |
| 9/30/2008 | 786.92 | -0.1771 | 1.0200 | -1.1971 |
| 8/29/2008 | 956.25 | -0.0822 | 1.6300 | -1.7122 |
| 7/31/2008 | 1041.86 | -0.0416 | 1.5500 | -1.5916 |
| 6/30/2008 | 1087.12 | -0.1016 | 1.6000 | -1.7016 |
| 5/30/2008 | 1210.04 | 0.0155 | 1.9800 | -1.9645 |
| 4/30/2008 | 1191.53 | 0.0787 | 1.1700 | -1.0913 |
| 3/31/2008 | 1104.58 | -0.0540 | 1.2200 | -1.2740 |
| 2/29/2008 | 1167.66 | 0.0725 | 2.0700 | -1.9975 |
| 1/31/2008 | 1088.72 | -0.1259 | 1.6400 | -1.7659 |
| 12/31/2007 | 1245.59 | 0.0028 | 2.7600 | -2.7572 |
| 11/30/2007 | 1242.06 | -0.0714 | 3.6300 | -3.7014 |
| 10/31/2007 | 1337.63 | 0.1102 | 4.0100 | -3.8998 |
| 9/28/2007 | 1204.90 | 0.1085 | 3.4300 | -3.3215 |
| 8/31/2007 | 1086.98 | -0.0232 | 4.0200 | -4.0432 |
| 7/31/2007 | 1112.77 | 0.0501 | 5.1300 | -5.0799 |

| | | | | |
|---|---|---|---|---|
| 6/29/2007 | 1059.69 | 0.0443 | 4.2800 | -4.2357 |
| 5/31/2007 | 1014.78 | 0.0462 | 4.7800 | -4.7338 |
| 4/30/2007 | 969.93 | 0.0440 | 4.8000 | -4.7560 |
| 3/30/2007 | 929.03 | 0.0374 | 5.0700 | -5.0326 |
| 2/28/2007 | 895.54 | -0.0066 | 5.2400 | -5.2466 |
| 1/31/2007 | 901.48 | -0.0122 | 5.0000 | -5.0122 |
| 12/29/2006 | 912.65 | 0.0441 | 4.7500 | -4.7059 |
| 11/30/2006 | 874.08 | 0.0732 | 5.2200 | -5.1468 |
| 10/31/2006 | 814.44 | 0.0466 | 5.1800 | -5.1334 |
| 9/29/2006 | 778.16 | 0.0065 | 4.6000 | -4.5935 |
| 8/31/2006 | 773.12 | 0.0229 | 5.1200 | -5.0971 |
| 7/31/2006 | 755.84 | 0.0111 | 5.0200 | -5.0089 |
| 6/30/2006 | 747.54 | -0.0046 | 4.5400 | -4.5446 |
| 5/31/2006 | 751.00 | -0.1076 | 4.7500 | -4.8576 |
| 4/28/2006 | 841.58 | 0.0683 | 4.6000 | -4.5317 |
| 3/31/2006 | 787.80 | 0.0073 | 4.6500 | -4.6427 |
| 2/28/2006 | 782.11 | -0.0021 | 4.4700 | -4.4721 |
| 1/31/2006 | 783.77 | 0.1094 | 4.3700 | -4.2606 |
| 12/30/2005 | 706.48 | 0.0576 | 4.0100 | -3.9524 |
| 11/30/2005 | 667.99 | 0.0819 | 4.0000 | -3.9181 |
| 10/31/2005 | 617.41 | -0.0664 | 3.7700 | -3.8364 |
| 9/30/2005 | 661.32 | 0.0909 | 3.1500 | -3.0591 |
| 8/31/2005 | 606.23 | 0.0061 | 3.4100 | -3.4039 |
| 7/29/2005 | 602.56 | 0.0662 | 3.2500 | -3.1838 |
| 6/30/2005 | 565.17 | 0.0310 | 2.9900 | -2.9590 |
| 5/31/2005 | 548.15 | 0.0304 | 2.8000 | -2.7696 |
| 4/29/2005 | 531.99 | -0.0304 | 2.7000 | -2.7304 |

| | | | | |
|---|---|---|---|---|
| 3/31/2005 | 548.69 | -0.0679 | 2.6300 | -2.6979 |
| 2/28/2005 | 588.68 | 0.0856 | 2.5100 | -2.4244 |
| 1/31/2005 | 542.28 | 0.0002 | 2.0600 | -2.0598 |
| 12/31/2004 | 542.17 | 0.0468 | 1.8900 | -1.8432 |
| 11/30/2004 | 517.95 | 0.0921 | 2.0700 | -1.9779 |
| 10/29/2004 | 474.27 | 0.0218 | 1.7300 | -1.7082 |
| 9/30/2004 | 464.15 | 0.0555 | 1.4700 | -1.4145 |
| 8/31/2004 | 439.75 | 0.0393 | 1.4500 | -1.4107 |
| 7/30/2004 | 423.14 | -0.0210 | 1.2700 | -1.2910 |
| 6/30/2004 | 432.20 | 0.0022 | 1.1700 | -1.1678 |
| 5/31/2004 | 431.26 | -0.0228 | 0.9400 | -0.9628 |
| 4/30/2004 | 441.30 | -0.0846 | 0.8300 | -0.9146 |
| 3/31/2004 | 482.06 | 0.0091 | 0.9600 | -0.9509 |
| 2/27/2004 | 477.73 | 0.0449 | 0.9500 | -0.9051 |
| 1/30/2004 | 457.19 | 0.0325 | 0.8500 | -0.8175 |
| 12/31/2003 | 442.78 | 0.0713 | 0.9000 | -0.8287 |
| 11/28/2003 | 413.33 | 0.0103 | 0.9600 | -0.9497 |
| 10/31/2003 | 409.11 | 0.0834 | 0.9600 | -0.8766 |
| 9/30/2003 | 377.63 | 0.0056 | 0.8700 | -0.8644 |
| 8/29/2003 | 375.52 | 0.0649 | 0.9800 | -0.9151 |
| 7/31/2003 | 352.62 | 0.0599 | 0.9100 | -0.8501 |
| 6/30/2003 | 332.68 | 0.0547 | 0.8100 | -0.7553 |
| 5/30/2003 | 315.44 | 0.0687 | 1.1600 | -1.0913 |
| 4/30/2003 | 295.17 | 0.0841 | 1.1300 | -1.0459 |
| 3/31/2003 | 272.27 | -0.0321 | 1.1600 | -1.1921 |
| 2/28/2003 | 281.29 | -0.0315 | 1.2100 | -1.2415 |
| 1/31/2003 | 290.44 | -0.0056 | 1.1700 | -1.1756 |

| | | | | |
|---|---|---|---|---|
| 12/31/2002 | 292.09 | -0.0340 | 1.2000 | -1.2340 |
| 11/29/2002 | 302.36 | 0.0678 | 1.2500 | -1.1822 |
| 10/31/2002 | 283.16 | 0.0641 | 1.4800 | -1.4159 |
| 9/30/2002 | 266.11 | -0.1097 | 1.6000 | -1.7097 |
| 8/30/2002 | 298.89 | 0.0145 | 1.7000 | -1.6855 |
| 7/31/2002 | 294.62 | -0.0786 | 1.7300 | -1.8086 |
| 6/28/2002 | 319.75 | -0.0766 | 1.6900 | -1.7666 |
| 5/31/2002 | 346.28 | -0.0186 | 1.7200 | -1.7386 |
| 4/30/2002 | 352.84 | 0.0040 | 1.7700 | -1.7660 |
| 3/29/2002 | 351.43 | 0.0561 | 1.7600 | -1.7039 |
| 2/28/2002 | 332.77 | 0.0153 | 1.7600 | -1.7447 |
| 1/31/2002 | 327.75 | 0.0326 | 1.6900 | -1.6574 |
| 12/31/2001 | 317.40 | 0.0780 | 1.6800 | -1.6020 |
| 11/30/2001 | 294.43 | 0.1033 | 1.8700 | -1.7667 |
| 10/31/2001 | 266.86 | 0.0615 | 2.1500 | -2.0885 |
| 9/28/2001 | 251.40 | -0.1569 | 2.2800 | -2.4369 |
| 8/31/2001 | 298.17 | -0.0116 | 3.4000 | -3.4116 |
| 7/31/2001 | 301.67 | | 3.6700 | |

**Table 4.9 : Inputs for Lambda calculation**

## Appendix A5 – Program codes in Matlab

### Blacklit.m file

```
% Black Litterman Method implementation in Matlab
clear all

format long g
% Import the Data file(which is in excel format)with titles
[data, hdr] = xlsread('BasicParas.xlsx');
% Determine the dimensions of the table ( number of rows and Columns)
size_of_matrix = size(data);  nrw = size_of_matrix(1); ncol =
size_of_matrix(2);

% Return calculation of each stocks
% (The function file findStkReturn.m is being called here)
%------------------------------------------------------
returnStk1 = findStkReturn(data(:,1));
returnStk2 = findStkReturn(data(:,2));
returnStk3 = findStkReturn(data(:,3));
returnStk4 = findStkReturn(data(:,4));
returnStk5 = findStkReturn(data(:,5));
returnStk6 = findStkReturn(data(:,6));
returnStk7 = findStkReturn(data(:,7));
returnStk8 = findStkReturn(data(:,8));

returns = [returnStk1' returnStk2' returnStk3' returnStk4'
returnStk5' returnStk6' returnStk7' returnStk8'];
%xlswrite('Retuns.xlsx',returns);
% (The function file findStkReturn.m is being called here)
mkt_ret = findStkReturn(data(:,9));
%disp(returns)
%disp(returnStk1')
%------------------------------------------------------

% Calculation of basic return parameters
%---------------------------------------------------------------------
------------------------------------
% (The Matlab inbuilt function "mean" is being used here)
av_rets = [mean(returnStk1),mean(returnStk2), mean(returnStk3),
mean(returnStk4), mean(returnStk5),...
    mean(returnStk6), mean(returnStk7), mean(returnStk8)];

% (The function file calVar.m is being called here)
var_Rtn =
[calVar(returnStk1),calVar(returnStk2),calVar(returnStk3),calVar(retu
rnStk4), calVar(returnStk5),...
    calVar(returnStk6), calVar(returnStk7), calVar(returnStk8)];

% (The Matlab inbuilt function "sqrt" is being used here)
stdDev_Rtn = sqrt(var_Rtn);

% Calculation of Beta
% beta = Cov(stk,mkt_rtn)/Var(mkt_rtn)
% (The function files findCov.m and calVar.m are being called here)
betaStk1 = findCov(returnStk1,mkt_ret )/calVar(mkt_ret);
```

```
betaStk2 = findCov(returnStk2,mkt_ret )/calVar(mkt_ret);
betaStk3 = findCov(returnStk3,mkt_ret )/calVar(mkt_ret);
betaStk4 = findCov(returnStk4,mkt_ret )/calVar(mkt_ret);
betaStk5 = findCov(returnStk5,mkt_ret )/calVar(mkt_ret);
betaStk6 = findCov(returnStk6,mkt_ret )/calVar(mkt_ret);
betaStk7 = findCov(returnStk7,mkt_ret )/calVar(mkt_ret);
betaStk8 = findCov(returnStk8,mkt_ret )/calVar(mkt_ret);

beta = [betaStk1 betaStk2 betaStk3 betaStk4 betaStk5 betaStk6
betaStk7 ...
    betaStk8];
%xlswrite('Beta.xlsx',beta);
%---------------------------------------------------------------------
------

% Calculation of Excess returns of each stock
% (The function file findDmeanStk.m is being called here)
%---------------------------------------------------------------------
------
dmeanStk1 = findDmeanStk(returnStk1,mean(returnStk1));
dmeanStk2 = findDmeanStk(returnStk2,mean(returnStk2));
dmeanStk3 = findDmeanStk(returnStk3,mean(returnStk3));
dmeanStk4 = findDmeanStk(returnStk4,mean(returnStk4));
dmeanStk5 = findDmeanStk(returnStk5,mean(returnStk5));
dmeanStk6 = findDmeanStk(returnStk6,mean(returnStk6));
dmeanStk7 = findDmeanStk(returnStk7,mean(returnStk7));
dmeanStk8 = findDmeanStk(returnStk8,mean(returnStk8));

A = [dmeanStk1' dmeanStk2' dmeanStk3' dmeanStk4' dmeanStk5'
dmeanStk6'...
    dmeanStk7' dmeanStk8'];
%---------------------------------------------------------------------
------

% Variable definition for Portfolio parameters
miu = av_rets';
obs = nrw-1;
% when portfolio is equally weighted
%-------------------------------------
weight = 1/8;
w = weight*ones(1,length(miu));

% S the Variance Covariance matrix
%-------------------------------------
S = (1/obs)*(A'*A);

% Calculation of portfolio parameters
Expected_Return = w*miu;
Portfolio_variance = (w*S)*w';
Portfolio_Stdev = sqrt(Portfolio_variance);

% Conventional Portfolio allocation methods
%(Accepting the inputs by command window)

rate = input('Enter the Risk free rate(%) : ');
prm = input('Enter Market risk premium(%) : ');

% Calculation of monthly rates
rf = rate/(12*100);% rf is monthly risk free rate
rp = prm/(12*100);% rp is monthly market risk premium rate
```

```
% Historical and CAPM returns
miu_hist = miu; % miu_hist is the expected returns when using
historical method
% miu_capm is the expected returns when using CAPM method
miu_capm = rf + beta'*rp;
% Calculation of Zs during Portfolio allocation calculations

% ex_miu_hist is the excess expected returns when using historical
method
ex_miu_hist = miu_hist - rf;
% ex_miu_capm is the excess expected returns when using CAPM method
ex_miu_capm = miu_capm - rf;
% Z_hist is the by result Z, during weightage calculation when using
historical method
Z_hist = inv(S)*ex_miu_hist;
% sumZ_hist is the sum of all Zs, during weightage calculation when
using
% historical method
sumZ_hist = sum(Z_hist);
% Z_capm is the by result Z, during weightage calculation when using
% CAPM method
Z_capm = inv(S)*ex_miu_capm;
% sumZ_capm is the sum of all Zs, during weightage calculation when
using
% CAPM method
sumZ_capm = sum(Z_capm);

% Portfolio allocations
% Portfolio allocation while using historical method
w_hist = (1/sumZ_hist)*Z_hist;
% Portfolio allocation while using CAPM method
w_capm = (1/sumZ_capm)*Z_capm;

% Portfolio expected return while using historical method
PER_Hist = w_hist'*ex_miu_hist;
% Portfolio expected return while using CAPM method
PER_Capm = w_capm'*ex_miu_capm;



% Portfolio variance while using historical method
PV_Hist = w_hist'*S*w_hist;

% Portfolio variance while using CAPM method
PV_Capm = w_capm'*S*w_capm;

% Sharp ratio while using historical method
SR_Hist = PER_Hist/sqrt(PV_Hist);
% Sharp ratio while using CAPM method
SR_Capm = PER_Capm/sqrt(PV_Capm);

% Calculation of lambda
ch = input('Do you want to calculate lambda(Y/N) : ');
if ch == 'Y'
    file = 'UT_EMRU.xlsx';
    lambda = calLambda(file);% (The function file calLambda.m is
being called here)
else
    lambda = input('Input your preference : ');
end
```

```
% Inputs for the calculation of implied equilibrium excess returns
% mkt_cap is Market capitalization of stocks
mkt_cap = input('Enter market cap for each stock as a vector : ');
% tot_mkt_cap is the Total market capitalization of stocks
tot_mkt_cap = sum(mkt_cap);

% w_mkt is stock weightages in the portfolio with respect to Total
market capitalization
w_mkt = (1/tot_mkt_cap)*mkt_cap;

% The Implied Equilibrium Excess Returns
Pi = 2*lambda*S*(w_mkt');

% View matrix Q and Link matrix P
Q = input('Enter the view matrix as a column vector : ');
P = input('Enter the link matrix : ');

% Calculation of omega
PO = (P*S)*P'
% The omega is the diagonal omega matrix
omega = diag(diag(PO))

% Calculation of BL Expected Excess Returns

% exp_ret_1 is the results of 1st part of Black Litterman
exp_ret_1 = inv(inv(S)+P'*inv(omega)*P)
% exp_ret_2 is the results of 2nd part of Black Litterman
exp_ret_2 = inv(S)*Pi+P'*inv(omega)*Q

% Derivation of portfolio weightages using BL returns
% ex_miu_bl is the excess expected returns when using BL method
ex_miu_bl = exp_ret_1*exp_ret_2
% Z_bl is the by result Z, during weightage calculation using BL
method
Z_bl = inv(S)*ex_miu_bl
% sumZ_bl is the sum of all Zs, during weightage calculation using BL
% method
sumZ_bl = sum(Z_bl)
% Portfolio allocation while using BL method
w_bl = (1/sumZ_bl)*Z_bl

% Portfolio expected return while using BL method
PER_BL = w_bl'*ex_miu_bl
% Portfolio variance while using BL method
PV_BL = (w_bl'*S)*w_bl
% Sharp ratio while using BL method
SR_BL = PER_BL/sqrt(PV_BL )
```

### getcolumns.m

```
function v = getcolumns( data, hdr )
%

for i=1:ncol
     v = genvarname(['' hdr{1,i}]);
     eval([v '= data(1:nrw, i);']);
end

end
```

### findStkReturn.m

```
function stk_return = findStkReturn( prices )
% Finding returns using monthly closing prices

N = length(prices);

for i=1:N-1
    stk_return(i) = ( prices(i)- prices(i+1) )/prices(i+1);
end

end
```

### findDmeanStk.m

```
function dRtnStk = findDmeanStk(stk,stk_mean)
% Finding the excess returns of return vector

for i=1:length(stk)
    dRtnStk(i) = stk(i)-stk_mean;
end
```

### calVar.m

```
function cv = calVar(data)
% Function to calculate population variance
N = length(data);

m = mean(data);
dev = data - m;
cv = sum(dev.*dev)/N;
```

**findCov.m**

```
function c = findCov(v1,v2)
% Finding the covariance of two given vectors

v1_bar = mean(v1); v2_bar = mean(v2);

sum = 0;
for i=1:length(v1)
    sum = sum + ( v1(i)-v1_bar )*( v2(i)-v2_bar );
end

c = sum/length(v1);
```

**calLambda.m**

```
function lambda = calLambda(file)
% Calculating the value of lambda using the given inputs

[data, hdr] = xlsread(file);
avg = mean(data);
variance = var(data);

lambda = avg/(2*variance);
```

```
% Calculation of basic return parameters
```

av_rets =

Columns 1 through 3

0.0100344277991056 0.0131344494113276 0.0306219269571734

Columns 4 through 6

0.0207401884631998 0.0177304540616442 0.0138071275335659

Columns 7 through 8

0.0074884030718853 0.0120659150986573

var_Rtn =

Columns 1 through 3

0.00515151092254157 0.00522887437331421 0.00612765016469282

Columns 4 through 6

0.0293993467934386 0.0126953226034172 0.0131276411075319

Columns 7 through 8

0.0137698590590917 0.0101423215123591

stdDev_Rtn =

Columns 1 through 3

0.0717740267961995 0.0723109561084226 0.0782793086625886

Columns 4 through 6

0.171462377195228 0.112673522193181 0.114575918532351

Columns 7 through 8

0.117345042754655 0.10070909349388

beta =

Columns 1 through 3

```
    0.383129361223695     0.56927156530342     0.599198238748652

  Columns 4 through 6

    0.914274974681077     0.891850820108116     0.840813883036107

  Columns 7 through 8

    0.680232269292539     0.755549387492689
```

**% Calculation of portfolio parameters**

```
Expected_Return =

    0.0157028615495699


Portfolio_variance =

    0.0036585346300807


Portfolio_Stdev =

    0.0604858217277462
```

**% Conventional Portfolio allocation methods**
**% accepting the inputs by command window)**

```
Enter the Risk free rate(%) : 8

rate =

   8

Enter Market risk premium(%) : 9

prm =

   9
```

**% Historical and CAPM returns**

miu_hist =

```
   0.0100344277991056
   0.0131344494113276
   0.0306219269571734
   0.0207401884631998
   0.0177304540616442
   0.0138071275335659
   0.0074884030718853
   0.0120659150986573
```

miu_capm =

```
  0.00951869306831321
   0.0109043411919664
   0.0111271162424262
   0.0134725568699829
    0.013305630793964
   0.0129257103109093
   0.0117303359847212
   0.0122909988608753
```

**% Historical and CAPM excess returns**

ex_miu_hist =

```
  0.00336776113243896
  0.00646778274466091
   0.0239552602905067
   0.0140735217965332
   0.0110637873949775
  0.00714046086689923
 0.000821736405218633
   0.0053992484319906
```

ex_miu_capm =

```
  0.00285202640164654
  0.00423767452529971
  0.00446044957575956
  0.00680589020331623
  0.00663896412729735
  0.00625904364424266
  0.00506366931805453
  0.00562433219420864
```

**% Zs during Portfolio allocation calculations using historical and CAPM methods**

```
Z_hist =

    0.184709165215852
    0.316624889862066
     4.03112493197849
    0.231135626498563
    0.647434854617294
   -0.778984375244907
   -0.644318746587318
   -0.204699906203325


sumZ_hist =

     3.78302644013672


Z_capm =

    0.336711193131969
    0.392323552701203
    0.303446399029215
   0.0951735777500381
    0.183282537972123
   0.0463739954765872
    0.121597922544881
    0.313395585731801


sumZ_capm =

     1.79230476433782
```

**% Portfolio allocation while using historical and CAPM method**

```
w_hist =

   0.0488257663906722
   0.0836961873971527
     1.06558201370456
   0.0610980732374182
    0.171142037958872
   -0.205915657099334
   -0.170318330253074
```

-0.0541100913362708

w_capm =

0.18786492109581
0.218893326909249
0.169305134409619
0.0531012245482708
0.102260810560217
0.0258739453240925
0.0678444452999074
0.174856191852834

**% Portfolio measures while using historical and CAPM method**

PER_Hist =

0.0270829576722346

PER_Capm =

0.00474781566022845

PV_Hist =

0.00715907173814409

PV_Capm =

0.00264900018941956

SR_Hist =

0.320086777220127

SR_Capm =

0.0922471280204709

Do you want to calculate lambda(Y/N) : 'N'

ch =

N

Input your preference : 2

lambda =

2

Enter market cap for each stock as a vector : [118.71 69.35 54.52 13.19 10.46 9.63 3.11 2.9614]

mkt_cap =

Columns 1 through 3

118.71 69.35 54.52

Columns 4 through 6

13.19 10.46 9.63

Columns 7 through 8

3.11 2.9614

tot_mkt_cap =

281.9314

w_mkt =

Columns 1 through 3

0.421059874848988 0.245981823947244 0.193380375509787

Columns 4 through 6

0.0467844305387764 0.0371012239147537 0.0341572453440801

Columns 7 through 8

0.0110310522346926 0.010503973661678

## % The Implied Equilibrium Excess Returns

Pi =

0.0102799802077666
0.00797050760314531

0.00917968287423137
0.0114849301564305
0.0121304593447166
0.0130006273990317
0.00701300563568469
0.00590675101589974

Enter the view matrix as a column vector: [0.025; 0.0025; 0.015; 0.005]

```
% View matrix Q and Link matrix P
```

Q =

0.025
0.0025
0.015
0.005

Enter the link matrix : [1 0 0 0 0 0 0 0; 0 -1 1 0 0 0 0 0; 0 0 0 -1 0 0 0 1; 0 0 0 -0.81 0.52 0.48 -0.19 0]

P =

Columns 1 through 3

| | | |
|---|---|---|
| 1 | 0 | 0 |
| 0 | -1 | 1 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Columns 4 through 6

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| -1 | 0 | 0 |
| -0.81 | 0.52 | 0.48 |

Columns 7 through 8

| | |
|---|---|
| 0 | 0 |
| 0 | 0 |
| 0 | 1 |
| -0.19 | 0 |

PO =

Columns 1 through 3

| | | |
|---|---|---|
| 0.00515151092254157 | 0.000721205948191193 | 0.000649077258394267 |
| 0.000721205948191193 | 0.00857800834652314 | -0.000568179390521442 |
| 0.000649077258394267 | -0.000568179390521442 | 0.0349678306517621 |
| 0.00203128776146087 | 0.000196855364190916 | 0.019477062889528 |

Column 4

| |
|---|
| 0.00203128776146087 |
| 0.000196855364190916 |
| 0.019477062889528 |
| 0.0245823983329143 |

**% The diagonal omega matrix**

omega =

Columns 1 through 3

| | | |
|---|---|---|
| 0.00515151092254157 | 0 | 0 |
| 0 | 0.00857800834652314 | 0 |
| 0 | 0 | 0.0349678306517621 |
| 0 | 0 | 0 |

Column 4

| |
|---|
| 0 |
| 0 |
| 0 |
| 0.0245823983329143 |

**% exp_ret_1 is the results of 1st part of Black Litterman**

exp_ret_1 =

Columns 1 through 3

| | | |
|---|---|---|
| 0.00254697988328331 | 0.000191699895535983 | 0.000371629683156413 |
| 0.000191699895535983 | 0.00435090583212981 | 0.00241493960575733 |
| 0.000371629683156413 | 0.00241493960575733 | 0.00475367188077008 |
| 0.000336050182385518 | 0.00264188933354992 | 0.00248884326052975 |
| 0.000793423679288044 | 0.00239435950230698 | 0.00258408701816257 |
| 0.00102834804800179 | 0.00250583071869122 | 0.00244259166840177 |
| 0.000591376827598647 | 0.000995528845737764 | 0.00167174248464609 |
| 0.000299444959849505 | 0.00236327074358634 | 0.00203109961446339 |

Columns 4 through 6

| | | |
|---|---|---|
| 0.000336050182385518 | 0.000793423679288044 | 0.00102834804800179 |
| 0.00264188933354992 | 0.00239435950230698 | 0.00250583071869122 |
| 0.00248884326052975 | 0.00258408701816257 | 0.00244259166840177 |
| 0.0149496279410606 | 0.00527005637424208 | 0.00531759688230397 |
| 0.00527005637424208 | 0.010808221004221 | 0.00821816921554128 |
| 0.00531759688230397 | 0.00821816921554128 | 0.010932790404184 |
| 0.00305445236985377 | 0.00439328142098321 | 0.00369003668779809 |
| 0.00429742941298034 | 0.00225858518630112 | 0.00170399598862285 |

Columns 7 through 8

| | |
|---|---|
| 0.000591376827598648 | 0.000299444959849505 |
| 0.000995528845737764 | 0.00236327074358634 |
| 0.00167174248464609 | 0.00203109961446339 |
| 0.00305445236985377 | 0.00429742941298034 |
| 0.00439328142098321 | 0.00225858518630112 |
| 0.00369003668779809 | 0.00170399598862285 |
| 0.0133850782785936 | 0.0022287421574331 |
| 0.0022287421574331 | 0.00895271920688432 |

**% exp_ret_2 is the results of 2nd part of Black Litterman**

exp_ret_2 =

6.53718465973852
0.69248435250794
1.06496444532018
-0.406580008065154
0.254171630246452
0.234259813303185
0.00547867130105299
0.470981595990388

**% excess expected returns when using BL method**

ex_miu_bl =

0.017628807549352
0.0080779021518575
0.0103470695218012
0.00522454143244795
0.0132142261687512
0.014369700499492
0.00819789850141035
0.00921187758477997

**% Zs during Portfolio allocation calculations using BL method**

```
Z_bl =

     3.115119208569
   0.957017479345412
   0.800431318482712
 -0.0296276814807528
  0.0853811517412901
  0.0784532177599586
  0.0671521153702468
   0.356952899385181

sumZ_bl =

    5.43087970917304
```

**% Portfolio allocation while using BL method**

```
w_bl =

   0.573593851343714
   0.176217764081381
   0.147385204855622
-0.00545541110599634
  0.0157214220003947
  0.0144457660565464
  0.0123648688548235
  0.0657265339135147
```

**% Portfolio measures while using BL method**

```
PER_BL =

  0.0141539068943343


PV_BL =

 0.00260619046126683


SR_BL =

   0.277251087929272

>>
```